Quantitative Finance

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Module - 08

Lecture – 47

So, welcome to this continuation on the var lecture. So, as I mentioned there were 4 properties, and 2 properties about combination being not equivalent to the concept of diversification was very important. And there is that if you take the linear combinations; the linear combinations of the overall risk may not hold true for those level of risk measures which do not follow these 4 properties under non normal distributions. So, var has a problem while cvar does not have the problem.

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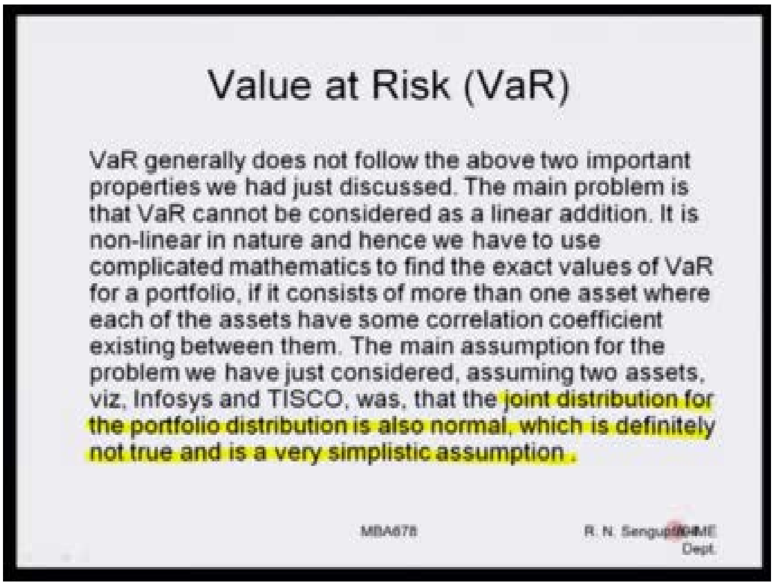

So, var generally does not follow the above 2 important properties, we have just discussed. The main problem is that var cannot be considered as a linear combination in order to find out we have to do some simple mathematics. If it consists of more than assets where each of the assets have these some sort of correlation coefficient existing between them. The main assumption for the problem we have just considered assuming 2 assets, say for example, for in one of the slides we discussed in the last class or last to last class, it was basically Infosys and TISCO was, that the joint distribution is a very important point which I will stress, and then again basically highlight accordingly. The joint distributions for the portfolio distribution is also normal which is definitely not true and is a very simplistic assumption.

If you remember as I mentioned time and again any utility being quadratic let to the fact for the distributions for normal, if they are normal then any convex combination of normal distribution would lead to a normal distribution, if we combine 2 normal distributions - it is a bi variant normal distribution, and so on and so forth. So, if these properties of normality holds then all the assumptions which you consider would hold that 4 assumption properties would hold for the var also for the standard deviation also. But they do not hold true, because the actual distribution of the return are not normal or class of multi variant normal distribution is not true.

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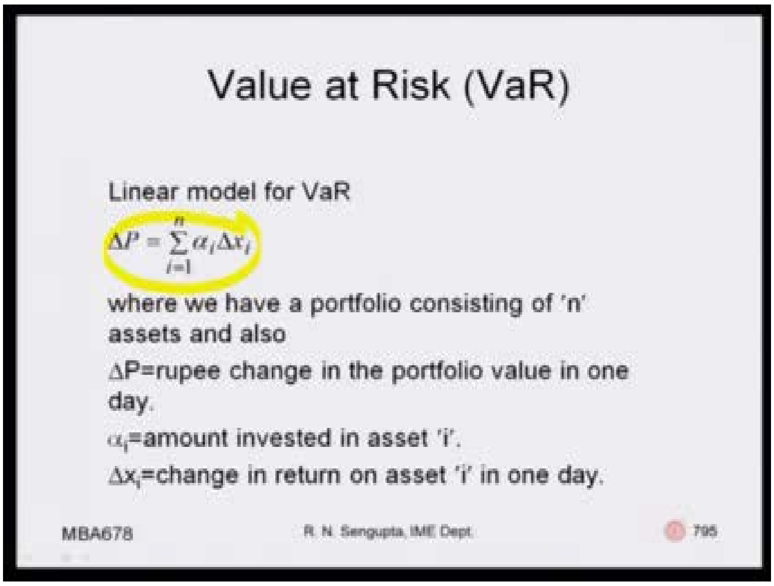

Value at risk basically linear model for the value at risk is basically very simply considered that the change of the portfolio which is on the left hand side is delta P, which is given by the change in the weights into the alpha value which is basically the amount invested in ith asset. So, where we have a portfolio consisting of n number of asset where delta P is the rupee change in the value of the portfolio del xi's are the amount invested in the ith asset, and delta xi is the change in the return in the asset i in one day. Now you may be asking that delta alpha i which I am taking is it fixed the answer is no, we are just simplistically checking that once you have fixed you portfolio, the value recontinues to be at the same level, which may not be true if you consider the dynamic programming, and basically it bring the concept of time into the picture.

