Quantitative Finance

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Module – 07

Lecture – 41

Welcome back, continue our discussion about the credit models. One model, which we thought and I particularly thought, I should discuss separately is the KMV model. So, before that, I give you simple background.

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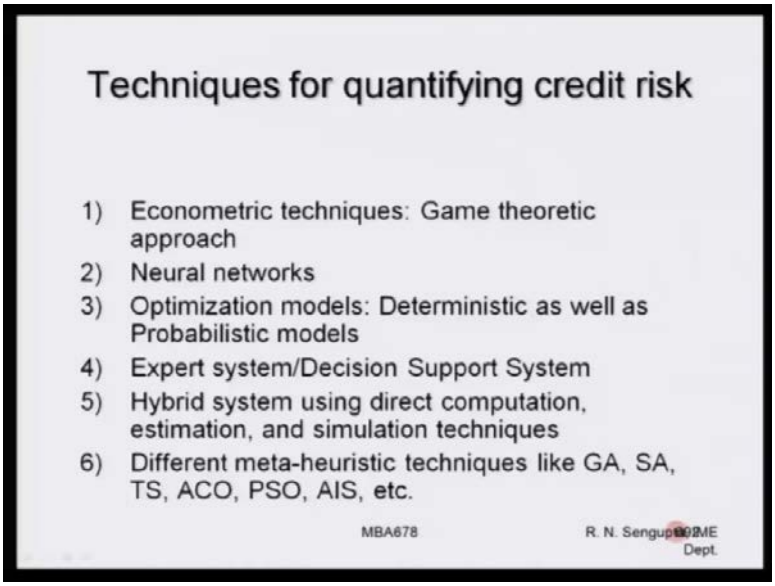

So, techniques for quantifying credit risk would be econometric matters, it can be game theoretic methods, it can be neural network. Neural network is utilized in a very big way and trying to analyze these models. You have the optimization models. Some of them I have discussed in very brief like safety first principle, ratio of x is retain to beta, ratio of x is retain to standard deviation, then the marc rich model, then the robust optimization model.

Though fitting the, I did not go into the actual mathematical little, because that is vary in depth analysis and that cannot be discussed in such a short time. Considering that we are trying to give you overall over view or overall very good knowledge about the different type of quantitative techniques, which are used rather than going to depth in any one of them.

So; obviously, that can be taken as a more specialized part of trying to analyze reliability models, robust models and statistics models. Here the expert decision system, support system, which can be build up depending on what you think or different type of ratios which should be consider and what weight age should be given. You have the hybrid system using direct computation techniques, simulation techniques; you have the different meta heuristic techniques, which is genetic algorithm, simulated annealing, tabu search and colony optimization and so on and so forth, which are given here.

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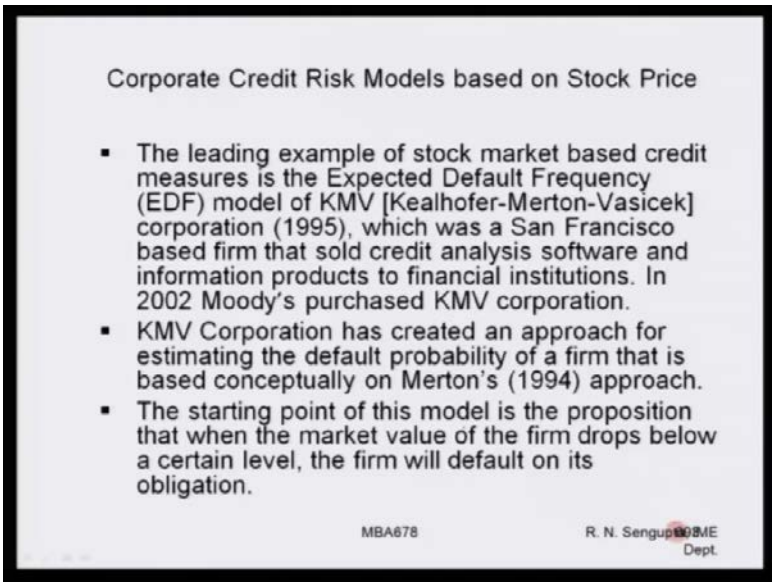

Now, the corporate credit models which will consider is the leading example of stock market based credit models is the expected Default Frequency Model or the EDF model of KMV corporation, which came off in 1995, which was basically a San Francisco based model. The firm that sold credit analysis software and information product to financial institution and in 2002, the company was brought over by Moody's. Now, it is basically if you do a search, it comes as an information as KMV under the Moody's.

KMV corporation has created an approach for estimating the default probability of a firm that is based conceptually on the Merton's 1994 approach will basically gives you that, how you can analyze the company given the different above fundamental and the financial ratios of the financial principle of the company. The starting point of this model is the proposition, that when the market value of the company drops below a certain level, the comp firm will default.

So, it is basically works as a very simple option. You exercise the option of a default if

your value falls below and if the value is more; obviously, you would not definitely exercise the option, so if you recall the concept of options.

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To understand this model we need a background on the option theory, which you have already discussed and we must remember, that the work of Modigliani and miller in the area of corporate finance is considered the base for option pricing theory. So, Black Scholes model if you remember, before that the concept of Modigliani and miller, concept wise may be utilize in order to analyze the Black Scholes model. So, again I am mentioning the EDF KMV model is basically like an option, conceptually like an option, which can be utilized in order to find out the price of that particular default.

