

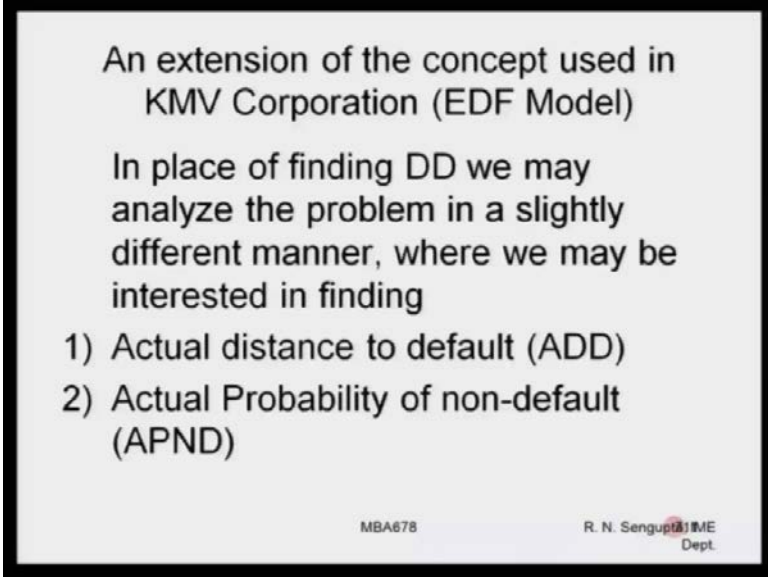
**Quantitative Finance**  
**Prof. Raghu Nandan Sengupta**  
**Department of Industrial and Management Engineering**  
**Indian Institute of Technology, Kanpur**

**Module – 07**

**Lecture - 42**

Continue our discussion about the credit rating models. We have considered initially the Z score, Zeta score, Zavgren's ((Refer Time: 00:19)) models and so on and so forth. Then, we moved on to a detail discussion about KMV Corporations EDF model. So, in the EDF model mainly it was distance to default, even though the underling concept was probability, which you want to find out.

(Refer Slide Time: 00:36)



An extension of the concept used in  
KMV Corporation (EDF Model)

In place of finding DD we may  
analyze the problem in a slightly  
different manner, where we may be  
interested in finding

- 1) Actual distance to default (ADD)
- 2) Actual Probability of non-default  
(APND)

MBA678 R. N. Sengupta IIME  
Dept.

So, an extension of the concept use the KMV model, you would be our discussion as of today and the relevant follow. In place of distance to default we may analyze the problem in a slightly different manner, where you want to find out the actual distance to default and the actual probability of non default. Because, why non-default is that, rather than trying to analyze for negative point of view, we will try to basically see, what are the positive point of view of using the credit risk model; such that we can also analyze the firms based on the positive statistics, it is like this.

We can rank a firm based on the minimum to the maximum level of risk, we can also rank a firm considering from the maximum to the minimum, considering the positive benefit or we can rank a firm considering the ratios of what. Ratios can be of return to risk, ratio can be of risk to return; where if it is return to risk, we rank it from the maximum to minimum, it is from risk to return it will be from the minimum to the maximum.

So, an example of the first type is return to risk would be excess return with respect to sigma, excess return with respect to beta, which we have already done when we were doing the portfolio analysis model.

(Refer Slide Time: 01:54)

### Actual distance to default (ADD)

$$ADD_t = (x_{1,t} - x_{2,t}) / (EMVA_t \cdot VA_t)$$

Where

- $x_{1,t}$  = Actual value of the assets at time t
- $x_{2,t}$  = Debt service requirement or DP at time t
- $EMVA_t$  = Expected Market Value of Assets at time t
- $VA_t$  = Volatility of Assets at time t

For the problem just discussed if at  $t = 2$  the actual value of the assets is 250 crores, then  $ADD \approx 0.037$

