Quantitative Finance

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Module - 08

Lecture - 46

So, once again welcome back I am sure till the almost for the 40 total number of lectures which would be given for this course. We are almost on the penalty made stage where about 3 classes are left, and we would in the last class we have just started about the value at risk concept is basically measure or risk like as you know I did mention about conditional valued at risk, that is also measure of risk I did mention you that conditional is the center of gravity of the overall distribution considering there is a loss distribution on the right hand side of var, it means you find out the expected value of that. And then we also did discuss in a very simple way the concept of standard deviation, concept of beta, concept of other risk measures in the swaps and also we did discuss, and how the risk concept would be brought into the picture.

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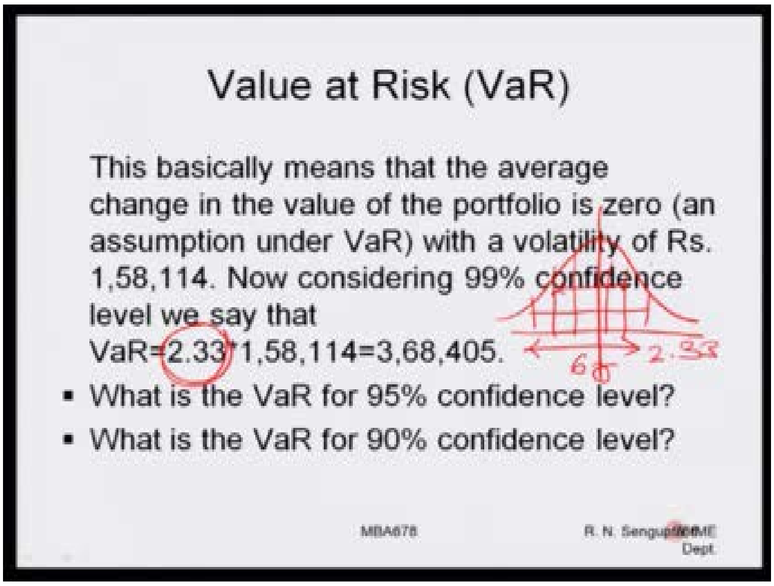

So, continue on discussion with value at risk, this basically means that if you consider the last line which we close in the last class, it means that the value of the portfolio would now loss would become more than 1,58,114 considering the level of confidence or number of days of var you want to have for your problem. Simply again with repetition I mention measuring that if you want to find out the var for more number of days. Obviously you have to multiply by the square root of t or multiply it by t depending on whether it is standard deviation, you are looking at or you are basically looking at the variance. So, that was basically the var which were calculated was 1, 58, 114, and if you consider 99 percent var it would basically give you the 6 sigma deviation 3 plus 3 minus over and above the mean value. So, the var value comes out to be 2.38 into that value which you calculated, it is 1,58, 114 that comes out to be about 368408. And if you want to find out the 95 percent confidence level var, then obviously what you will do it rather than you take 2.33, you multiply it by the corresponding value of the variable k. If you want to find out 99 percent again we will multiply by a corresponding value of k. So, what we mean by that k is this, again I am reiterating; this is the normal distribution. So, if you go 6 sigma, it is 3 sigma here, 3 sigma here and you have to basically multiply it by 2.33.

If you go 4 sigma that is 2 sigma on to the right 2 sigma on to the left or 2 sigma; that is sigma to the left and sigma to the right then the corresponding values can be found out from the standard normal deviation based on which you can find the overall area.

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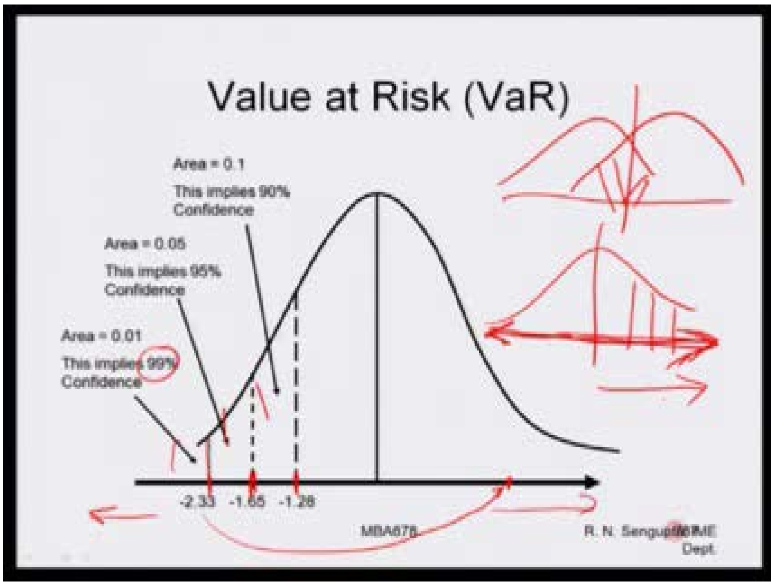