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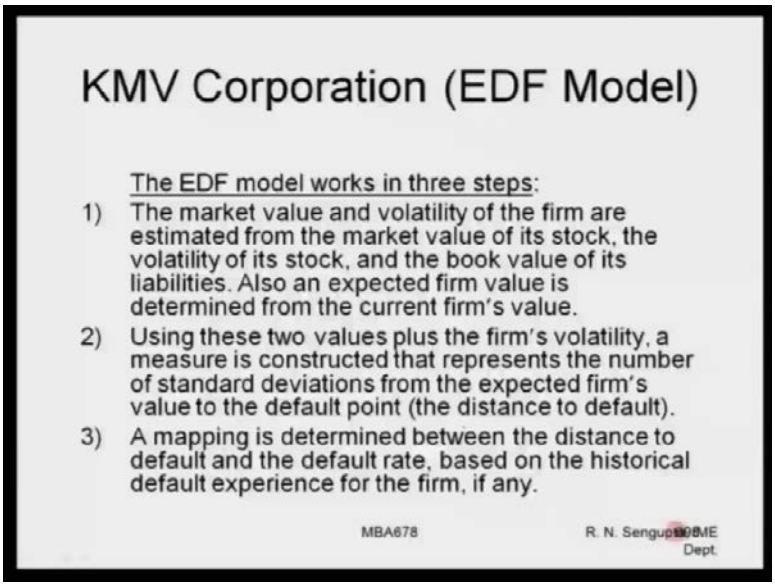

The EDF model works on three steps. The market value and the volatility of the firm are estimated from the market value of the stock; that means, whatever information you have from the market. The volatility of it is stock and the book value of the liabilities are also taken into account to find out the market value and the volatility. Also an expected form value is determined from the current value, firm's value.

Using these two values mention in step 1; the firms volatility, a measure is constructed, that represent the number of standard deviation from the expected firms value to the default point. This is what is important. If you remember, in options we had a strike price. So, if the strike prices are more or less, depending on that basically the person, who is the buyer and who has the obligation that he can exercise that, the option will exercise that. So, that is the triggering point on that or the default point, which you are considering here. A mapping is determined between the distance to default and the default rate based on it is historical default experience of the firm is calculated and can be analyzed.

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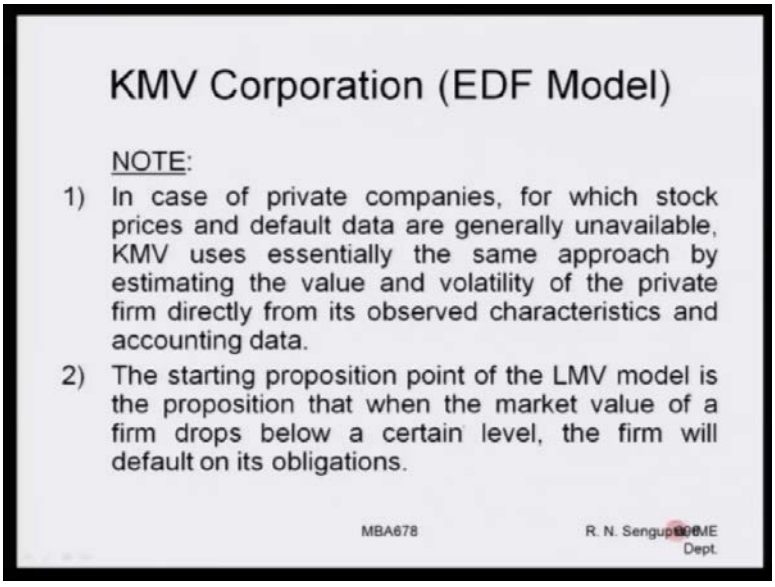

In case of private companies for which stock prices and default data's are generally not available. KMV uses essentially the same approach by estimating the value on the volatility of the private firm directly from the observed characteristic of the accounting data, which means again balance sheet profit and loss of account trial balance, are important.

So, if a company is in a market, where the actual information from the financial ratios or

the profit and loss account, the balance sheet are giving the exact information, then without analyzing them we can use the stock market. In case in the stock market, the information is not available, then he will sit down with this ratios or this information, which is given in the balance sheet and the profit and loss account and then, try to analyze that. The starting proposition point of the KMV model is that the, that when the market value of a firm drops below a certain level, the firm will default in it is obligation.

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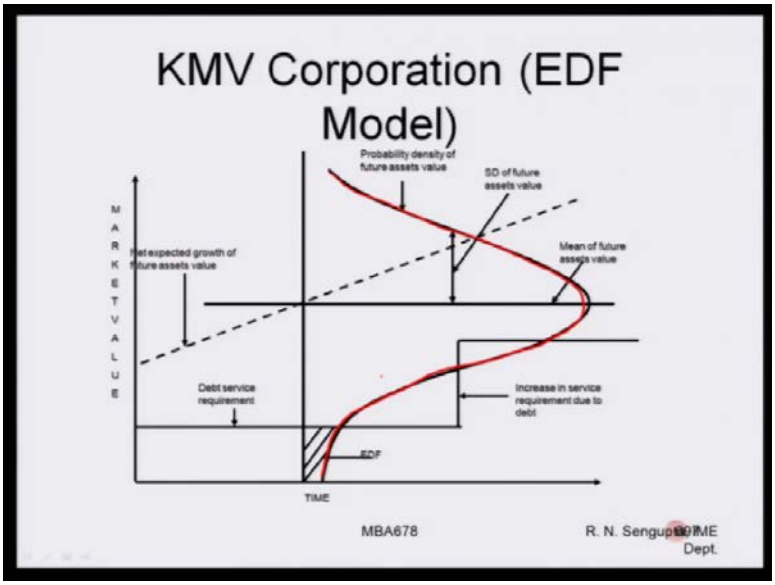

So, this is the graph which is there, so let us spend some time in this graph. Again, the distribution which we have analyzed is a normal distribution. What is normal you may be asking? This is the normal distribution which I am talking about. So, this is normal distribution. So, you will be, I am talking about with what is this normal distribution. So, I will just turned it by 90 degrees that is a reason for that, so let we first erase this.