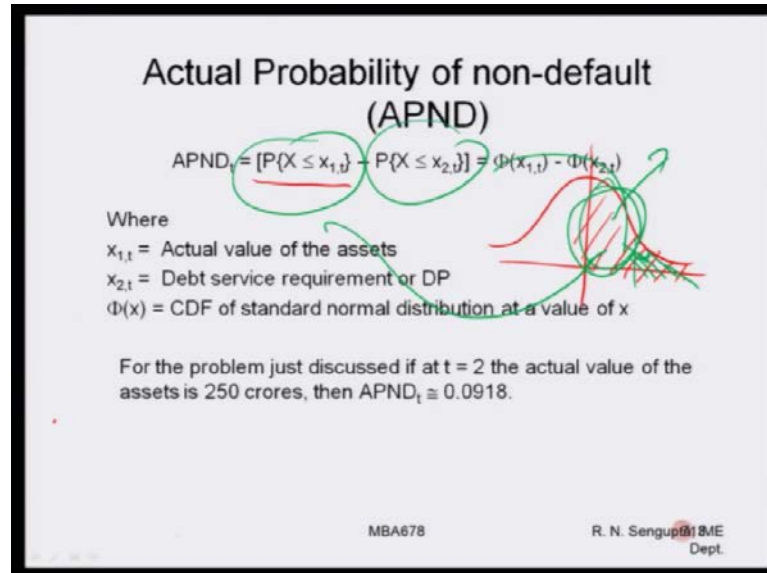
MBA678
R. N. Sengupta IIM  
Dept.

So, here actual distance to default would be consider as  $x_{1,t}$ , which is the actual value of the asset at time t and  $x_{2,t}$  is the debt service requirement or DP at time t. So, as we move; obviously, this debt service ratio as well as the actual value of the asset may change depending on the time hour they are changing. So, what we take in the numerator is the distance or the difference of that and in the denominator in order to normalize, we divide both by the expected value and the variance.

So, where  $EMVA_t$  is the expected market value of asset at time t and  $V_t$  is the volatility of assets at time t. For the problem just discuss as we discussed in the last day, if at time t is equal to 2, the actual value of the asset is 250, then the actual distance to default comes

out to be 0.037 and we can find out the actual distance to default at different points of time and then, rank them accordingly how the company is doing.

(Refer Slide Time: 02:53)

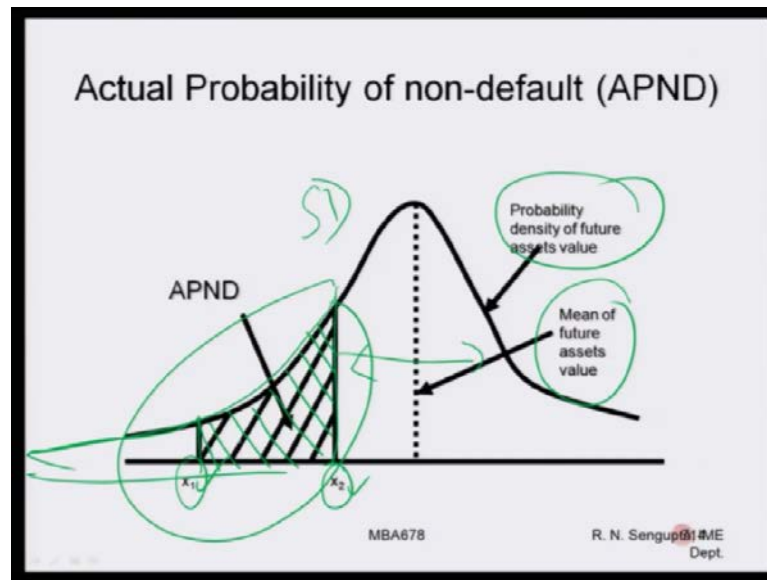


Now, actual probability to default would be given by the difference. Difference mean, what is the probability of the actual value of the asset in the absolute sense and what is the probability of default in the debt service requirements. So, you find out the probability and find out the difference, this is what is exactly is given. In the sense, so this is the probability, so if I have the distribution like this, let me turn it on it is original phase.

So, say for example, this is the probability in first case and if you have the color scheme as possible for the second case that we see. So, this would be the case if a distance to default, another would be the distance to default, so it will be this case. So, we will try to find out the actual distance to default, so you find out both the probabilities. So, this one is given by the red one, this one is given by the green one and based on that, we find out the difference.

