Quantitative Finance

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Module - 08

Lecture - 45

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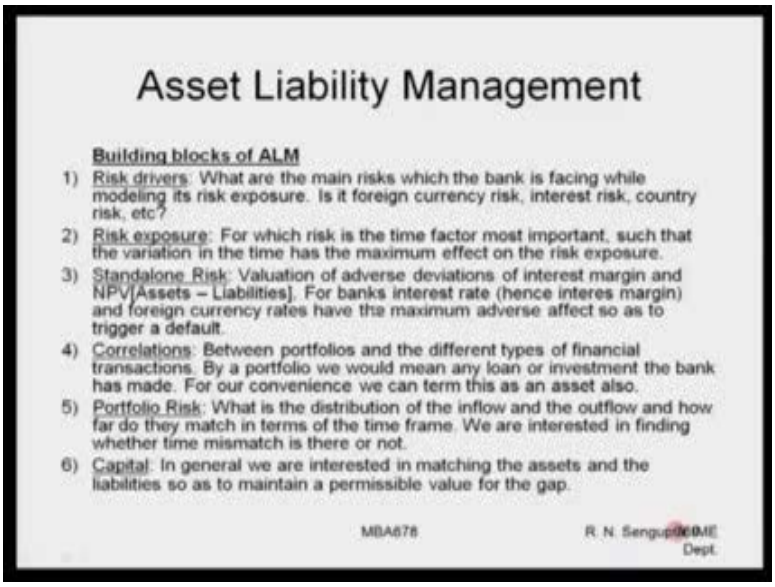

So, continue our discussion we were in the area of asset liability management. So, what are the building blocks? So, as you are discussing the risk drivers till the capital. So, risk drivers are the main risks variables parameters, which the bank is facing, which the financial institution facing thus that the overall risk exposure increases. So, you have to find out whether it is foreign currency, whether it is interest rate, whether it is oil price, whether it is gold price and all those things. Then you are also considered what is the risk exposure risk variables would be there, but you have been exposed to that. So obviously, that would be the overall risk exposure. Say for example, if I am a bank, I know the change of the dollar to yen value, really affects my overall risk exposure, but if my overall risk concept and I am not invested in any dollar denominated assets or on any dollar denominated liabilities or not into yen, so obviously those risk exposures would not come. So, any change in the Dollar and Yen would affect those dollars and yens asset

and liability not the other one. So, if I have only dollars, and only assets, and assets in yens. So obviously, risk exposure is the highest.

So, you try to understand what are the parameters which are effect, and what is the actual exposure you are trying to face. You will try to basically make a one to one corresponds and then try to analyze your overall risk. You also find out the stand alone risk valuation of adverse deviation of interest margins, if you are consider im and the concept of interest rates how they affect your stand alone on a one to one basis.

Then if standalone basis is found; obviously, there would be correlation between them, because there are random variables. If you consider then you have to find out the correlation which is the covariance variance metrics is the ultimate goal. Now if as I have mentioned finding out the correlation coefficient on the covariance metrics becomes difficult, because data is not available. So, there you have to be careful are you find out the correlation coefficient or whether you are able to find out proxy to find out the covariance variance metrics. Given this correlation, given this risk exposure, given the risk diversify, you find out the overall portfolio risk.

So, you want to basically find out the portfolio risk, balance it with the return and then basically make a decision, whether that investment is really worth well, considering your main aim is again I am mentioning is to minimize the risk. And you want to find out the what is the capital? So, in general we were interested in matching the assets and the liabilities. So, as to maintain a permissible level of gap, because gap is also dependent on time, if you remember marginal gaps and all these things, if you bring into the picture overall capital which is basically a concept of gap how it is being managed within a permissible region is also building block for the alm.

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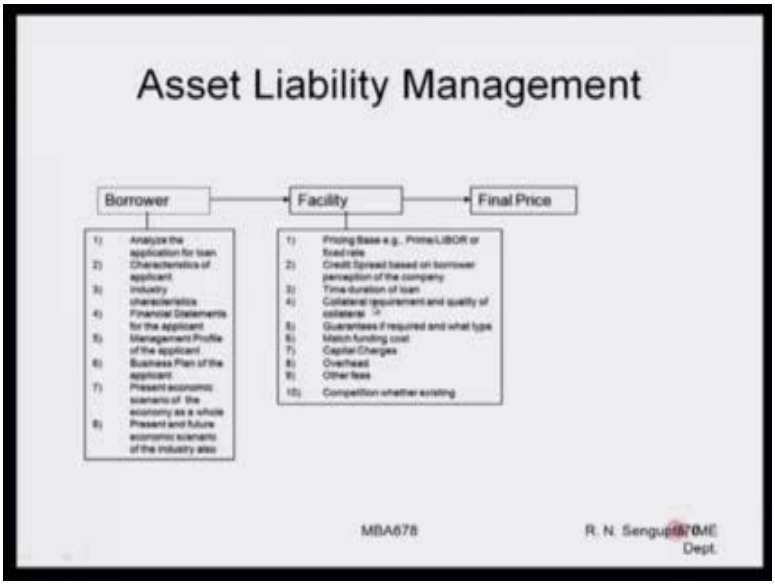