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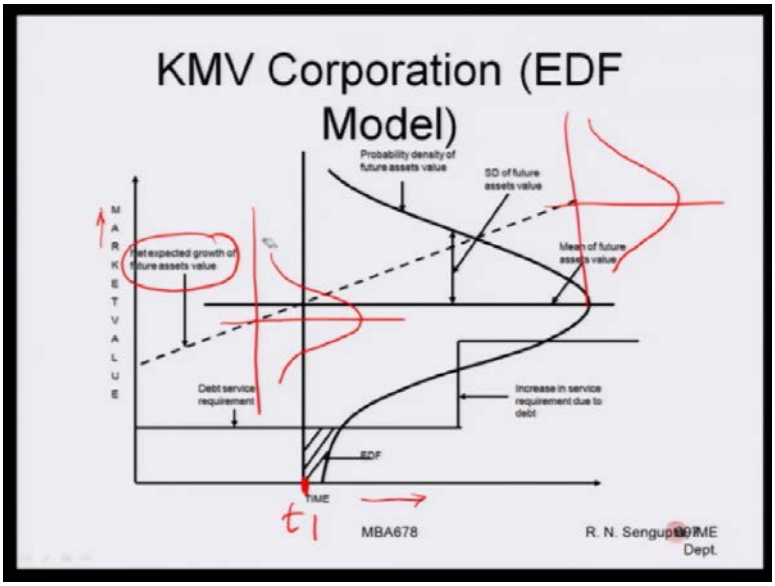

So, now, consider the time frame along the x axis. This is time and the market value of the firm along the y axis, so the market value being more or less than a default criteria or default point, which below that the firm will default over that of firm own default. Now, consider the firm's business is increasing, which means that on an average the firms net expected growth of the future asset is happening at a standard rate, which is the dotted lines.

Now, if you see the dotted lines is passing through the mean value or the mode value or the medium value of the firm at time T is equal to 1, which is I am trying basically trying to highlight here. So, consider this is time 1. So, if I have the performance of the firm as it proceeds with the time, then it may be possible that the firm, overall expected value here is like this or if you are consider the firms overall performance, let be consider here, this is like this. So, they are normal, they may not be the same variance, but I am trying to give you a picture, that how the overall performance is. So, if I let me again erase it…

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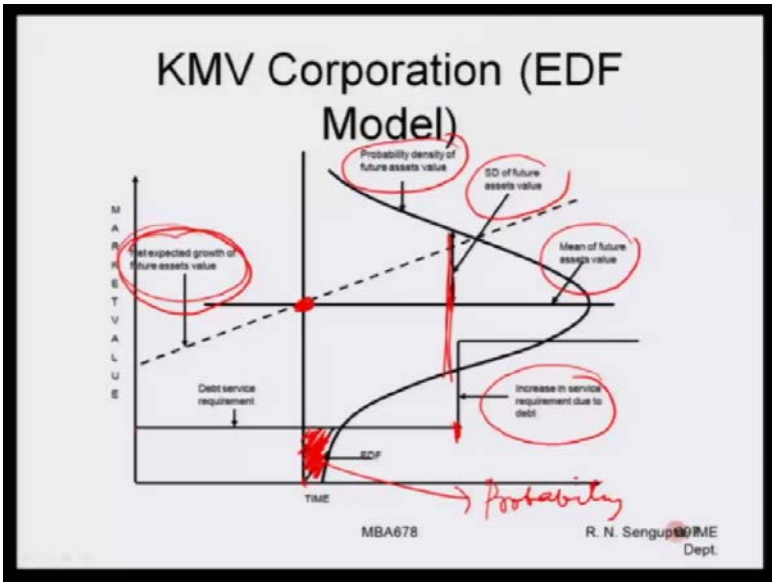

So, now, what we are saying, we were taking a snapshot of the company at time T is equal to T 1, this value and as the company progresses this normal distribution considering the variance is same will shift. It will shift up and down depending on what is the net value growth of the future of the firm. Now, consider this. The depth service requirement is we are taking as a step function, this one. Why does it can may not be a step function, but we are considering the companies borrowing at a certain interval of time.

This is the interval time, we need borrow this second time; at some time in future it is borrowed some money. And an interested payment are based on a fact that they have been fixed at a fixed interested. Hence, we are considering this is the increase in the service requirement and due to an extra burden of a debt, the company has taken as the liability as the company has taken. Now, this curve, which I denote is the probability density of the future asset of the value of the company and this area, which you are considering is the standard deviation.

So, now, we are considering very simply the standard deviation is not dependent on time. So, the curve remains as it is, but only shifts and the mean value of the asset value at time T 1 is this and the mean value is changing depending on the net expected root of the future asset of the company. So, now, if I draw it, so this has area, which I marking now with the red bold part is basically the default, because if the net value of the company is below this debt service requirement, the company will default.

So, what I need to find out is that with respect to them with the probability distribution, what is the probability of default. Now, in case if I need to find out the probability of default of the company, considering that the variance has increased and the increase in the debt service requirement has also built in there, I will try to draw a graph now on this same one. First let me delete it.