So, this capital phi, this CDF or the cumulative distribution function or standard normal distribution at a value of  $X$ , so it will be  $x_1$ ,  $x_2$  depending on wherever you are. For the problem, just discuss if at time  $t$  is equal to 2, the actual value of the asset is 250, then the actual probability to default is given by 9 percent, which is 0.0918.

(Refer Slide Time: 04:18)



So, this is the actual probability to default. So, this is the difference between the probability on this side, total one and the probability on this side, so minus of that is this total area. So, the probability density futures value of the asset is given mean value of the future is given, so you have that standard deviation which will be given by SD and then, you can find out  $x_1$  and  $x_2$  are the values.  $x_2$  is the value of the asset at any point of time,  $x_1$  is the dead service coverage ratio of the debt level.

(Refer Slide Time: 04:49)

### Consumer finance model

Consumer Credit: It means both the short term and intermediate term credit that is extended through regular business channels to finance the purchase of commodities and services, for personal consumption, or to refinance debt incurred for such purposes. Example being automobile loans, housing loans, personal loans etc. It is important to note that the total consumer credit outstanding as it predicts the **turn in the credit cycle**. An important ratio which is of interest is the **credit change ratio**.

MBA678

R. N. Sengupta BME Dept.

Now, you will consider few simple consumer finance models. In consumer credit means that both the short term and intermediate term, credit that is extended through the regular business channels to finance, the purchase of commodities and services are considered for personal consumption. Example being automobile loans, housing loans, personal loans, it is important to note that the total consumer credit outstanding as it predicts the turn of the credit cycles. So, how fast you are able to get back the money; that means, either the credit card company or the banks which are trying to give a loan on different aspects.

(Refer Slide Time: 05:24)

**Consumer finance model**

$$CCR = (CICO/PDI)_t - (CICO/PDI)_{t-1}$$

Where:  
CCR = Credit Change ratio  
CICO = Consumer installment credit outstanding  
PDI = Personal Disposable Income

Another ratio of interest is debt service payment ratio.

$$DSPR = (PM + PC + PP + PA + PO)/PDI$$

Where:  
DSPR = Debt Service Payment Ratio  
PM = Payment for Mortgages  
PC = Payment for Credit Card  
PP = Payment for Personal Loans  
PA = Payment for Auto Loans  
PO = Payment for Other Loans  
PDI = Personal Disposable Income

MBA678 R. N. Sengupta  
Dept.

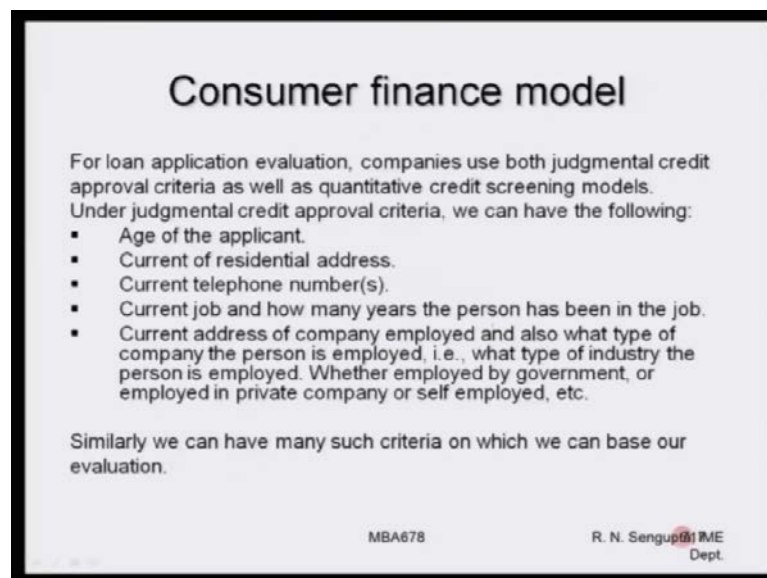
So, we will consider the CCR or the Credit Change Ratio is basically how the credit risk value is changing on the ratio concept, is basically given by the ratios of consumer installment credit outstanding and the personal disposable income. So, we have a personal disposable income, based on that you spend your money on different account. So, you want to find out that, what is the change of the credit change ratio, change of that depending on two different time period.