So, the value of the portfolio would be given by this which is basically the multiplication of each and every unit which you have multiplying for, like say for example, a 3 reliance stocks, 4 tisco stocks, 5 telco stocks. So, alpha 1, alpha 2, alpha 3 would be accordingly 3, 4, 5 and whatever it is. And the delta xi's are the change of the values of those assets which is happening in the return form. That means, either small r or capital R.

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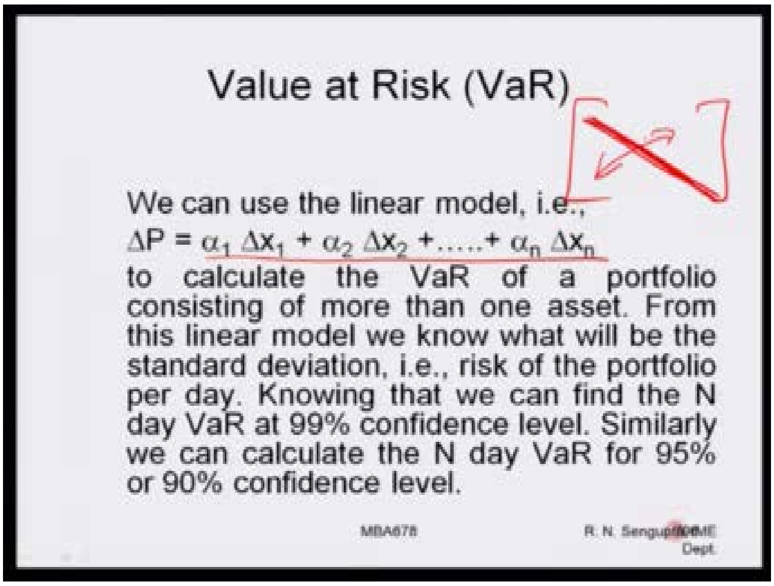

So, we can use the linear model, such that if you have say for example, n stocks it would be alpha 1 into del x 1 plus alpha 2 into del x 2 so on and so forth till alpha into del x n. To calculate the value for var of a portfolio consist of more than 1 asset from a linear model we know that they would be standard deviation, that is the risk of the portfolio knowing this we can easily find out the value of the var at 2.33 value which is 99 percent or 95 percent or at 90 percent. The main factor is that if you consider these equations what you have, you have basically n assets. So, when you multiply them what you need again going back to the simplistic rule, the variance covariance matrix where along the principle diagram, we have the variance of the diagonal element of the covariance's. So, they are a linear model you can solve it problem relatively simple, because you would be having some information about the linear relationship which is basically the correlation coefficient.

Now remember one thing, in statistics if you do some problem you will come across the fact that correlation coefficient covariance is a liner relationship. Hence the linear relationship if it holds for the case of the assets also considering the var model is true, there the covariance and or the correlation coefficient values can be used to find out the change of the portfolio, hence the var value quite accurately. Obviously, considering the fact that the normal or symmetric distribution holds true.

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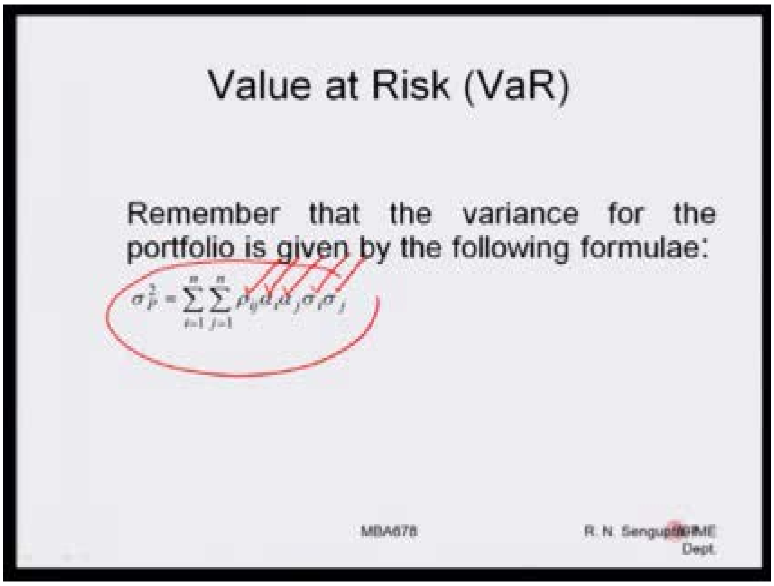

Remember that the variance for the formula is given for the portfolio will given by this formula which is right. So, what you have with the correlation coefficient, the investment in ith asset, investment in jth asset, the number standard deviation i or standard deviation of j.