So far the asset liability management we analyze the borrower, the facility and the final price under the borrowers, you have to do two different types of qualitative studies, which analyze the application for the loan characteristic of the applicant, what are the interest industry characteristic, what are the economy characteristic, what is the financial statement, profit and loss, balance sheet of that company. What is their business plan, who are the main person, who is running the company, what is the present economic scenario which I have already discussed.

What is the present and the future economic scenario, how the country is doing, how the market is doing, what is the market share and all those things are concentrated. Facilities is that when you facilitate you find out the pricing base, where this is a lybor one, this is the mybor one, whether this is lybid one, mybid one, whether this is a fixed rates, you find out the time duration of the loan is that long term, short term, you find out what are the collaterals, you find out what are the guarantees, you find out the matching funding cost has been made. So, if they are taking a loan; obviously, collateral is important, but whether they are trying to match that overall loan with their own amount; that means, the company which is trying to borrow, such that in case of risk they would also face the risk and that is a much better position in order to tackle the risk.

What are the other overheads, what are the other fees? So, whether there are competition from other lenders. So, if a bank has another competitor's bank. So obviously, you have

to bit tackle it very, very, very delicately; because if the interest rate if the overall benefits being given by the other bank is on the positive side for the other company which is taking the loan; obviously, you will you as a banker face a loss of business, and then based on all these things you find out the final price.

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So, this is for the asset liability management I am just giving you a simple matrix - transition matrix, but note down the word transition; it means that in the first row you have basically A B C which are the different types of assets or portfolios and similarly in the first column. So, if you find out the matrix value, it will basically gives you that given one is in A, the probability of shifting from A to A is 90 percent. Then again this means that given C probability that will continue remaining in C is 0.93 percent, but if you see the off the diagonal elements, it gives you this that if you are in B then it is flowing down or going down to basically C is 0.06 percent, but remember one thing see this value, they are different say if I am considering that I am at B moving to C is 0.65 and in this case if I am at C then moving up to B is 0.06.

So, this diagonal which I draw are the probabilities of staying at the same level, and the off diagonal element are moving from i to j or j to i, and obviously it would very simply and very logically mean that such movements are not of same probability. So, if you see the matrix the off diagonal element, they are not symmetric, not symmetric, not symmetric, not symmetric; that means, moving from B to C or C to B, moving from A to

B B to A are not of the same probability, and this transition probability matrix given is for only one time. So, as you move from time one to time two, time two to time three, this transition probability matrix also changes.

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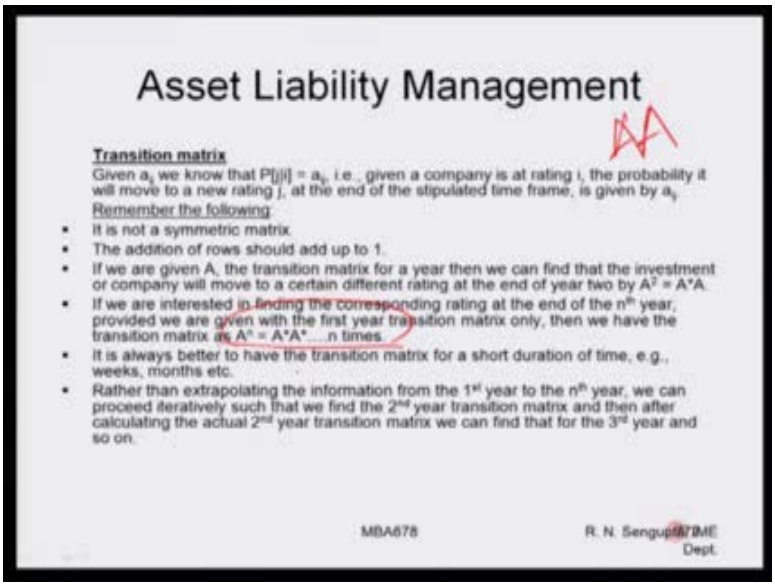

So, this is the just, given a ij we know that probability j, given it was an i was given by aij, that is given a companies you are rating I that probability that it will remain continue to remain go to a new rating j at the end of the stipulated time; the time factor is given by aij.