So, if you find out the ratios at t and t minus 1 or t and t plus 1, you can find out the differences, which will give you the CCR. Another ratio of interest is the debt service payment ratio, it is value is given by the ratio, where in the numerator you have the payment of mortgages, payment of credit card, payment of personal loan, payment of

auto loans, payment of other loans and the total value is the disposable income; that means, in the denominator.

What you are doing here in the numerator, you are trying to basically break down all the loans in different concepts based on the fact, whether they are personal loan based on credit card, personal loan based on automobiles or other loans or educations, whatever they are. They are trying to basically build a block; such that you are able to concentrate on each and every level and find out the DSPR, which is the Debt Service Payment Ratio.

(Refer Slide Time: 06:47)



**Consumer finance model**

For loan application evaluation, companies use both judgmental credit approval criteria as well as quantitative credit screening models. Under judgmental credit approval criteria, we can have the following:

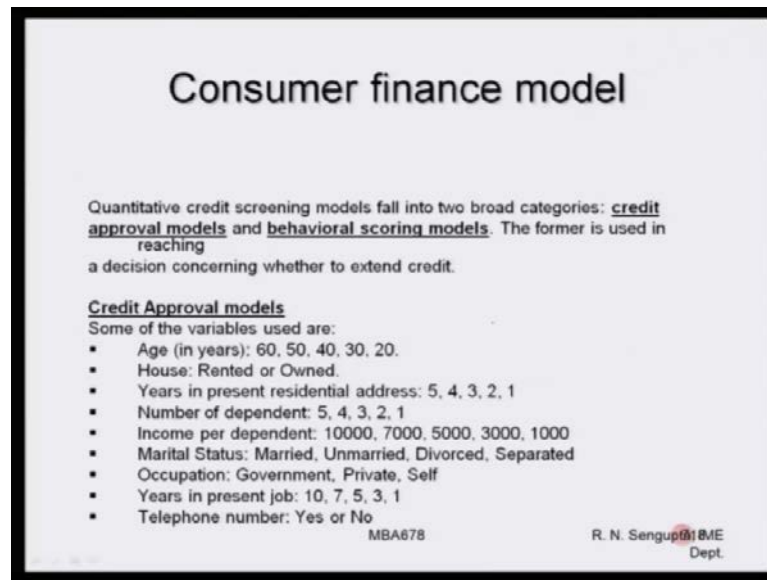
- Age of the applicant.
- Current of residential address.
- Current telephone number(s).
- Current job and how many years the person has been in the job.
- Current address of company employed and also what type of company the person is employed, i.e., what type of industry the person is employed. Whether employed by government, or employed in private company or self employed, etc.

Similarly we can have many such criteria on which we can base our evaluation.

MBA678 R. N. Sengupta ME Dept.

For consumer finance model for loan application evaluation, the companies use both judgmental credits approval criteria as well as quantitative credits screening models. Under the judgmental one, he have the age of the applicants is important to know that, current residential address of the applicant is important, current phone number, in what type of job the person is working, whether the person has taken any loans, what is the salary, whatever business the person is, all these are taken in to account. Similarly, we can have many such criteria on which we can base our actual analysis.

(Refer Slide Time: 07:20)



## Consumer finance model

Quantitative credit screening models fall into two broad categories: credit approval models and behavioral scoring models. The former is used in reaching a decision concerning whether to extend credit.

**Credit Approval models**  
Some of the variables used are:

- Age (in years): 60, 50, 40, 30, 20.
- House: Rented or Owned.
- Years in present residential address: 5, 4, 3, 2, 1
- Number of dependent: 5, 4, 3, 2, 1
- Income per dependent: 10000, 7000, 5000, 3000, 1000
- Marital Status: Married, Unmarried, Divorced, Separated
- Occupation: Government, Private, Self
- Years in present job: 10, 7, 5, 3, 1
- Telephone number: Yes or No

MBA678

R. N. Sengupta IIMB  
Dept.

So, quantitative credit screening model falls under two broad categories, one is credit approval models and the one is behavioral scoring models. The former is used in a research and decision making model whether to extend the credit, yes or no. And in the credit approval model, there are different type of criteria's or parameters; one is age, one is house with rented or owned, one is the years present at the residential address, number of dependent, what is the income per dependent, what is the disposable income, what is the marital status whether the occupation is government, private, business or doctor whoever it is, years in the present jobs, telephone numbers and all these things are noted down. So, that it gives you much better and holistic information about the credit approval models you are going to base based on these rights.