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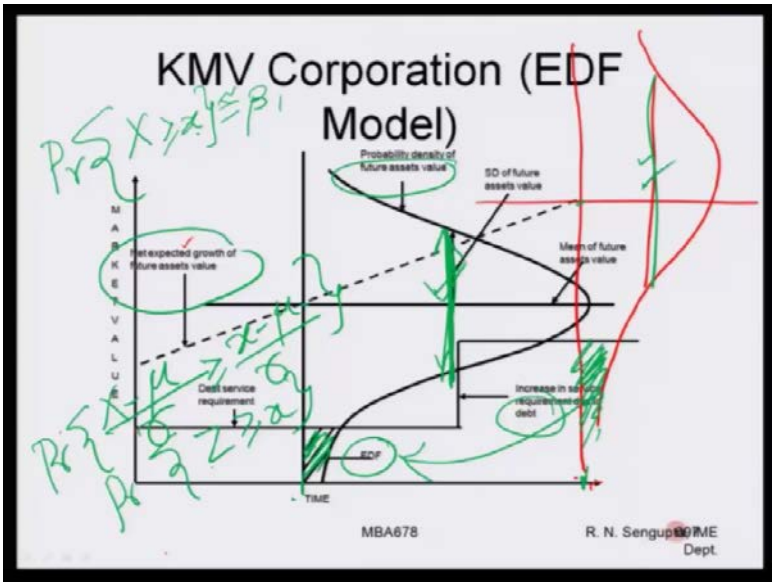

So, consider the company's net expected value has increased, so let me draw the curve. Consider although, also very simply the variance is also increase. So, let me draw first the mean and the standard deviation and this is the variance. So, now, if you consider this value, the variance here of two standard deviation plus 1 and minus 1 and if I want to have a study of the variance of this portion, so this is the standard deviation. So, let us consider this standard deviation and that one which you have here are different.

Now, if you consider the EDF, which a value of here and the expected value, which you are here they are different, which means the value of the expected probability the probability default is different for the case for three different reasons number 1 net present is increasing, number 2 variance at different, number 3 detect service ratio is changing are the debt service requirement.

So, what we need to find out is the probability at any instant of X, X is the random variable, which denotes the net the X this value of the of the future of the assets X is greater than equal to sum x, where the small x is basically that service requirement and you want to find out, what is the probability value, which is beta, which we need to find

out. Now, what it can be done very simply is probability of X minus the mean value, mean value would be here for the time period T 1 mean value would be here for the time period of T 2.

So, remember that the mean values are changing divided by sigma, what is sigma again this the ratio this the standard deviation, which you have at time T 1 and this is the standard deviation standard deviation for time T 2 this is the standard deviation is greater than equal to some x minus new this x value at time T 1 as this x value at time T 2 is this. So, that will changed depending on, where you my standard deviation.

So, this is what we known under standard normal deviate and using the standard normal deviate table. So, you can find out the value very simple for the table, so it means that given is value you can easily find out, what is the probability of default. This in a very simple not sell is the way we basically try to use the EDF model of the KMV corporation and try to analyze, how what is the probability of debt for company or what is the overall default of a company.

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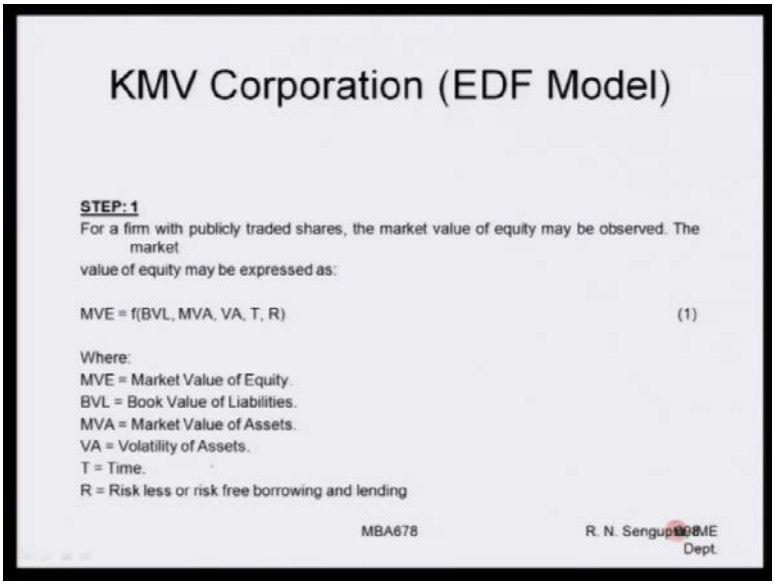

So, now, coming to the actual steps this was, what I was trying to explain step one of firm with public literature the market value of the equality maybe observed. And the market value of the could maybe a expressed as a function of this book value of liabilities the market value of assets volatility of assets the time and the risk does lending and borrowing of the risk of the total amount of money based on which, we are trying to analyze your debt are you are trying to analyze you default.

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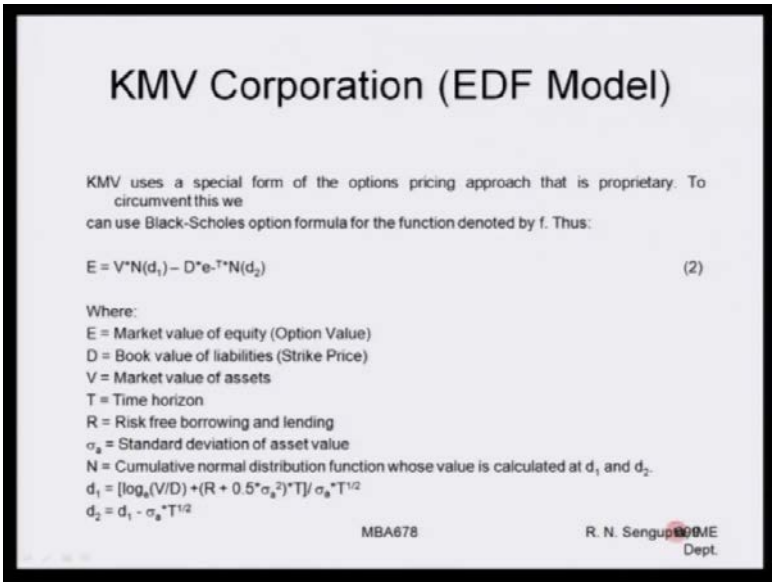

KMV uses a special firm of the options pricing approach that is proprietary as you remember we did mention about the concept of that being a concept of option. But, the way they try to price this option is not known to us I was not able to find it any place and the use the Black Scholes model in order to find it. So, once you analyze the Black Scholes model in a very simple manner your market value the equities given depending on the variables, which are market value assets time are asset risk free lending borrowing standard deviation assets and the cumulative normal distribution factors and d 1 and d 2, which are given. You can find out any only based on that you have the market value of the equity, which is the option values based on which, the option will be exercise if it is defaulted or not.