Point one - remember is not symmetric addition of the rows add up to one, because is very simple to understand. If you are A, a you can either move to A go to B or go to C those. So, there are no other positions. So, some of them should be 1. Similarly some of them should be 1, because out of that position you can move either stay at the same level or can move up and down, there are no other positions, so obviously, the total probability is one. If you are given A, the transition matrix for year then you can find out the investment or transition moved matrix for the second year considering the correlation coefficient remains to be needs to be the same.

What you do is that, multiply AA, AA is the transition probability matrix for one year, A square is the transition probability matrix for two years. Similarly An would be the transition probability matrix consider that you have started at one, and you want to continue and find out what is the transition probability value after n years. So, this is the

very simplistic assumption, you multiply A n number of times find out an, but the fact remains in practical sense that it is not true, because what we are trying to do at priority you are considering the one time period information is being transferred to the n minus 1 time period at a constant level. Hence the end transition probability matrix which you multiply is multiplication n values. So, all these n values assumes the later n minus 1 are all dependent on the first one and they are constant, which is not true, because if you consider the transition probability matrix for second year, it is dependent on the first year as true, but other factors would also help change the values.

Similarly, if you consider the transition probability matrix for third year, it is dependent on second year, it may also be dependent on first year, but there are other factors also. So, what you should be aware is that as the transition probability matrix changes from the year one to year two for time period one to two, how is the dependent structure known we have to use that to find out the second year part transition probability matrix given the second year transition probability matrix, you will find it for the third year, so on and so forth. But if data is not available in a very simple case you can assume an which is a multiplication of n number of times of AA matrix to find out the transition probability matrix for n number of years rather than extrapolating the information from the first to the nth year, we can proceed it iteratively as our same. Such that you find for the second year, given the second year you have to find out the third year, given the third year you find out the fourth year, so on and so forth. Bit again remember in transition probability matrix, if you want to find have that information to find out the standard deviation, then standard deviation is dependent on a function of square root of t which is a very important fact to remember.

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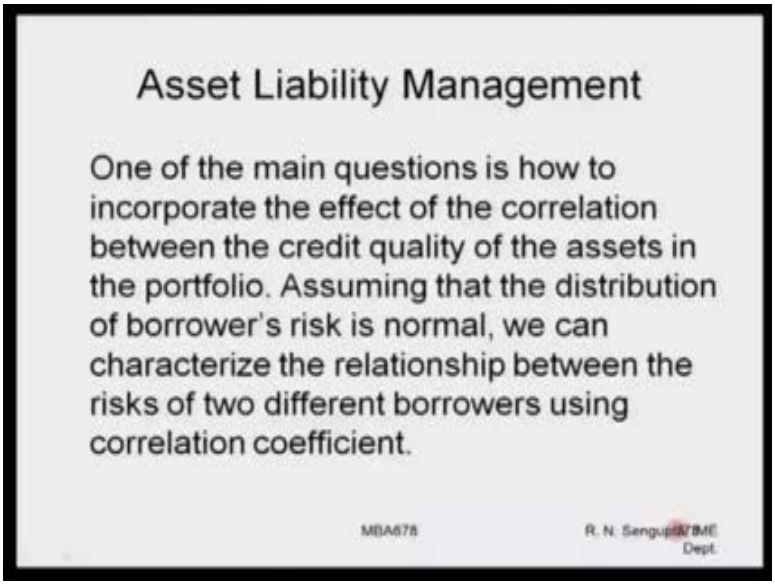

One of the main reason is how to incorporate the correlation between credit quality of the assets in the portfolio assuming that the distribution of borrower risk is normal. We can characterize the relationship between the risk in two different borrowers using the correlation coefficient matrix of the value.

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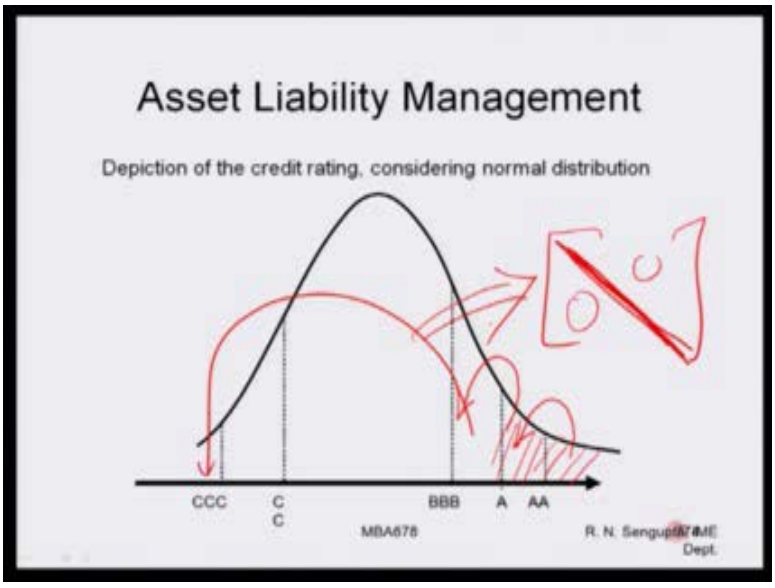