(Refer Slide Time: 08:10)

### Consumer finance model

Depending on the criteria mentioned above we can assign a score to each of them. Like if the house is a rented one, we can assign a score of 0 and if owned then +1. Similarly for occupation. If employed by government company, then the score can be +3, if employed by a private company, then the score is +2 and if self employed it is +1. The system for scoring is totally dependent on the company which is doing the credit screening. The importance of a criteria, discussed above can be categorized according to its level of significance depending

MBA678 R. N. Sengupta ME Dept.

Depending on criteria mention above, we can assign a score for each of them. Like, if the house is rented, you can assign a sign of 0, if it is owned house it is basically plus 1. If it is business, we give a score of say for example plus 1; if it is that of a government one we give a score of plus 2, if the person has a car or has his rented one or on mortgage on, then we give different scores. If the person's age is very high it is a older generation, then we go less score, if a younger generation is a more score; which means that it gives you some information about the credit ((Refer Time: 08:48)) of the person and also reflex what is the overall so called credit risk the company would face in case there is a default.

(Refer Slide Time: 08:55)

### Consumer finance model

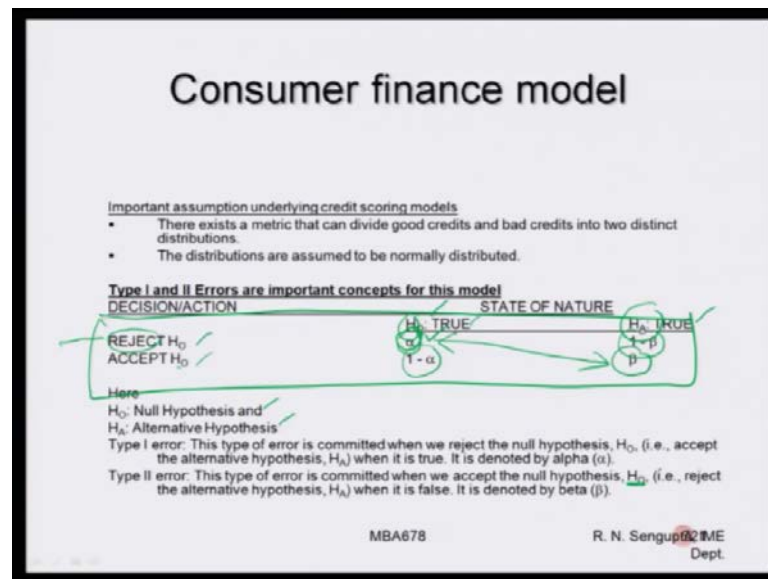
On the other hand behavioral scoring models are used to improve the profitability of accounts by subjecting them to different treatments with regard to credit lines offered and the collection methods used. Some of the variables used are:

- Number of credit requests.
- Number of credit inquiries conducted.
- Number of credit approved, i.e., the number of persons to whom credit was extended.
- Total number of credit approved vis-à-vis the requests.
- Total transaction approved and disbursed.
- Average transaction volume, i.e., the ratio of total volume of transaction to number of persons to whom credit was approved.
- Number of items default. Example what was the percentage of persons who had defaulted.
- Average transaction volume defaulted.
- Average number days past the due date of payment.

MBA678 R. N. Sengupta ME Dept.

On the other hand, behavioral scoring models are used to improve the probability of accounts by subjecting them to different treatment with regard to credit terms. It is based on the fact or the information needed or number of credit request, credit enquires contacted, total number of credit approval given, total transaction approved, so on and so forth as stated in this slide.