Now, this is the corresponding picture which I want to discuss. So, it is basically 2.33 or plus 2.33 whichever direction you are considering, so 2.33 would basically have a counterpart plus on the right hand side. Similarly 1.65 and 1.28 would be the corresponding values of 99, 95 percent, and 90 percent. So, if I am considering ninety 9 percent then the overall area left would be 0.01. So, 0.01 would be basically if it is a 2 side, if you consider the very simple example of hypothesis testing. And when if you remember we have done the hypothesis testing of these cases where alpha and betas were there. So, generally if it is 2 sided tail test, you w ill have basically equal proportions being divided of the total of 1 percentage; that means, 0.05 0.05 on to the right or the left. Similar to that 90 percent, so 10 percent would be divided by 5 percent and 5 percent on to the left right and to the left.

But now in the var case you are only interested to find out the overall risk or overall loss you are facing for either the left tail or the right tail depending on what you are trying to basically specify. Generally if you want to specify the losses then obviously the losses are the gains, it can be the other way around also the gains, so obviously the losses would increase on the right hand side, if you are doing that nomenclature one; that means, all losses are more on the right hand side and losses are less on the left hand side, which means profits are more on the left hand side, profits are less on the right hand side.

So obviously, you can do the nomenclature accordingly, but in this problem if you see the overall var values would be given here for 90 percent, for 95 percent, for 99 percent and the overall area which you are considering based on which you would predict the overall loss is going given on the left hand side of this diagram. That means, losses are increasing in the want reasons more on to the left, and if you go on to the right it would be gains.

So, similarly you can have diagrams like this, where this is 90 percent, 95 percent, 99 percent where the losses are increasing in the valued terms on to the right while on the left the losses are decreasing. So, whichever nomenclature you follow very careful that how you are trying to basically draw the losses, and how you are trying to basically draw the value of var based on which you are trying to find out the overall risk.

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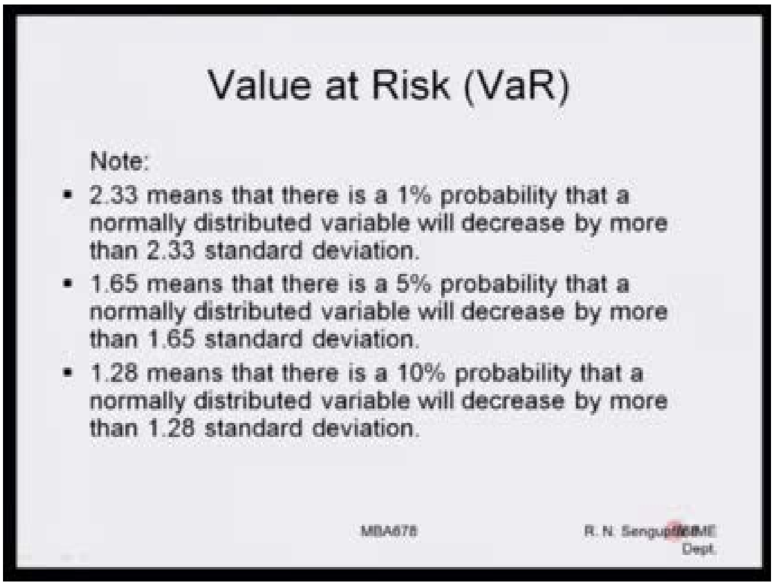

Note: 2.33 means that there is 1 percent probability that are normally distributed variable will decrease by more than 2.33 standard deviation on to the right or on to the left depending on how you are trying to draw the overall loss. 1.65 means that there is 5 percent probability that a normal distributed variable will decrease by more than 1.65 standard deviation on to the right or the left, similarly plus or minus would be see will come into the picture; that is minus 1.65 or plus 1.65 or it will be minus 2.33 or plus 2.33 whichever nomenclature you are using. While 1.28 means that there is 10 percent probability that a normal distributed variable will decrease by more than 1 . 28 standard deviation on to the left or the right as the case may be.