Now, this is the graph which you see. So, if you have these as double A, these as single A. So, what you want to find out what is the transition probability happening from AA to A. Similarly this is from AA to BB plus, similarly this is from BBB to triple C. So, if you

have the transition values the principle diagonal means same state half diagonal elements means from different state. So, this can be found out in the matrix formula for only one time period. So, what you will have is for different time period you have the distributions, if you have the different distribution the underlying distribution properties may be different, but they are of the same class of distribution. So, if they are skewed they will remain continue to be skewed, but the parameters the location parameters the scale parameter on all these things values for the distribution will change which will have an effect on what is the expected value what is the variance of that particular distribution.

So, if the distribution by itself changes the matrix based on which you are trying to do your calculation would also change. So, what we meant in the initial slide was as a changes hence finding out an may not be right.

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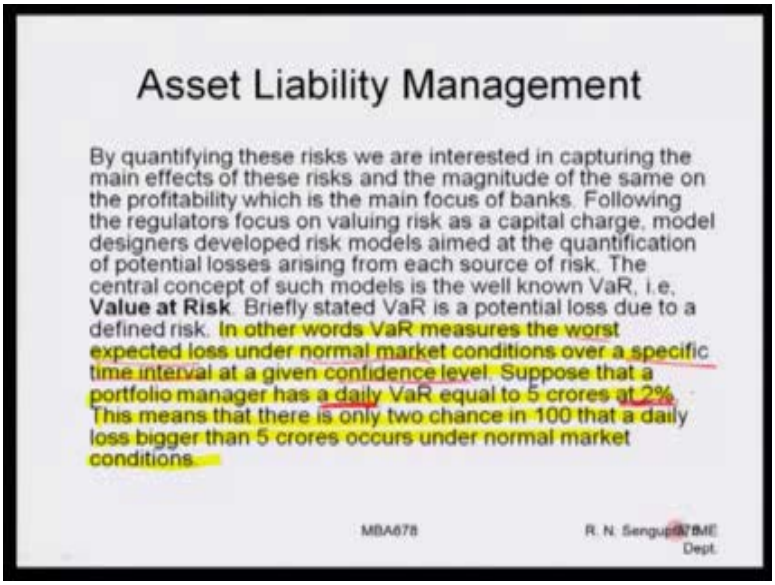

By quantifying these risks we are interested to in capturing the main effects of these risks, and the magnitude of the same in profitability, which is in the main focus of the bank following the regulators risk are very important concept of risk comes into the market which is known as value at risk. Briefly stated value at risk is a potential loss due to a defined level of risk a company or a fi or a bank is facing in other words war measure the worst expect. So, this is very important for us to understand I will just highlight it. So, I will use the highlighter this is what is very important for us to

understand, and as I go through the concept this will come let me first highlight. So, it means in other words war measure the worst expected loss under normal market conditions over a specific time interval at a given confidence level.

So, what are the keywords here? Keywords are worst expected loss under normal market condition over a specific time interval at a given specific level. So that means, time and level of confidence are very important parameters to define war. So, let us consider the statement and understand the risk suppose that a portfolio manager has the daily war daily; this is important, because time factor is coming to the picture, daily war value of 5 crores at two persons again, this is important this means that in only two chances. So, two persons means out of the 102 chances in 100 number of days that are daily loss greater than 5 crores under normal market conditions will occur. So, if I say that I keep that amount of 5 crores it means and a two person confidence level for a daily value; it means if I continue doing the taking the scenario then out of such 100 number of days there would be instances two number of times where my overall loss would be greater than 5 crores now.

You may be thinking that why I am mentioning daily cannot it be done weekly with this same definition, answer is no, because if you remember again I am repeating we mentioned that the concept of risk is dependent on time, how standard deviation function of square root of t variance function of t. So, as you increase the gap using day, if you want to find out for two days given day, if you want to find out for three days then the time factor would also be there coming. So, the hence the overall risk is not anymore the overall war, that is the war or the value of risk. We are trying to keep is not no more 5 crores per day, it will basically change, it won not be twice, it won not be 10 crores, it will be much more, because the time factor would also be coming into the picture.