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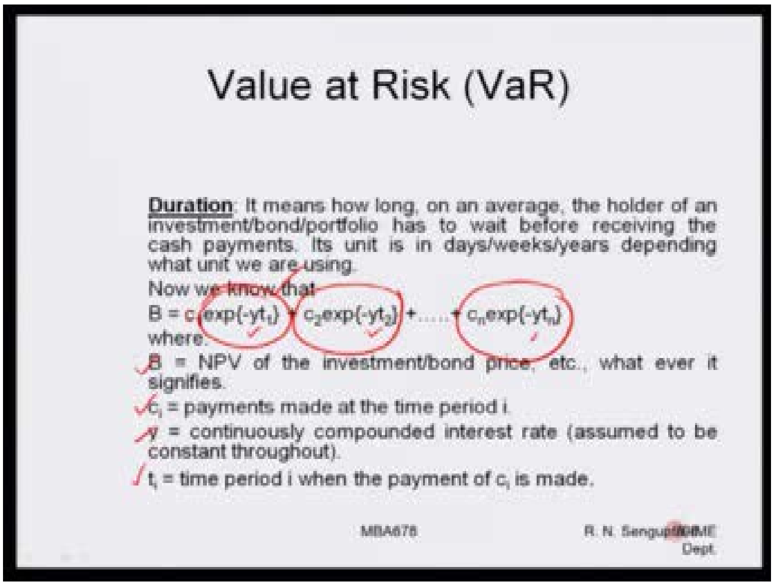

Now, you want to consider what is duration? So, duration it means how long on an average the holder of an investment bond portfolio has to wait before receiving the cash payment. So, if we have the duration concept, if you have done in the bonds what you want to do is that basically find out the net present value. So, c 1 is the value which is being paid after say for example t 1 time, and the net present is given by this where i y would be the rate of return continuous compounding rate of return. c 2 is the amount which is being paid at time t 2 then its present value is c 2 multiplied by that factor and so on and so forth, where B is the net present value as I mentioned ci's are the payments made at the time period i, y is the continuous compounding interested.

Now look very carefully, y's are same technically means they need not be same, but we are considering they be same, and t i is the time period at which the time payment for c i is made.

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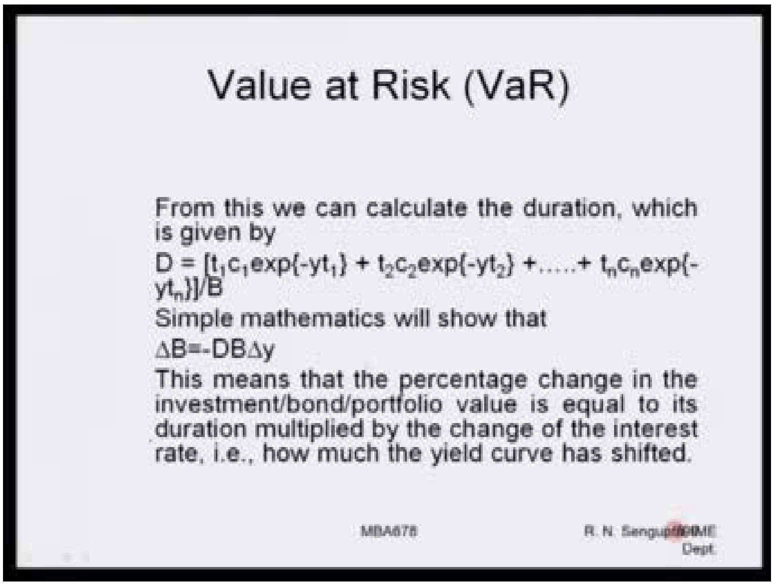

From this we can calculate the duration, which is given by the total value multiplied by each corresponding time period divided by the value of the bond. Simple mathematics will show that the delta B is the change would be given by minus D is the duration into B into delta y which y is basically the rate of change of the interest rate. This means that the percentage change in the investment or the bond or the portfolio is equal to the duration multiplied by the change of the interest rate that is how much the yield curve has shifted technically, we have already done that.