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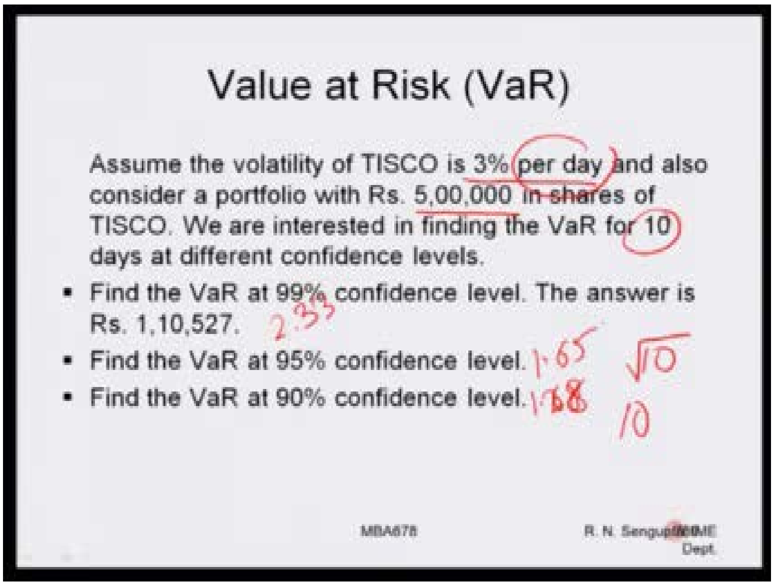

Assume the volatility of TISCO is 3 percent per day, and also consider portfolio consisting of 5 lakhs in shares of TISCO. So, this shares which I am considering does not mention whether they are options, whether they are stocks, whether they are bonds, whether they are swaps, whatever I am just collectively considering them as the the portfolio, very interested to find the var for 10 days at different confidence levels. So, this is 3 percent per day var. So, find the var at 99 percent confidence, the answer is 110527. So, what you have to do is that, this is 3 percent per day and the value of the portfolio is given.

So, what you will consider is that per day has to be converted if you are finding the var at 10 days. So, it would be multiplied by square root of 10, because you are considering only on the standard deviation - it was variance, so obviously, it would be multiplied by 10 only. And then considering the actual formula, you can just find out the var values as it is given. So, here it would be multiplied by 2.33, here it will be multiplied by 1 point, and 1.65, 1 1 point; let me check. 1.28, 11.65, 1. 28, and 1.65 as the case may be and you can find out the var accordingly.

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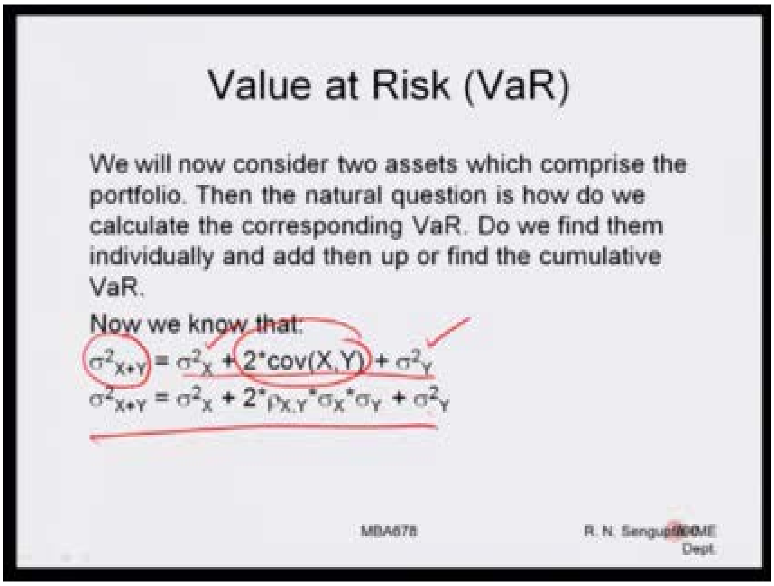