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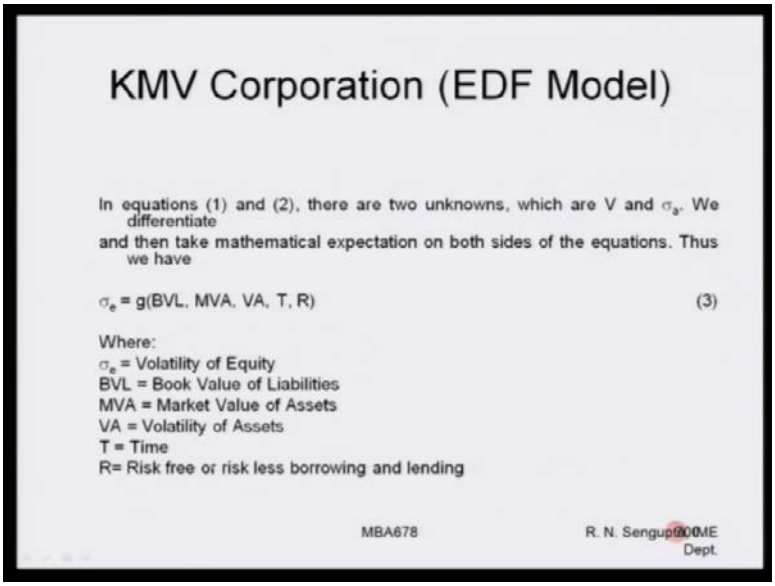

In equations 1 and 2, which we have discuss there are two unknowns, which have V and sigma a we differentiate and then, take the mathematical expectation on both sides thus at the end will be basically have the sigma, which is the sigma of basically the equity and the equity values would be given by depending on book value of liabilities market value of assets volatility of assets a time to expiration when it will basically default; that is according to the concept of option if you try to analyze and we have the risk this interested.

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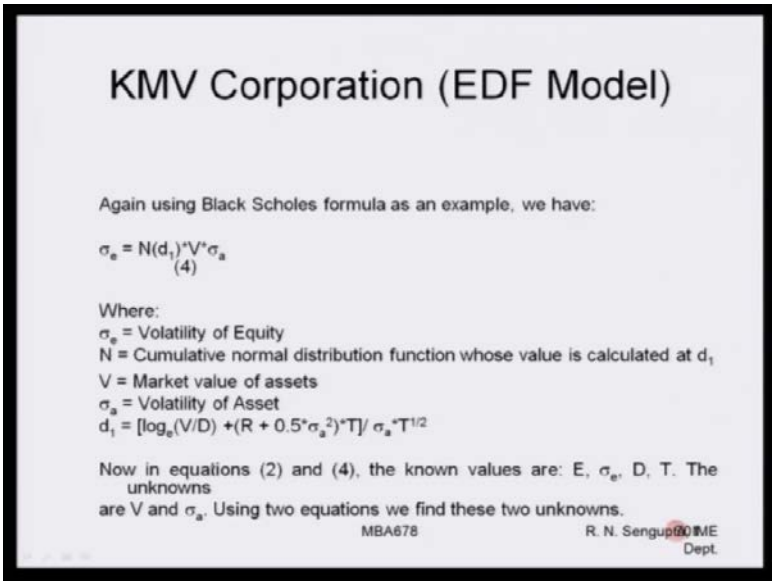

Again using Black Scholes model you find out different way of trying to analyze the standard deviation by equities and they are dependent on the factors, which is cumulative

normal distribution market value of assets cumulative normal distributions function, which is calculated using d 1 or d 2 whatever it is their the last two slides.

The volatility of the assets, now in equation 2 and 4 the unknown values are e standard deviation equity D and T and this unknowns can we found out using the simple information, which you have already from the mark this fundamental principles of the balance sheet profit and loss account on this the trial balance of the company.

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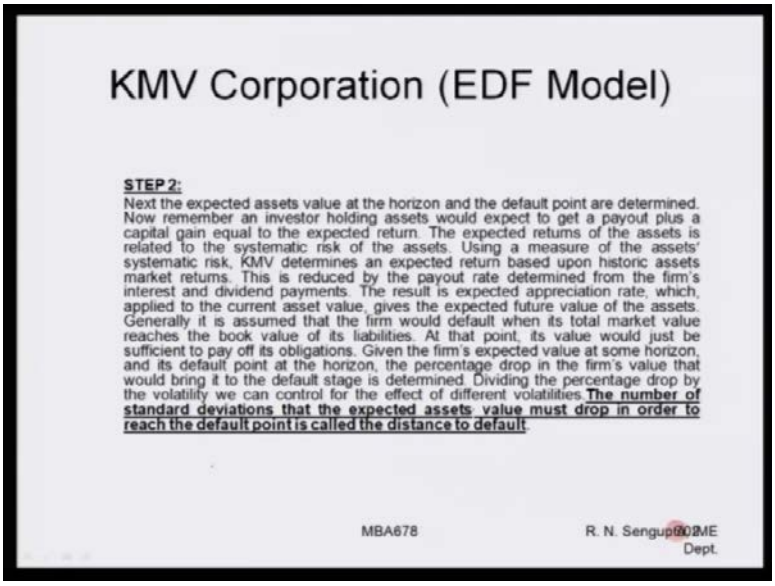

Next, the expected assets region value at the horizontal and the default point are determined. Now, remember an investor holding asset would expect to get a payout plus a capital gain because; obviously, it mean price, which has been paid, which is the price small p small c if you remember is basically somewhere based on the triggering point, where it the default will happen. Using a measure of the asset systematically risk, KMV determine an expected return based on historical asset market return this is reduce by the payoff rate as determine.