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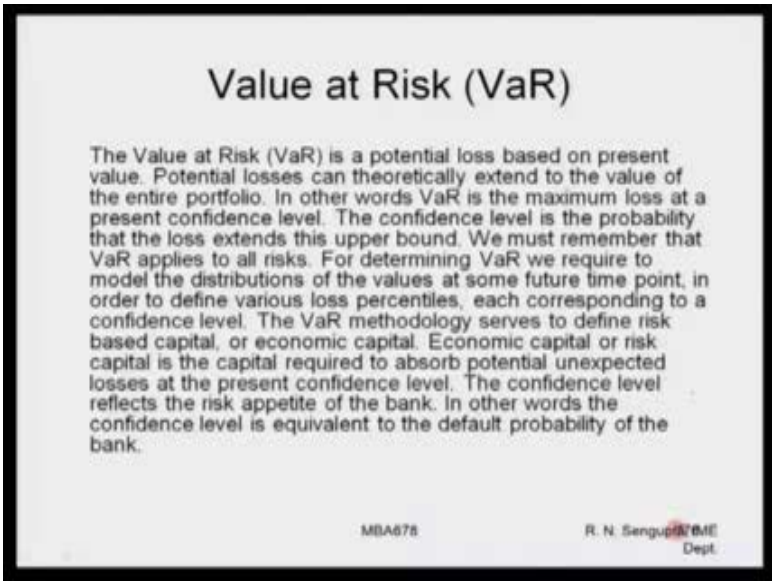

The value of war is a potential loss based on present value portfolio loss can theoretically extend the value of entire portfolio. In other words war is a maximum loss at a present confidence level, the war methodologies serves us to define this. So, this is the just the simple background and the definition which I am giving. So, now I will basically state it very simply using the graph

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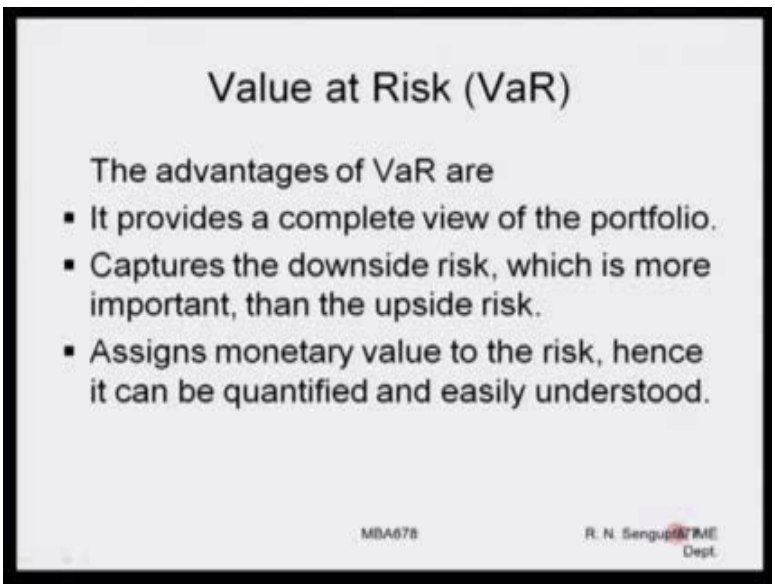

So, it advantages of var are it provides a complete overview picture captures the downside is very well practically definitely feasible, but it is technically in the actual sense if you go into the 90 gritties, it is not right remember it is very simple.

The concept of normality we have considered a is the underlying assumption factor for var. So, if you are considering normality of distribution, which means we are considering utility function to quadratic. Then all these things statements which you have made is true, but if the distribution is not normal then var is used a measure, but it would not give you the actual picture, and another advantage is that assigns a monetary value to the risk hence it can be quantified and easily understood by people.

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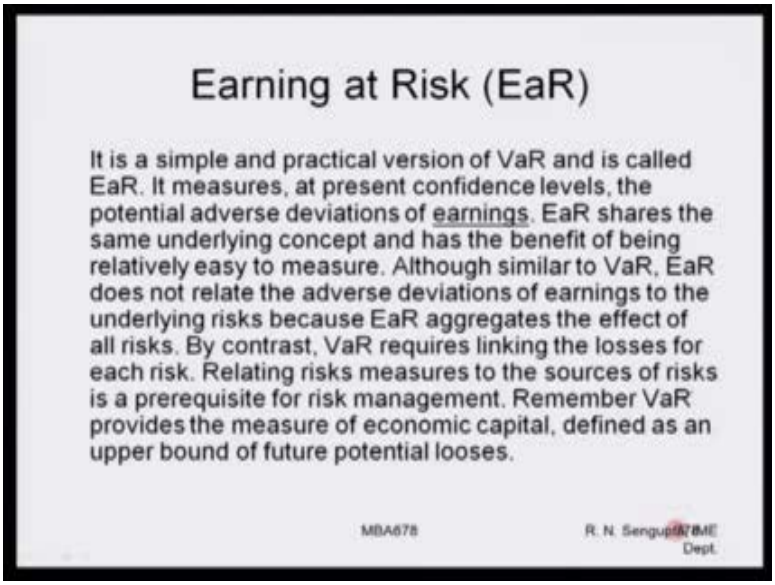

It is simple var is also called the is called the expected value or just which you are basically facing earning at risk. Earning at risk says the same underlying concept as the benefit has been relatively easy to measure. Although similar to var, the Ear which is earning at risk does not relate the adverse deviation of the earnings to the underlying risk. Hence or aggregates the effect of always, but contrary to that var requires the linking of the loss to a risk. So, there may be other losses also what I want to find out is on an individual basis what is the in a portfolio overall effect which is happening and that would be given much better by value at risk.