Now, we will now consider 2 assets. So, till now we were only considering a portfolio which was considered as a single asset. So, the word which I mentioned few minutes back that we would not consider, what is there in the portfolio whether there is an option or a swap or different types of stocks, we are not going to differentiate that, you only consider that as a block based on that we found out the var. Now what we will comes in to the mind is that what we have a portfolio, and we want to find out the var of the portfolio given the var for each and every individual financial asset which is in the in the portfolio. So, that is the main concern considering that we have understood the concept of var in a little bit more, more objective manner. So, now we consider 2 assets - only 2, it can be increased to n also. 2 assets which comprise the portfolio then the natural question is how do we calculate the overall var. The corresponding var has to be found out, but when you want to find out the var; obviously, the question would come that if there are 2 assets they are random, if they are 2 assets which are random; obviously, it would mean that there would be a covariance variance matrix also there between these 2 assets which will be of size 2 cross 2, answer is right.

We have to use that concept in order to find out the overall covariance of the portfolio. So, this is how you do and we have already covered that topic considering that you are doing the portfolio application problem in the first few lectures. So, now we consider the covariance variance being given for 2 assets is given by this formula, whether first is the covariance of the first asset with itself which is the variance, third term is the covariance of the second asset with itself which is the variance, and the middle term which is off the diagonal element multiplied by twice is basically the covariance existing between x and y. So, basically we have the overall covariance, given and once the overall covariance is given if you go back to the actual formula, this the var value which was given considering 2.33 standard deviation or 1.65 or 1.28 depending on the level of confidence you want to have you can definitely find out the var, but there are few pitfalls which are going to be mentioned very categorically to all of you.

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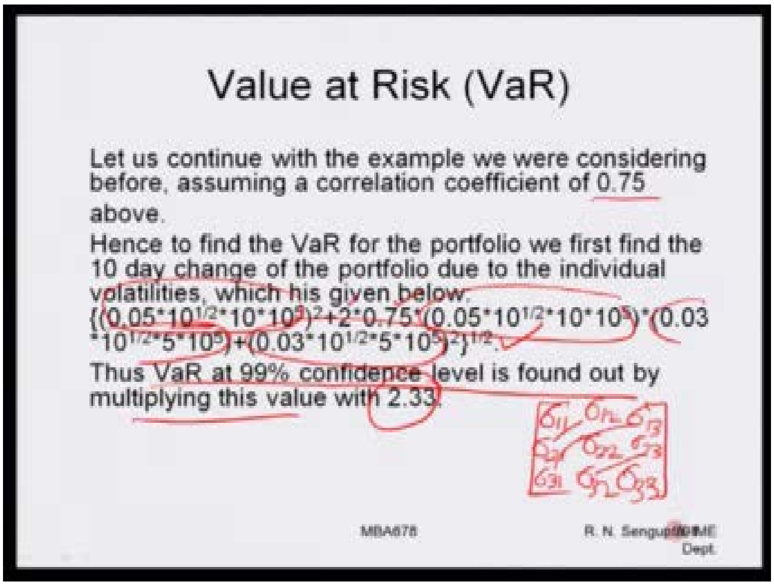

So, let us first complete the problem in a very simplistic sense, then I will come to the pitfalls. Let us continue the example we will consider that as before that the overall correlation existing between these 2 assets is 0.75. Hence, to find out the var for the portfolio, you first find out the 10 day change of the portfolio due to the individual volatilities which would be given by this. So, this is 10 day. So, basically this is being multiplied by square root of 10, and these values which you see this is sigma square. So obviously, sigma square would mean that the value of standard deviation which you have for the first asset is this, value of the standard deviation of the second asset is this, while this is 2 into variance this the correlation coefficient which is 0.75 into the values which you have is this. So, this is the standard deviation of the first multiplied by the standard deviation of the second, but only frank being very important to note is that you always multiply it by the square root of 10 considering that you want to find out the var given the var is given for 1 day. So, that thus the var at the 99 percent confidence level you found out by multiplying the value which you have found out here by 2.33 .

So, similarly if you want to find out the var at 99, 95 percent confidence level, 90 percent confidence level you can find it accordingly. Similarly just before I go to the next slide. You want to you have a question that you want to find out the var for more than 2 assets, what you will do? You will follow the same exactly the same principle have the variance covariance matrix. So, this is sigma 1 1 which is the variance of the first which is the covariance of the first with itself. Sigma 2 2 is the covariance of the second with itself, sigma 3 3 is the covariance of the third with itself and these are the covariance values between 1 and 2 or 2 to 1 this is between 1 and 3 and 3 to 1, and this 1 is basically 2 to 3 3 to 2. So, these are basically symmetric that is why you have been multiplying by2's. Again you follow the same fundamental principle, if you have the var for 1 day you want to find out the var for 10 days multiply them by the square root of 10, which is the important factor which I am trying to stress time and again.