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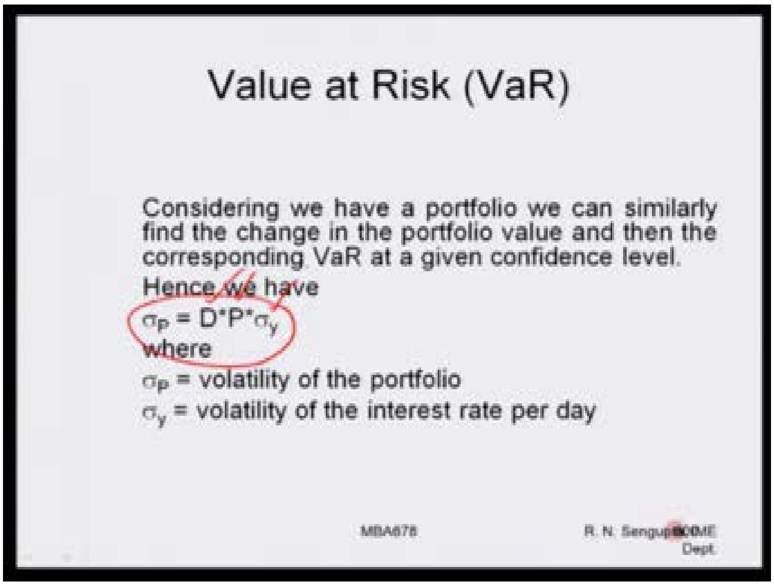

Now considering we have a portfolio, we can simply find the change in the portfolio and then the corresponding value in the var terms which would be this. So, change in the portfolio is the standard deviation would be given by the portfolio value into the duration into the value of the interest standard deviation of the interest rate. So, given this you can find out delta p, once the delta p is given which is the overall standard deviation you can again go back and find out the value of the var at 95 percent at 90 percent at 99 percent, and if it is given for one day, you will again go back to the same fundamental principle multiplied by the square root of t and find out the number of days. So, these 2 important factors are very important; one is for how many days the var is given, for how many days you are trying to calculate, and what is the overall the var value at what percentage, is it 99 percent, 95 percent or 90 percent. So, once you are aware of these 2 answers, then doing this calculation becomes simple.

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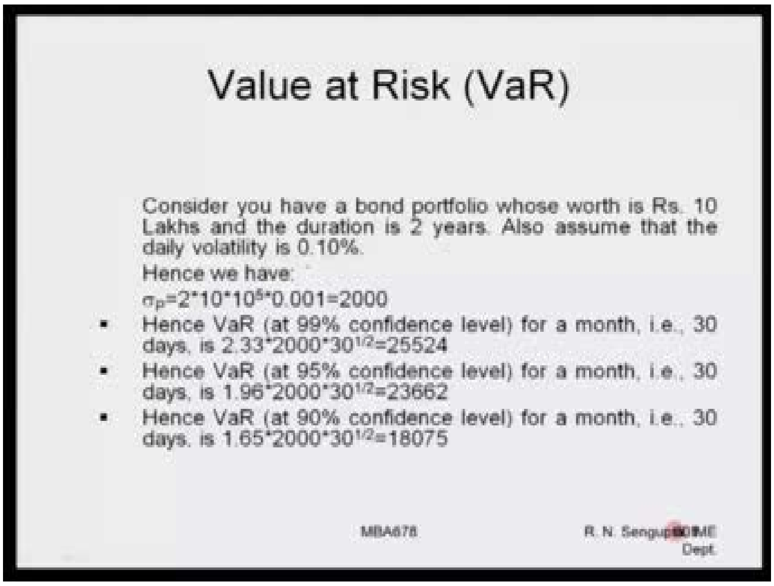

Consider that we have a bond whose worth is 10 lakhs and the duration is 2 years also assume that a daily volatility is given daily remember that per day. Hence you will basically have sigma p would be given by this, where this 0.1 comes out from this and this value of 10 into 10 to the power 5 is basically being given by the value of the bond. And this 2 is basically given by the number of years. So, this is ok.

So, you have basically the number of years considering this of a annual rate and based on that you find out the sigma P given by 2000. So, this is not var, this is basically the standard deviation of the overall portfolio. So, 2 is the number of years, this is the value of the bond which you have and this is the volatility which you have. Hence the var for a 1 month level would be what? At a 99 percent would be 2.33 that is the val that the standard deviation shift 2.33 plus or 2.33 minus, this is the value of the standard deviation which you have already found, and this square root of 30 is basically the number of days which you have, which is t to the power half considering this a the standard deviation.

