Quantitative Finance

Prof. Raghu Nandan Sengupta

Department of Industrial and Management Engineering

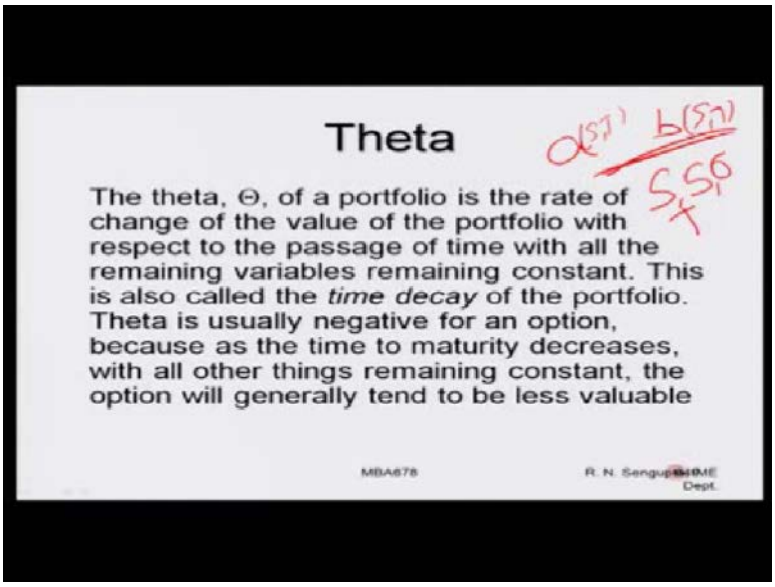
Indian Institute of Technology, Kanpur

Module – 07

Lecture - 38

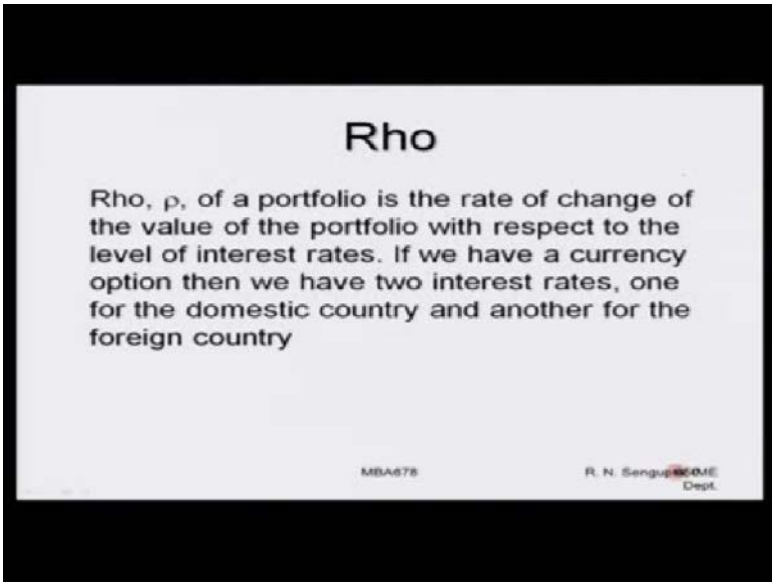
So, we were discussing about theta in last few slides in the last class we had discussed what is delta. Delta was rate of change of that portfolio with respect to stock price and was gamma. Gamma was the second derivative and we have basically vega. vega was the rate of change of that portfolio with respect to standard deviation.

(Refer Slide Time: 00:33)



Now we are considering theta which is the change of the portfolio with respect to passage of time. So, this is also called the time decay of the portfolio theta is usually negative for an option as we had discussed. Because as it comes to with all remained things remaining constant option is generally tend to be less valuable as time approaches comes to t k to its expiration.