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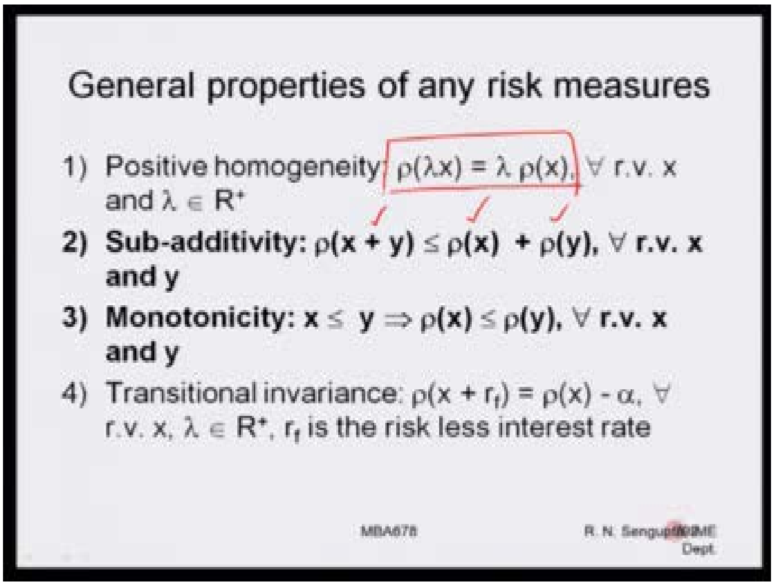

Now, the pitfalls which I mentioned, generally the properties of any risk measures. Now, I am taking making a very general statement and it is very important in finance, the properties of any risk measures be it beta, be it standard deviation, be it correlation coefficient, be it the value of var, be it the value of conditional risk whatever you have, there should basically of 4 properties. One is the positive homogeneity which means that if you multiply the overall. So, I am not going to the mathematical term, I am basically explain it from the qualitative point of view then coming to the quantitative point of view. See if you see the first formula, the positive homogeneity it means for any delta value; that means, any increase and decrease of the overall portfolio value or overall value of the asset which you have in your in your basket or you want the portfolio into formulate. If that can be formulated by a single asset that this means single smallest unit of the asset, and if you find out the risk of that and multiply by the number of times it will decrease or number of times it will increase that will basically give you the overall value of the of the risk measure, which if the property holds then the property of positive homogeneity is true which means I have 1 asset. And I have the standard deviation of that.

Now, in the second instant I say that I take such 1 asset 10 number of times. So, lambda would be 10. So, if I find out the variance covariance or say for example, whatever risk measure I have and rather than basically finding out all of them 10 collectively, what I do is that I take one of them find out the variance covariance of whatever risk measure is there and multiply it by 10. So, if there is 1 to 1 correspondence; that means, it can either be increase or decrease and the exact value of the risk measure can be found out then the positive homogeneity property holds to; that means, there is a linear relationship between the increase and decrease.

Second point is the sub relativity which is very important. Consider that you have 2 different financial assets, and you have some measure of risk. Now, rather than finding out the collective combination of these 2 assets, and then finding out the risk. Another nice thing which can be done is that, find out the individual level of risk which is this row x or row y, and if you basically add them, considering this additivity between them on the left hand side which means adding the risk values of 2 different assets when combined should definitely give you the collective risk of the portfolio whether portfolio now consist of those exact 2 assets in the same proportion.0 In case if I say for example, there are 3 different individual assets and I want to find out the risk of all of them 3 individually.

Then if I combine them; then the combined risk of all of them 3 combined together is the exactly the risk of the portfolio which would be there, if the portfolio consists of those 3 assets combined together as 1 unit. Now this property of dia and this sub adidtivity means that adding or subtracting of this financial risk of 2 or more different type of financial assets should hold to whatever the distribution is, but the caveat or the important point is that this only holds true for the case when you have normal distribution or a class of utility functions which are quadratic in nature. If you remember very clearly I had mentioned that in the in the beginning of the class and I did mention 1 or 2 points later on also that utility function being quadratic which would imply that the written being normal would have a huge amount of simplification, and overall understanding of the overall portfolio in a very simple sense.