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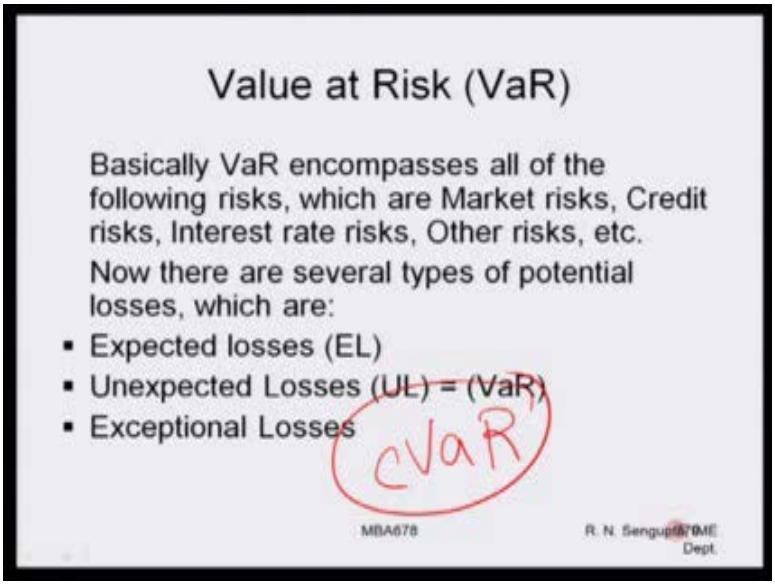

So, base considering the concept of var, we will with a simple diagram, consider the expected loss or type of risk, unexpected loss and exceptional loss, and that also will lead us to the concept of conditional value at risk which we have not written, but I will discuss with a very simple diagram

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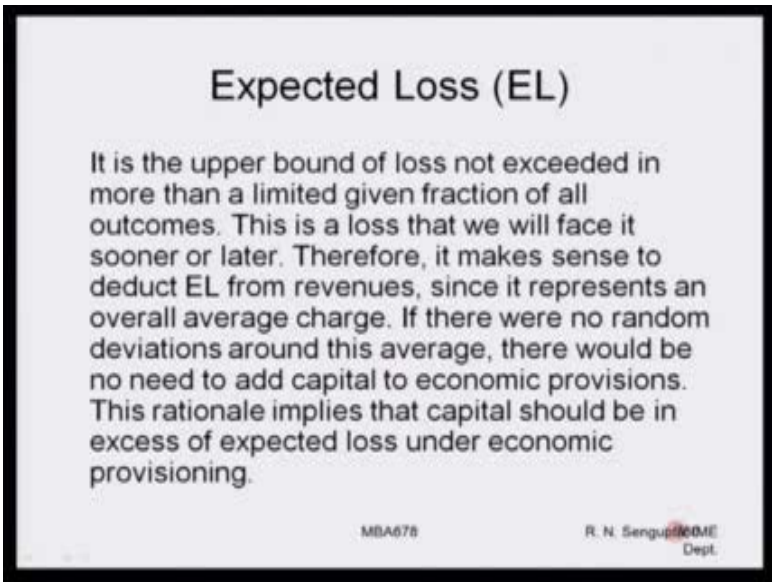

The expected loss - it is the upper bound of the loss not exceeded in more than limited given fraction of all outcomes. This is a loss that will face sooner or later. Therefore, it

makes sense to detect expected loss from all the revenues, since it represents and overall average change.