Now, continuing given the firms expected value at some time horizon as default point he horizon the percentage drop in the firms value, that would be to it is default stage is determine dividing the percentage drop we trying to find out the number of standard deviation it is away from the default point. So, how for it is from the default point, which are main concept; obviously, probability default is important considering they are trying to find out the price pricing from the option point of view, then finding of the pricing on the option point of view you try to analyze the distance to default.

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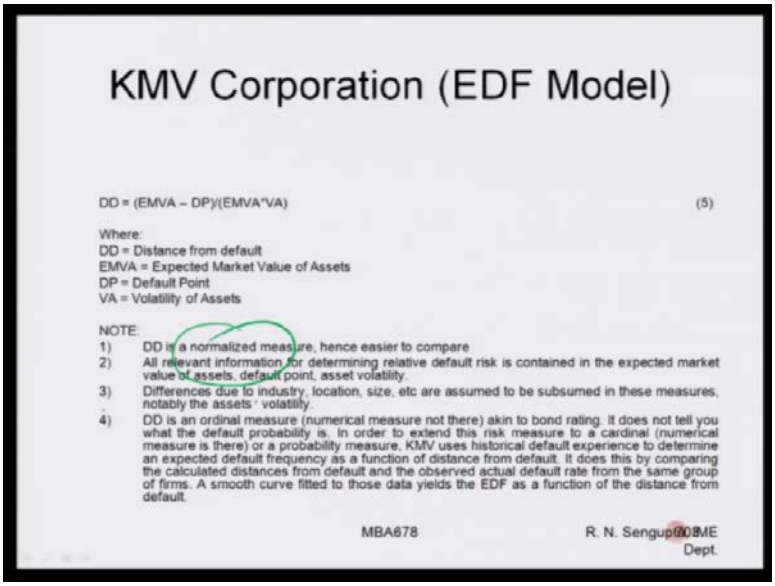

So, which is what where mentioning D, D is the distance to default it is given by the expected value market value of the firm by the default point minus the by divided by the volatility of the assets. So, what you are trying to do you are trying to find out the normalized distance to default given the normalized distance default there are different type of firms in different sectors you can find out the, what is the normalized distance to default and try to base on that you can find out how for or how close the firm is form the default point given all the set of information's have been normalized using the distance default concept.

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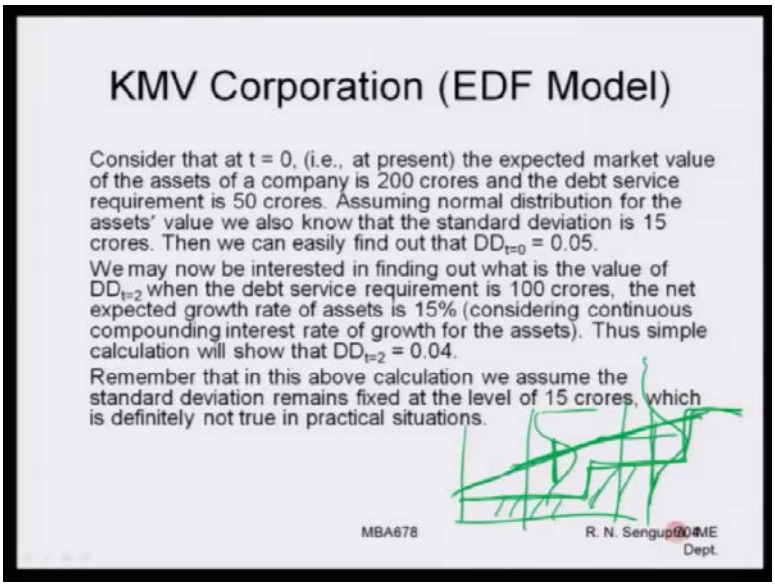

Consider that at time t is equal to 0, which is as present the expected value. So, for this

always consider the graph, which we have discuss the average value is increasing this way, so you will try to analyze where that the variance is increasing not decreasing the mean values increasing or decreasing and what is the default value, which is there.

So, these are the changes of the default, which you already discuss the expected market value of the asset of a company's 200 crores and it is debt service requirement is 50 crores assuming normal distribution for the asset value we also know that the standard deviation is 15, then we can also find out the distance to default very simply as 0.05. We may, now we interested to find out what is value of the distance should be default at a time t is equal to 2 when the debt service default is 100 crores the net expected value know the asset has been given by 15 percent.

So, simple calculation will give you; that is destination default is now 0.04, because it was decrease and they maybe instance, where their rate of change of the net present value are the rate of change of the net default would increase a decrease depending on; that is distance would be increase or decrease depending on whether the debt service values are increase or decreasing or whether the standard deviation is an increasing or decreasing or whether the average value is decreasing or increasing by the average value I mean this by standard deviation I mean this and the debt service basically means this.

So, these are the important points and then; obviously, you consider from the simple Black Scholes model, what is the actual value of the option based on, which that triggering point will happen. Remember that in this above calculation we assume that standard deviation we may fixed as a value of 15 crores, which is definitely not in the practical sense, because it may change depending on how the companies doing on how the overall market share of the company changing, how the overall business scenario is doing and so on and so forth.