Similarly for 95 percent we have the same thing 2000 into square root of 30 into 1.96, and similarly for the last example it is 1.65 into 2000 into 30 to the per square root of one half.

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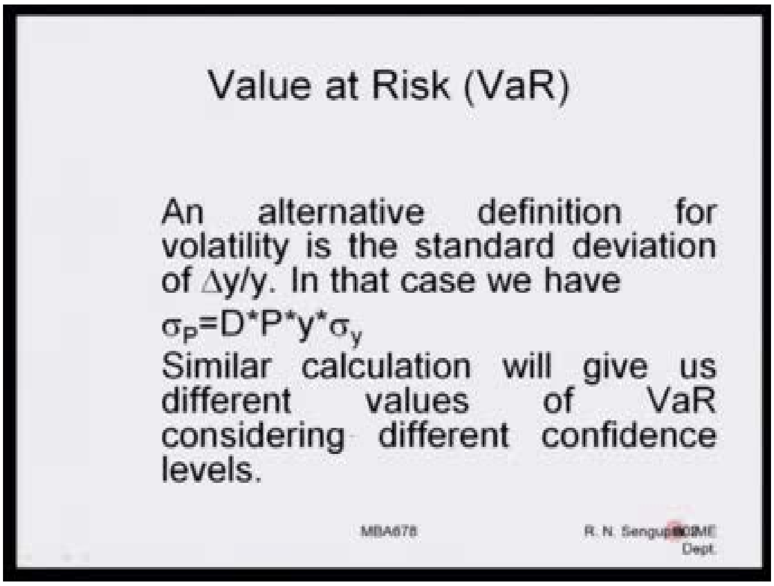

An alternative definition for the volatility is the standard deviation of between del y and y in that case we have already calculated, it would be given by D into P duration into portfolio value into y yield into the standard deviation of it.

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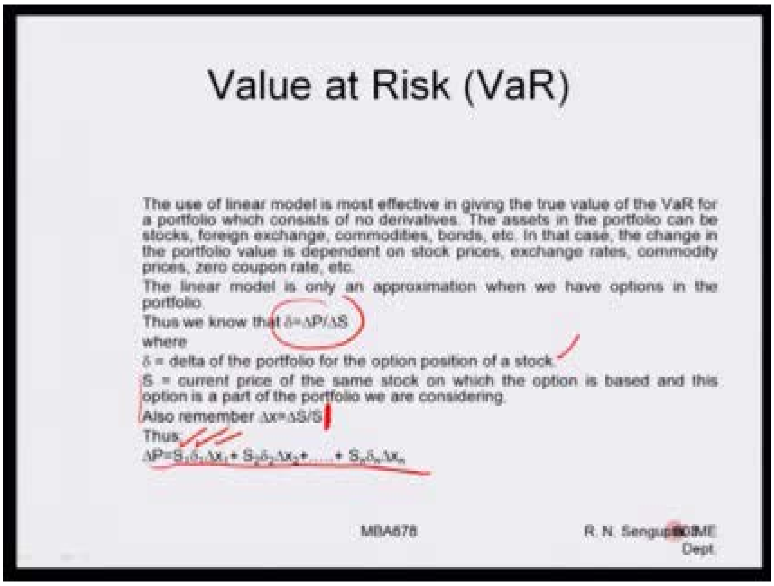

The use of the linear model is more effective in giving the true value of the var for the portfolio which consist of no derivatives, remember that because derivatives pricing on non-linear. Here we are still now considering linear functions. The assets in the portfolio can be stocks, can be foreign exchange, can be commodities, can be bonds, etcetera. In that case the change in the portfolio value is dependent on the stock price or the exchange rate or the commodity price whatever the underlying variables are. The linear model is only an approximation where we have the options and when we have the options then obviously it becomes a non-linear function.

Now if you remember that the concept of delta, wega, then all these omega, and all these things. So, why we are using the delta concept? Delta concept is the change of the portfolio with respect to the change of the stock price. So, here delta is the value of the portfolio for the options consisting of a stock. So, we will find out the delta value for each and every stock and how the portfolio changes with the rate of change of the stock price. S is the current price of the same stock on which the option is based and this option is a part of the portfolio we are trying to calculate. So, remember that x is basically changing the stock price, that is del S by s and given del P you can find out this.

So, what you have is stock price s 1 multiplied by delta for 1 multiplied by basically delta x which is basically same definition as we have done. So, now, it being more more of a sort of linear combinations considering that we have already have the options in the picture, but remember delta is the dy dx, which means that now you are trying to slowly being into the rate of change of that portfolio value with respect to the spot price considering that they are more than 1 option which is there in the picture.