Now standard deviation which you are using, variance which you are using, var you are using that sub additivity property for those risk measures would hold, if and only if they are symmetric distributions or normal distribution. In case you have non symmetric distribution or extreme value distribution, then these properties if you are considering these types sort of risk which is var which is standard deviation which is variance would not hold true, but if you ask that question will these properties of properties hold true, if we consider this extreme value distribution or asymmetric distribution for the conditional value at risk the answer is yes, because conditional value at risk basically follows all the 4 properties which we are going to discuss 2 of them which we have already discussed which is positive homogeneity and sub additivity.

The next one is monotonicity which means that if there are 2 different assets in different proportions then an or if the overall risk measure. So, called risk measure whatever it is, if you think the overall risk measure of the one asset is less than the other, then this property of monotonicity, monotonicity would also hold true for different combinations of the assets you are going to take. This property also does not hold true for the var standard deviation and variance, if you consider any distribution which is non normal, but it holds for the property of cvar, even if you consider non normal distribution.

So, that is why if you see the point 2 and 3 when you refer to the slides they are marked in bold, which means these two are a very important properties which would hold for any risk measure, but it does not hold for the case of var in case the distributions are non normal. The transitional invariance property basically means that whenever you add or subtract a certain proportions from your assets in terms of risk free interest rate is basically a simple addition and subtraction, which is going to take place for your overall risk measure. This property generally means that if you remember in one of the problems we considered the ratio of the excess return divided by beta or excess return divided by sigma which means that you are trying to basically analyze your problems from the excess amount you will get from any particular asset considering that you are doing a benchmarking with risk free interest rate. So, which means that any one of the risk measures whichever you are considering, whether is cvar, whether is standard deviation, whether is basically beta, whether is conditional value of this var value var which you are using should basically have that properties such that you will always peg your actual asset or a portfolio with respect to risk free interest rate.

So, amongst all these 4 which are the general properties of risk measures, the second and the third are very important, which should be noted that for var they do not hold to considering the fact, if the distribution is non normal

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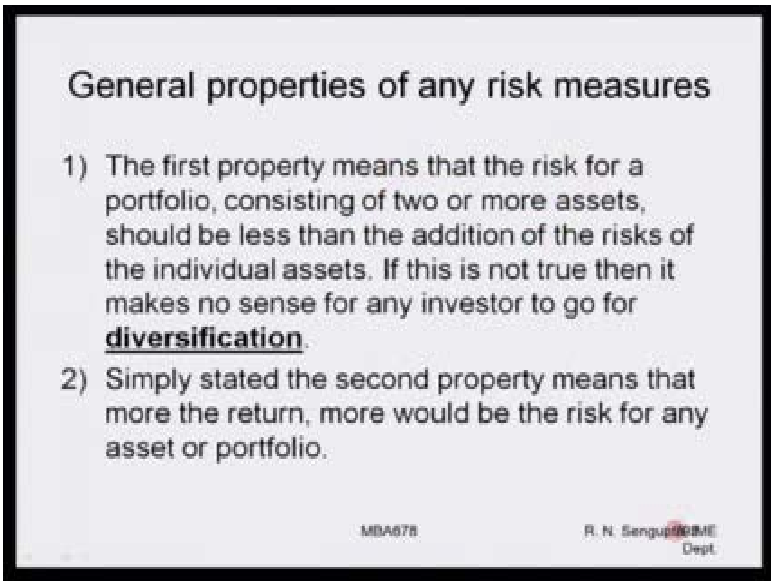

Now, coming to the first property; first property basically means what I actually mean technically is a combination of the first and the second in this slide. The first property means that the risk for a portfolio consists of 2 or more assets should be less than the additional value of the risk, which we have already done. How you have done the question is, you remember we have done diversification which means that more you basically put your risk in baskets the overall combined risk basically decreases for the overall collective portfolio. This property may not hold true if you have a non normal or asymmetric distribution remember that.

And the second simply states that second property means that more the return is more would be the risk; that means, there is a commensurate 1 to 1 correspondence, may not increases the same proportion, but this property also does not hold true for those class of risk measures. One of them being var in the case the distributions are normal normal or asymmetric distribution. So, with this I will close today's class and continue the discussions more in the next class which would be considering the credit matrix on the different concepts of var.

Thank you.