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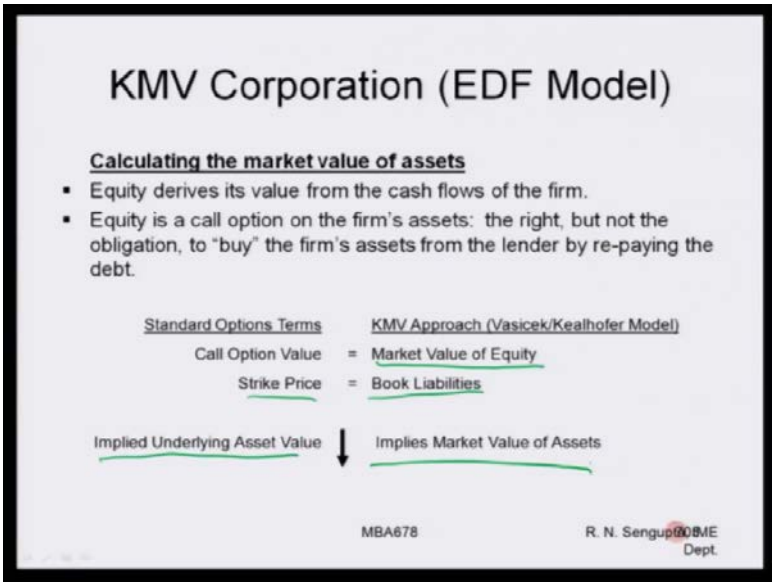

Calculating the market values assets, so equity derives from the value of cash flows of the firm's equity is basically type of call option in the firm's assets the right, but not the obligation to buy the firms as at from the lender by repaying the debt. So, on a standalone basic standard option terms would we are trying to analyze and try to basically bring a one to one similarly with the KMV models are call option prices is the market value of equity and the strike price is basically the book liabilities book value of the liability based on which, you are trying to do the calculation. And the implied underlying asset value is equal to the imply market value of the assets.

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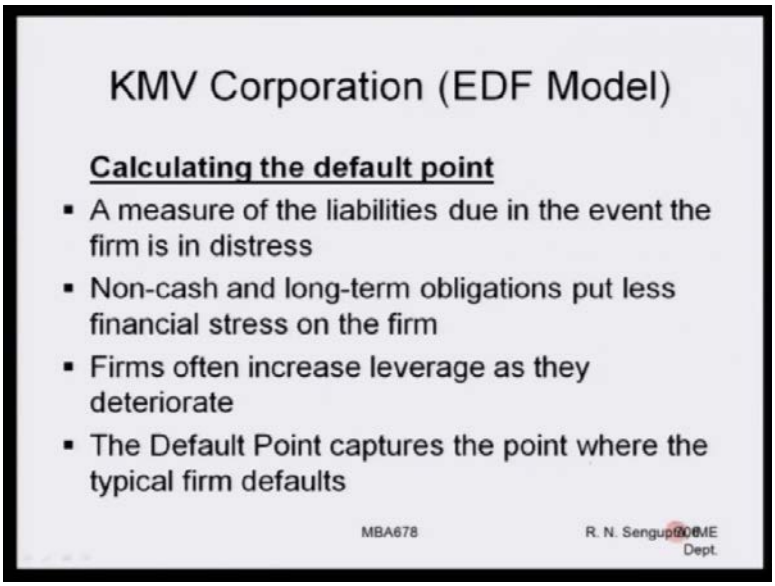

So, why do we calculate that distance default it is basically measure of the liabilities due

to the event in the firms if it is in distress non cash and long term obligations put less financial stress on the firm and it can be analyze in a very nice manner in the distance default in the probability default firms of an increase the average as leverage as they deteriorate. So; obviously, they try to change the overall liabilities overall scenarios long term and short term, but this when it is normalized you use a better picture.

The default font capture is that the point, where the typical firm defaults and you can find out in a very nice manner in even in the case if the firm is changing is overall leverages, that normalize score would give you normalize means distance score would give you much better picture, how the scenario even they are changing that can be normalize and you can find out the distance to default.

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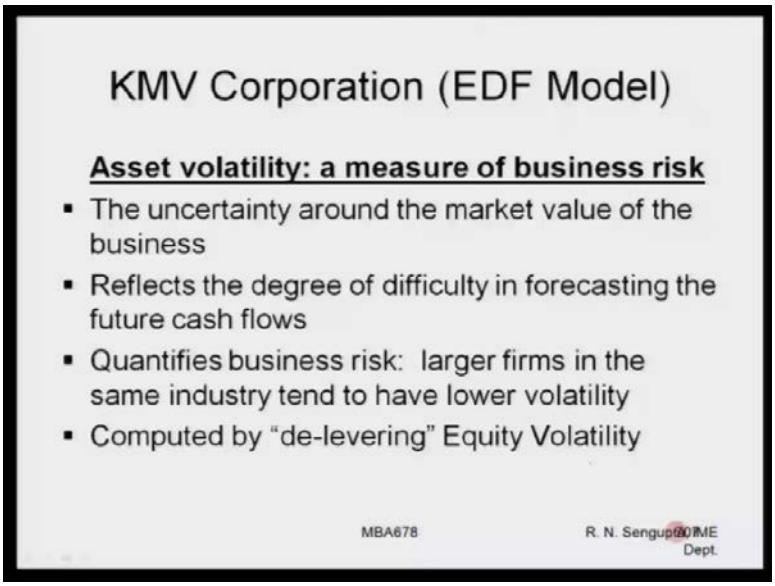

The assets volatility basically measures the uncertainty around the market value of the business reflex the degree of difficulty in forecasting the future cash flows quantifies business risk as larger firms in the same industry tend to lower volatility and computed by de levering the equity volatility as that the distance default is normalize in a, now rational manner.