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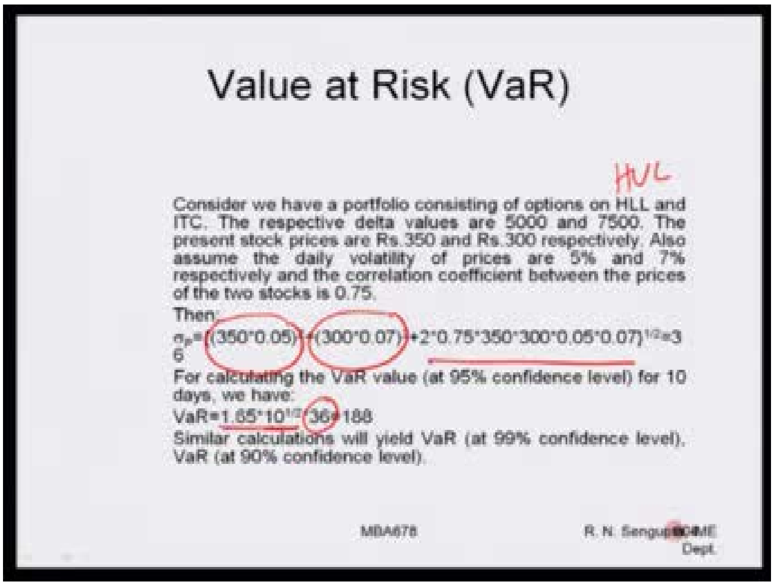

Consider we have a portfolio consisting of options on HLL or HUL whatever you say Hindustan Unilever limited, and ITC the respective delta values are 5000 and 7500 which means rate of change of the portfolio with respect to HUL is 5000 rate of change of that portfolio with respect to ITC is 7500. The present stock prices are given as 350 and 300; that means, 350 for HUL and 300 for ITC. Also assume the daily volatility prices are 5 percent and 7 percent and the correlation coefficient is given as 0.75, then again you do the same thing. You find out the variance here, the variance here, and the covariance here. So, when you find out the covariance variance here, you can find out the delta P. Once you find out the delta P, that is for the overall portfolio, you need to find out the var. And once you need to find out the var, again is the same thing, this value is 36 is given, you know it you have to find out for the 10 days which is square root of 10, and you know that you have to find for a level of confidence which is basically 95 percent and find out the corresponding k multiplied by square root of t multiplied by the sigma P, which is the change of the portfolio risk of the portfolio which you have considering that linear model holds considering the deltas are utilized in our calculation.

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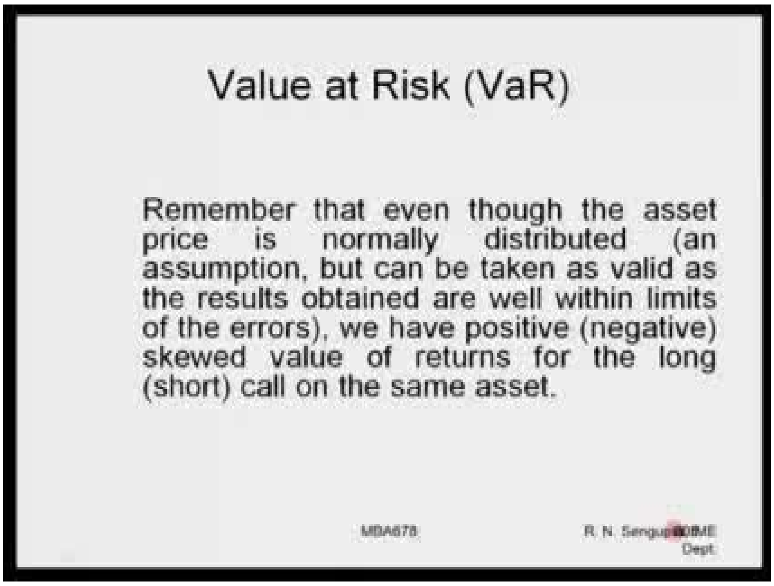

Remember, that even though the asset price is normally distributed and assumption, but can be taken as the valid as the results obtained or close to the actual reality, we have positive or negative skewed values of the return for the long and short call on the same asset. Now it is coming to the picture that as you have the long and short call, and as if the losses and the and the gains are considered in the picture in the realistic sense then it will have a skewed distribution and not a normal or a symmetric distribution as we have been considering till now.