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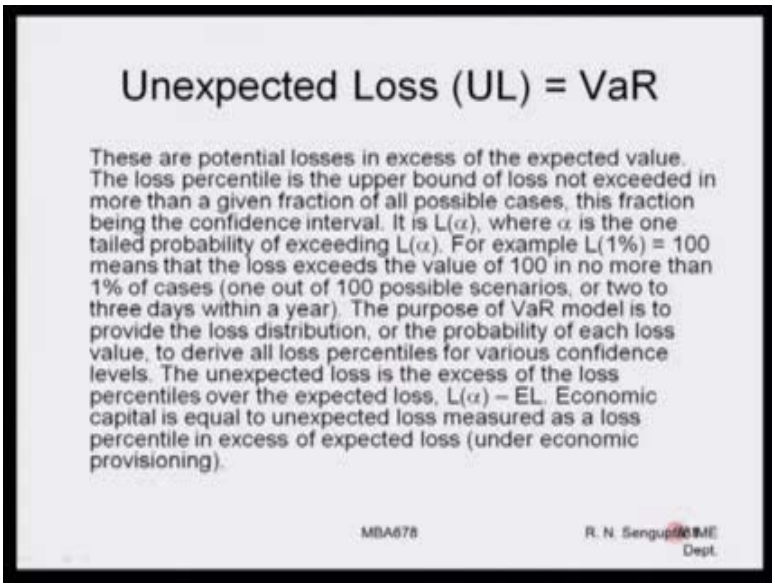

So, how it is done, let us continue. Unexpected loss which is var as I discussed is the overall risk which you are facing per day at a certain level of confidence. For example, if you say it means 100 means that if it is given as 100, it means that loss exceeding the value of 100 is more than one person of the cases, because the confidence level is one which means that out of that 100 cases; one case would be whether loss would exceed that value.

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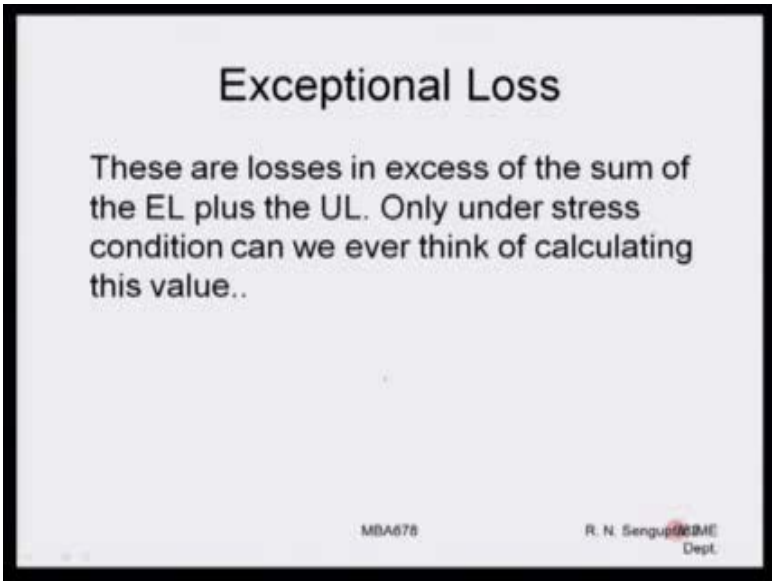

And exceptional loss is the loss which is excess on the sum of the EL plus the unexpected loss which is var. So, only stress conditions can under stress conditions can be think how to calculate the exceptional loss.

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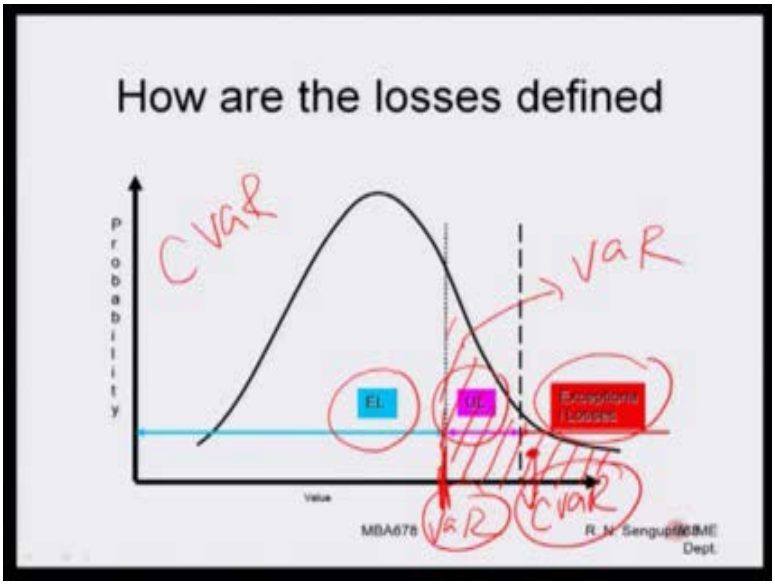

So, now with this diagram, let us consider. EL is the expected loss which you already have to keep, UL is the unexpected loss from where you will get the concept of var and this is the exceptional loss which is over on the var.

So, now if you remember I did mention what is cvar, which is conditional value at risk. So, conditional value at risk is very simple, if you have the value of var here, say for example, var if here. What you want to find out is what is the overall area of the right hand side, and what is the center of gravity of that? So, what you want to find out is that, you find out the overall distribution on the right hand side of var considering that losses are increasing on the right hand side. And find out the center of gravity of the expected value.