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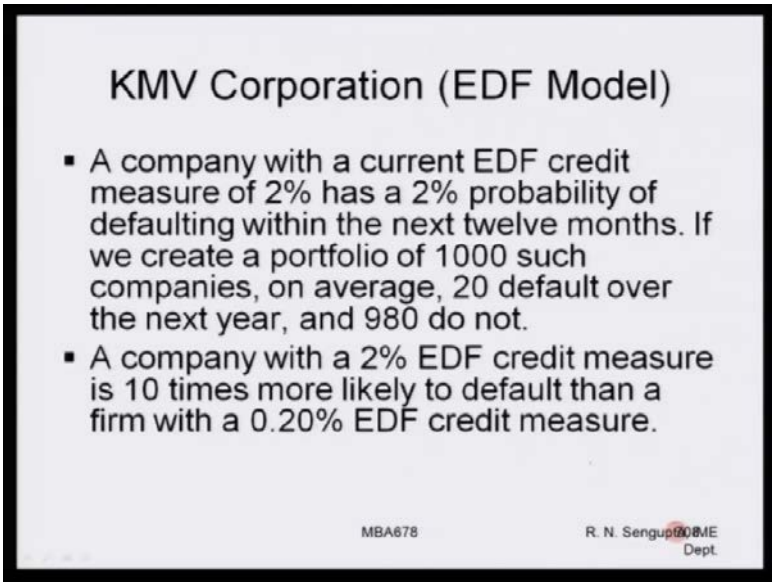

A company with a current EDF credit of 2 percent has a 2 percent probability of default within next 12 months if we create a portfolio of 1000 companies and an average 20 default; that means, 982 do not; that means 2 percent basically give you the probability of differ a company with 2 person EDF credit market and 10 time as more likely to default than a firm with 0.2 percent EDF credit by; that means, the distance, which we are trying to use has been normalized.

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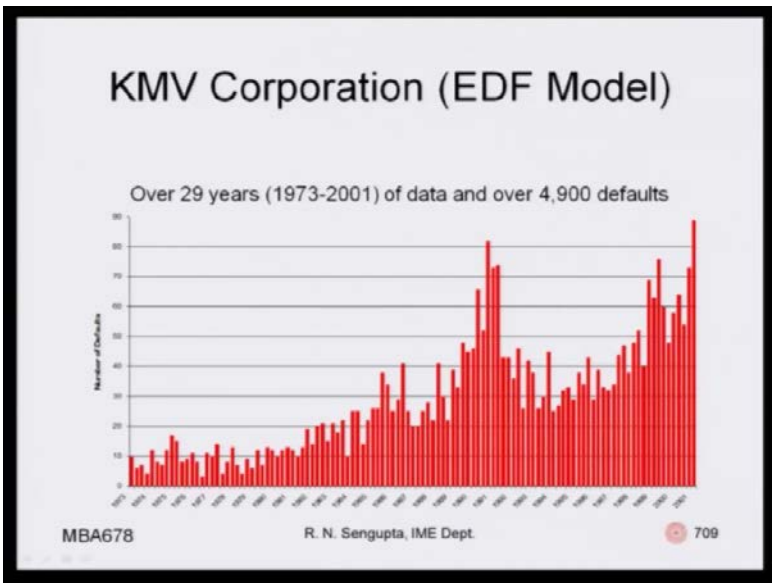

So, this we are taking from the last 29 years staring from 1973 to 2001 the data, which I have found the KMV Corporation these are some of the data for 4900 defaults. So, the numbers of defaults using the years are given, such that they give you good picture.

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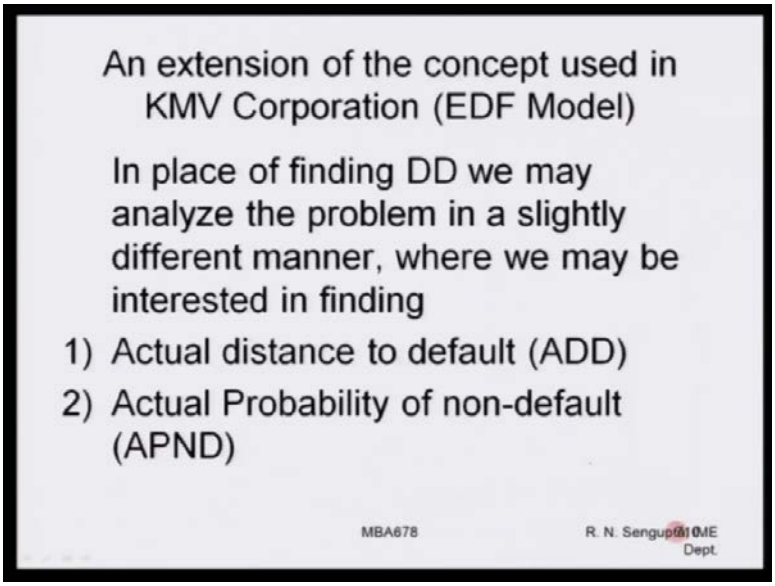

So, now, before we go into the extension of the KMV models ((Refer Time: 21:28)) I will try to basically again wrap of this session the remember the credit models, which you consider the ((Refer Time: 21:34)) models the zeta score the Z score all these things give you a very good picture that how you can analyze in a very nice manner giving the firms fundamental principles. And then, KMV model which is by Moody's and now, being utilize in a big way if we can analyze the firm, but the underlying fact which I want to stress is that if able to find out the one to one similar between the option pricing concept and the KMV model that will give you much good better picture. And obviously, probability to default and distance default are consider as the main focus area, which are the bullet points of the statistics based on which, you can analyze a firm whether is defaulting or not.

So, I leave the class today and thank you very much and will continue the discussions with the credit models and other topics as usual.

Thank you.