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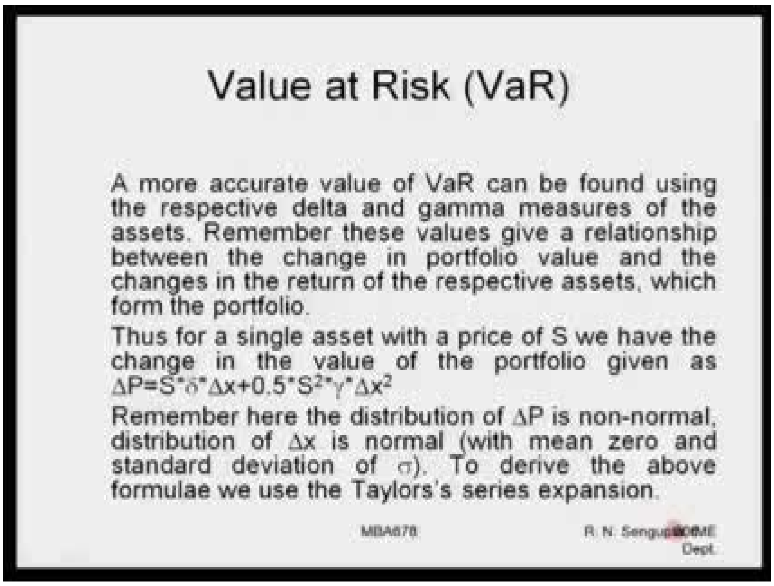

A more accurate value of var can be found out using the respective delta and gamma values. So, now you understand why we have been considering the delta, gamma, wega, and theta values. So, if we have the delta and the gamma measures of the assets remember these values give a relationship between the change in the portfolio values and the change in the return on the respective asset which forms the portfolio. So, whatever the characteristics of the portfolio are. Whether you think spot price important, whether you think rate of change is important, whether you think standard deviation is important, whether you think the risk free interest rate is important, whatever is important you have to find out the rate of change of that portfolio with respect to these variables and you utilize them accordingly. Thus for a single asset with a price of S, we are only considering 1 asset, it can be expanded accordingly. S we have the change in the value of the portfolio are given by this.

Now if you look at this formula this is exactly going to the Taylor series expansion we consider. Now if you have only 1 asset, if you expand the Taylor series - the first term is basically the s which is the spot price multiplied by delta multiplied by delta x; delta x has the same assumption. And if you go to the second derivative second term is basically d 2 y dx 2 which means the rate of change. So, the rate of change of what when you find out the rate of change of the portfolio there you see for the first time gamma is coming.

So, now you will see that concept. So, deltas, gammas, and so on and so forth can be brought into the picture, as you basically make your problem much more realistically considering the datas are available and you can do your calculations accordingly. And this half basically comes from that factorial 1 by 2. And this delta x is basically the h square expansion or h square term, which we have in the Taylor series expansion. Remember here the distribution of delta P is non normal distribution of delta x is technically normal with 0 mean as standard deviation to derive this above formula we use this simple Taylor series expansion, if there in 1 variable or multi variable this depending on how you are able to solve the problem. Say for example, you have gamma, delta, wega, omega. So obviously, we see the number of variables which are coming to the picture as spot price time standard deviation risk free interest rate, so obviously we will use the multivariate Taylor series expansion to solve your problems, but remembering the fact the higher terms have to be neglected depending on the how the problem has been formulated.

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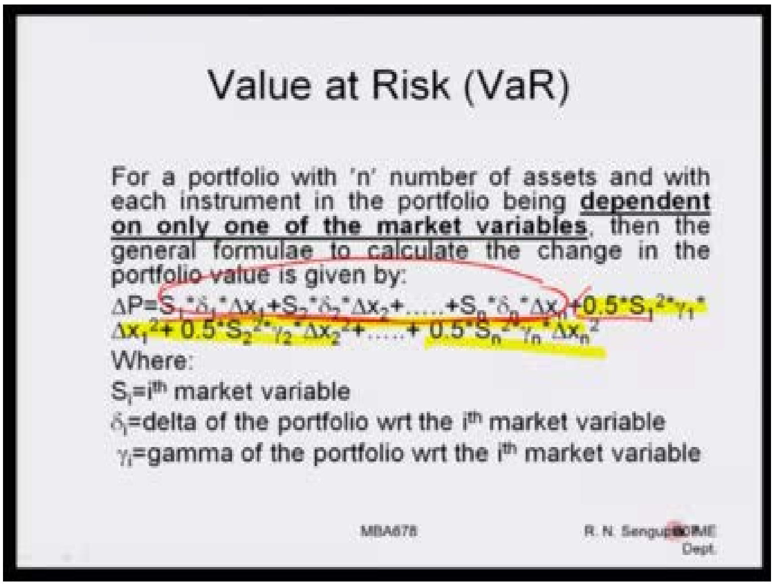

For a portfolio with n number of assets and with each instrument in the portfolio being depending on only one of the market variables, that is important. Then the general formula to calculate the change in the portfolio is given simply by this. So, what you have, the first term - this is delta 1, the second one which is delta 2 so on and so forth till the nth. So, these are the first term expansion for the n number of such assets which are there. And if you consider the next set which is here. So, I will use a different color here if possible and highlight it.