(Refer Slide Time: 09:17)



Now, one thing is very important to understand is that, for any credit rating models there are some sort of risk. Risk is not the point which you already discussed; we have been discussing some sort of uncertainty. So, what is that uncertainty, we will discuss. So, consider from the very simple statistical point of view there are two type of hypothesis, one is the null hypothesis, one is the alternative hypothesis. Null hypothesis is the  $H_0$ , alternative is  $H_A$ . So, these are the one which are considering  $H_0$  and  $H_A$ . So, this is the null, this is the alternative.

Now, in this case we can have two type of errors. Why two type of errors and what the implications are? I will come to that later, but first let us note this matrix. So, consider that we are taking a decision and there is one state of the nature, which we are not aware, that consider that state of the nature has a only important god or whatever supreme power, who has all the information and we are taking a decision and we are trying to bench mark our decision based on the only important decision, which may be true may be right.

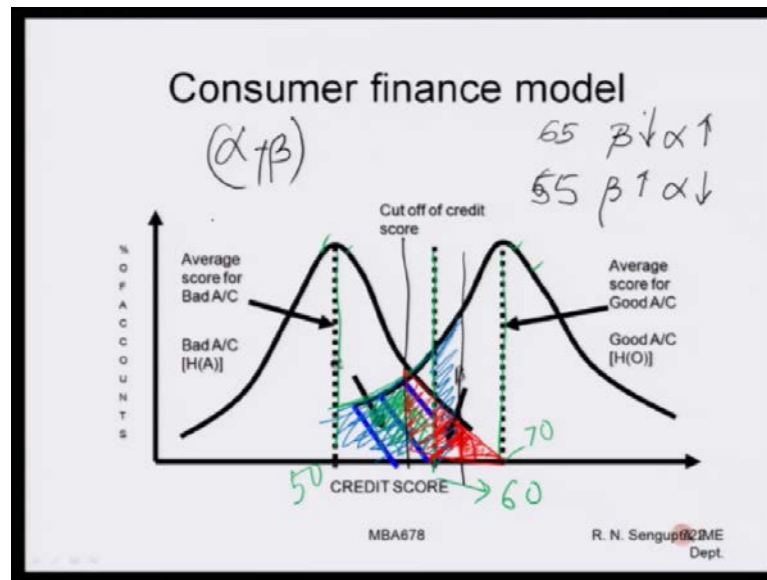
Hence, whatever we are taking may be right or wrong, so consider that these are alternative one, which is  $H_A$  and this is null hypothesis, which is  $H_0$ . Now, consider that we due to some reason, the first row we consider you reject  $H_0$ . So, if you reject  $H_0$  and actually  $H_0$  is true by Mother Nature, then there is obviously, error in our part. So, consider that as type 1 error and the error is given by  $\alpha$ ; that means, we are earning we are basically making a mistake.

On other hand, if you reject  $H_0$  and Mother Nature also saying that  $H_A$  is untrue; that means  $H_0$  is actually true, underline fact then; obviously, there is no error as such and we are in line with whatever we are saying. In the second case, consider that you accept  $H_0$  and  $H_0$  is true, which means the overall error which you are going to make is basically  $1 - \alpha$ . Error means we are going to consider both on the positive and negative side, I may be using the word error, but error would be positive or negative depending on whenever we are taking decision, which is in line with nature and which is in line with not in Mother Nature.

So, what is important to notice is that, if you reject  $H_0$ , but still  $H_0$  is true; this is the error we make in the first instant and in the other case, if you accept  $H_0$ ; that means, we are rejecting  $H_A$  and  $H_A$  is true, this is the error. So, what our main concern are these which is  $\alpha$  and  $\beta$ . These are just the counter part of that  $1 - \beta$  and  $1 - \alpha$ . So, type 1 error basically means, out of  $H_0$ , when it is true it is denoted by  $\alpha$  and type 2 error is the type of error is committed when you accept the null hypothesis  $H_0$ , when it is false it is denoted by  $\beta$ .

So, in the case when it is type 2 we accept the null hypothesis 1 and we try to find out what is the error in that. So, if you want to see from the pictorial point of view, let us draw the grand diagram as follows.