So, this is the cvar which you want to have and cvar and var at one go would give you a lot of information, if your distribution is actually normal, but if I am talking about the non normal distribution I did mention var would not be right, but still it can be proved mathematically that cvar would give you much better results, because it has some very nice properties under symmetric and asymmetric distribution, which is not held by var. Var has only these properties under symmetric distribution while under asymmetric distribution, it does not follow all those properties while cvar has those properties.

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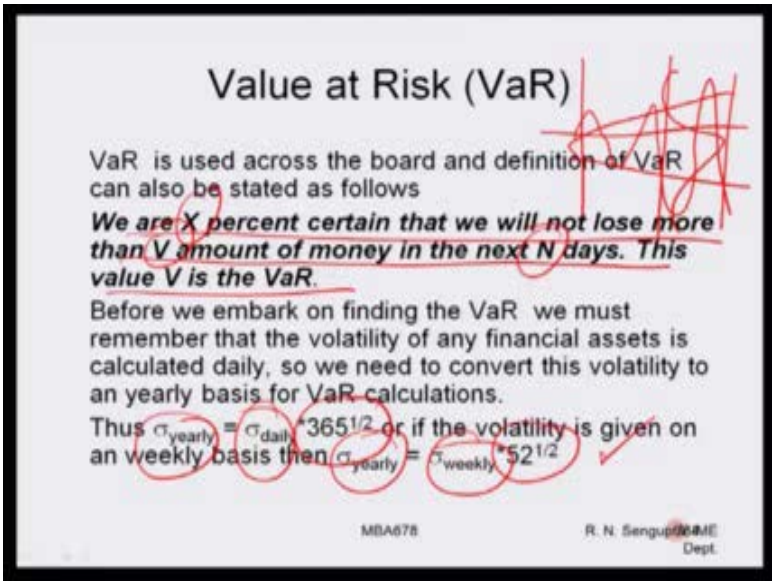

So, again coming to var, if we mention by x as the value of var. So, it means as the v value as var and with certain percentage and certain this number of days. So, when we state we are x percentage certain that we will not lose more than v amount of money in the next n days. So, this value of v is the var, this is the number of days based on which you are trying to find the var and this is the percentage of the confidence level. So, again

coming back to the same thing, which I am repeating sorry for that. So, if you have the yearly one, and if the daily one is given you would basically multiply by the square root of 365. If you have the weekly one multiply by square root of square root of 52, you find out the yearly one, if it is monthly one multiply it by the square root of 12, but this is the simple method. Actually it would not give you the result, because it is dependent on time which is exploding in the sense, if I consider the variance it will explode. So, if you none of the normal distribution overall variance will increase and depend on time.

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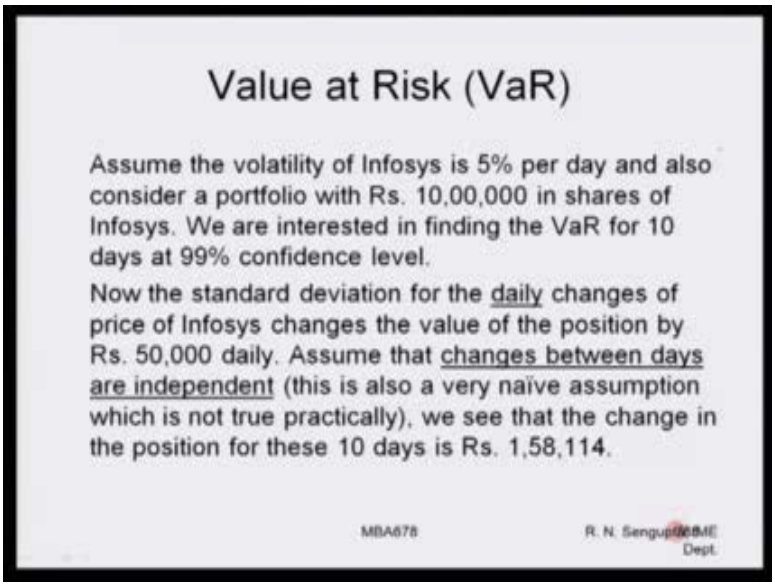

Assume a volatility of Infosys 5 percent per year and also consider portfolio with one million 10 lakhs in shares of Infosys we are interested to find the variance value of risk for 10 days. Now the standard deviation on the daily change is given on Infosys is the value of proposition is basically 50000, assume that the changes between days are independent, that is very important, we can find out the change in 10 days is given by158114. So, you use the same calculation and you can solve your problem. So, with this I will just end the discussion on simple concept of var and continue discussing the actual intrinsic properties of var and cvar in the next class.

Thank you.