So, if you consider this set of terms, there the second state expansion of the Taylor series expansion, but they also consider the same number of assets which I have considered. So, this is for the first one, this is for the second one, and so on and so forth till the nth one. So, what you are doing, doing is that for the Taylor series expansion; the first set of terms are for the m corresponding to the dy dx only. Second set of terms are for again for m, n number of assets, but for d 2 y dx 2. So, what you are trying to do is that you are trying to basically club them according to the number of assets and according to which stage of expansion you are, what you will notice that if we go to the higher terms d 3 y dx 2 or d 4 dx 2 for the Taylor series expansion. They can be neglected, because the overall implication in our problems would be very negligible.

So, here Si is the ith market value, delta i is the delta the portfolio with respect to the ith market variable and gamma i is the gamma the portfolio with respect to the ith market variable.

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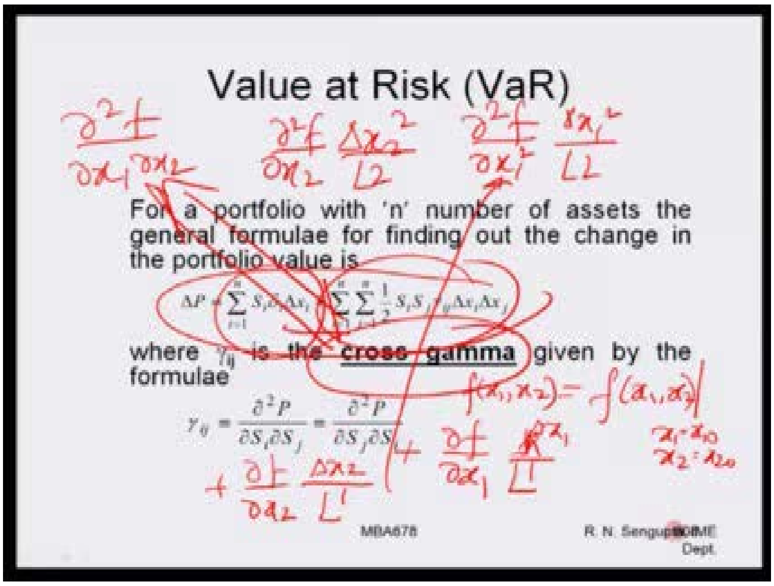

Now for an asset if we go to the next stage of sophistication, you will have to consider the cross gamma. So, now if you consider the Taylor series expansion, let me write the Taylor series expansion for 2 terms. So, what you have is f of x x 1, and x 2. So, these are the 2 variables. So, basically you will have x 1, x 2 at x 1 is equal to x 1 0, x 2 is equal to x 2 0; this is the first expansion. Then if you have basically it will be del f by del x 1 and this would be multiplied by h by factorial 1. Now in this case let be technically this h is del x 1 and the next term would be del f by del x 2 into del x 2 by factorial 1. So, now you see in the first explanation is coming in 2 terms, because there is x 1 and x 2.

And if you go to the higher 1, let me write it here it will be del del 2 y del x 1 square, so whatever the terms would be, this would be true, the second term I am writing del 2 f del x 2 delta x 2 square by factorial 2. Now the interesting part comes is there are also terms which are del 2 f del x 1 del x 2, now it can also be del 2 f del x 2 into del x 1.

So, these were you get are the cross gammas which are coming into the picture. So, the cross gammas are corresponding to the rate of change of the function second time with respect to the first and the second or a second to the first, which means they should be basically symmetric values. So, for a portfolio with n number of assets, the general formula for finding out the change in the portfolio is given by this, where the first set of terms are the linear expansion for all the n terms, and the second set of expansions are the rate of change of the second derivative with respect to x 1 or x 2 taken in different combinations. So, where as I mention gamma ij is the cross gamma given by the formula. So, it can basically have the data and solve your problems accordingly. So, with this I will close today's class and continue the discussions of var and such small concepts accordingly in the next class.

Thank you.