(Refer Slide Time: 12:33)



So, consider I am using the scales of the x and the y axis as for financial concept, even though it can be done in any concept. And rather than doing from the statistical point of view, I am trying to basically give the information based on the very simple credit rating or credit finance model. So, consider now you are a banker and as a banker, you have been given the task to analyze whether the loan should be given or not.

So, in a bank you have pay set of papers, there is a form and in the form, there are different type of scores, which have assigned, whether the person what is his or her age, what is the type of business the person does, whether the person has his or her own house, whether the person has higher education, whether the number of dependence is high or low, whether the person has a government job, so on and so forth.

So, based on that you have different asset of criteria's and consider the overall score, based on which you will try to analyze the person who has given a request for a loan is 100, consider it is hypothetically and also consider hypothetically that you have agreed that if the score is 60 and more, you will give that loan; if the score is 59 and less; obviously, you would not give the loan; that means, there is a cut of value and let us consider the straight and dotted lines, which you have is the 60 score, here in this diagram.

Now, what are the two graphs? Graphs means, these are normal distribution, even though they have not been drawn as normal, but I have tried my level best to draw it is as a

normal distribution, but they are normal distribution. So, if you consider the first graph it means this is the distribution on the people to whom you are willing to give the loan. So; obviously, the average score you will consider is greater than 60, consider it is 70; hypothetically consider it 70.

So, the distribution is basically gives you the type of account work good in nature, in the sense that if you give the loan, they would return the loan. But, as you are trying to analyze the problem from the probabilistic point of view, if you check the area which is on the left which is the blue one, blue hashed area; it means those are the people who have the propensity to return the loan, but you have analyze them to be on the lower scale, where the overall score is less than 60, which means that you would not give them the loan, even though if we had given the loan they would not have defaulted.

Now consider; that means, this type of this green one when I am basically putting all the green color, which was actually blue earlier or the set of people who are your potential customer, who would not have defaulted, but you are losing your business. So, this is a type of error you are doing, where your loss of business is there. Now, consider the other graph. Again it is a normal, but their average score group of people who will default is 50 so; obviously, you would not like to give them a loan, but see very carefully let me use the color as red.

So, these are the people where you would not give the loan actually, but unknowingly depending on different criteria's based on which the firms have been filled, you give the loan. So, if you give the loan there is a probability that they would definitely default. So, this is the level of beta which is the default risk, which is the type 2 error which you have committed for those people, who should not have been given the loan, but you have given the loan.

So, if you see combine effect of alpha, which is there on the right hand side and beta, which is on the left hand side, these are two type of errors which you have committed for one set of people, you should not have given the loan, you have given the loan, one set of people you should have given the loan and he have not given the loan. So, if you combine both of them, this is the overall risk you are facing.

Now, consider another important concept. Consider that you are trying to change this score, middle score of 16; consider you make it 65. So, 65 means it will lift shift to the

right or you want to make it 55 to shift to the left. But, if you see that you will very clearly understand that trying to minimize both alpha and beta, which should be done is simultaneously not possible; that means, you cannot decrease alpha you cannot decrease beta at the same time.

Because, if you shift it more to the right to 55 or you shift it move to the left which is 55 and you shift move to the right, which is 65 will see the values of alpha and beta changes differently. How? Make it 65. So, consider that you are making it a value of 65, which is here, so in this case the overall risk coming from the red area has now decreased. So, it is decreased, but if you consider the overall area from the point of view for the blue one it is increased, which means this is the extra part which you are trying to do.

Now, consider the picture from other point of view; that means beta is decreased alpha is increased, now consider the picture where your actual score is now 55. In this case, beta has now increased and while alpha has decreased so; that means, for the situation which we are trying to analyze, changing score to 65 or changing to 55 would have two effects. So, the first instant beta decreases alpha increase, in this case beta increase alpha decrease, which means individuality cannot decrease them. So, what is to be done?

Best answer is decrease the value of sum of the error which is alpha and beta. So, you want to bring it in to such a level that the overall risk, which is emanating from the case of type 1 and type 2 errors would be decrease to the maximum possible extents; such that the sum is decreased and made to the minimum level.

(Refer Slide Time: 19:14)

### Consumer finance model

**Remember the following**

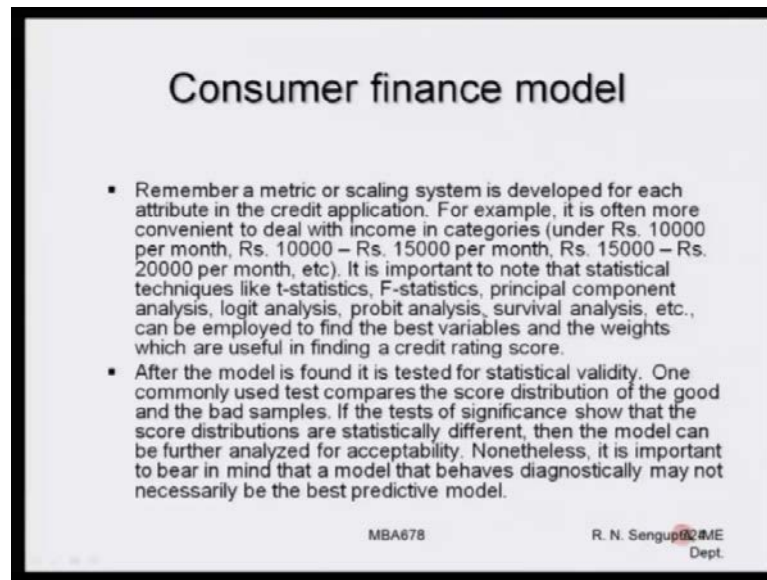
- $P_{H(O)}[\text{Reject } H(O)] = \alpha$  ✓
- $P_{H(A)}[\text{Accept } H(O)] = \beta$  ✓

Power of the test =  $\gamma = 1 - \beta$ , i.e.,  $P_{H(A)}[\text{Reject } H(O)]$

MBA678 R. N. Sengupta  
Dept.

So, remember the following, you reject  $H_0$  considering the  $H_0$  to be true is alpha, you will accept the  $H_0$ , that is you will reject  $H_A$ , considering  $H_A$  to be true is beta. Power of the test would be given by 1 minus alpha, which is basically rejecting  $H_0$ , given  $H_A$  is true. So, but our main concern always remember is alpha and beta, which is type 1 and type 2 errors.

(Refer Slide Time: 19:39)



**Consumer finance model**

- Remember a metric or scaling system is developed for each attribute in the credit application. For example, it is often more convenient to deal with income in categories (under Rs. 10000 per month, Rs. 10000 – Rs. 15000 per month, Rs. 15000 – Rs. 20000 per month, etc). It is important to note that statistical techniques like t-statistics, F-statistics, principal component analysis, logit analysis, probit analysis, survival analysis, etc., can be employed to find the best variables and the weights which are useful in finding a credit rating score.
- After the model is found it is tested for statistical validity. One commonly used test compares the score distribution of the good and the bad samples. If the tests of significance show that the score distributions are statistically different, then the model can be further analyzed for acceptability. Nonetheless, it is important to bear in mind that a model that behaves diagnostically may not necessarily be the best predictive model.

MBA678 R. N. Sengupta ME Dept.

So, remember a metric or scaling system is developed for each attribute for credit application. For example, it is often more convenient to deal with income in categories under 10,000 rupees, between 10,000 to 15,000, 15,000 to 20,000 per month, so on and so forth. It is important to note that different type of statistical analysis, logit probit model, logit model, principle component analysis, survival analysis, etcetera can be utilized to basically mark and find out good consumer finance model.

After the finance model is found, it is tested for statistical validity. One commonly used best compare test would be to compare the score distribution for the good and the bad sample. If the test of significance shows that the score distributions are statistically significant, then definitely you utilize that model for future statistics. If it is not, then; obviously, we will try to analyze, recalculate your model, make your model differently and do the calculation silently.

So, with this I end today's lecture and then, tomorrow we will start of the extension of this ALM model and other things, which would be up an extension on the consumer model, but in a different way, where we will consider the portfolio model extension also.

Thank you.