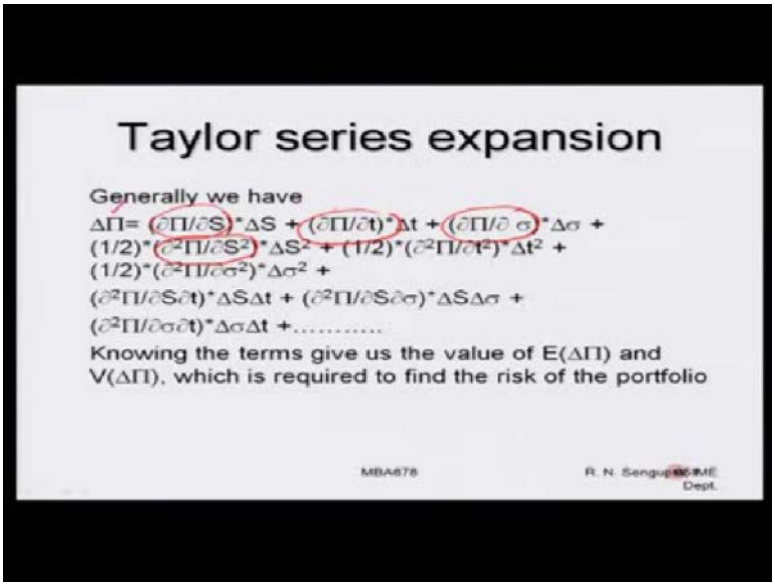
(Refer Slide Time: 00:57)



Rho is the value of the portfolio which is rate of change of the value of the portfolio with respect to level of interest rates. So, initially again I am repeating first one with respect to spot price second one would also with respect to spot price was the second derivative. Then it was respect to standard deviation. Then it was respect to the time and now, it was respect to the interest rate.

So, if we have currency rate option then we have two interest rates; one for the domestic currency country and another for the foreign country. So, if you remember the domestic and the foreign currency with respect to the share plus r and r star and based on that we basically formulated different swaps. So, here also row would be the rate of change of the value of the portfolio with respect to interest rate.

(Refer Slide Time: 01:36)



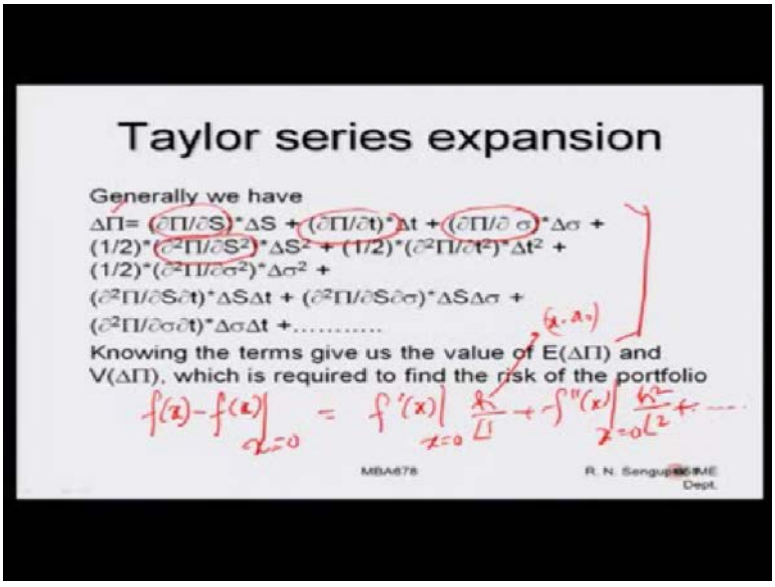
Now, in general, if you consider all these five or six rate of change then how would we express the rate of change of the portfolio in totality? So, if you check the rate of change of the portfolio it will be with respect to the rate of change of the portfolio with respect to spot price which was basically, the first one was the delta. Then we consider this is the second one which was basically, the gamma. Then if we consider that we have the vega. Then we consider if we have the respect to the time and then we consider we had with respect to the interest rate. Interest rate, I have not drawn it here; let me check. No I am not showing interest rate.

So, basically, it can be the interest rate also can be drawn. So, if you see this is the rate of change of the portfolio with respect to spot price; multiply it by the quantum change. This is the rate of change of the portfolio with respect to time; multiply with the quantum change means quantum change on that variable. Rate of change with respect to standard deviation into the quantum change; this is the second derivative. This is basically a second derivative. This is the second derivative, but the second derivatives are with respect to spot price with respect to time with respect to standard deviation. This is basically again the second derivative, but the second derivative being taken two different times by two different variables; first with respect to spot; then with respect to time. It could also been with respect to time and then spot. This is with respect to spot and standard deviation. It could also have been with respect to the standard deviation and the

spot. Then again with respect to standard deviation and time and it could also have been with respect to time and the standard deviation.

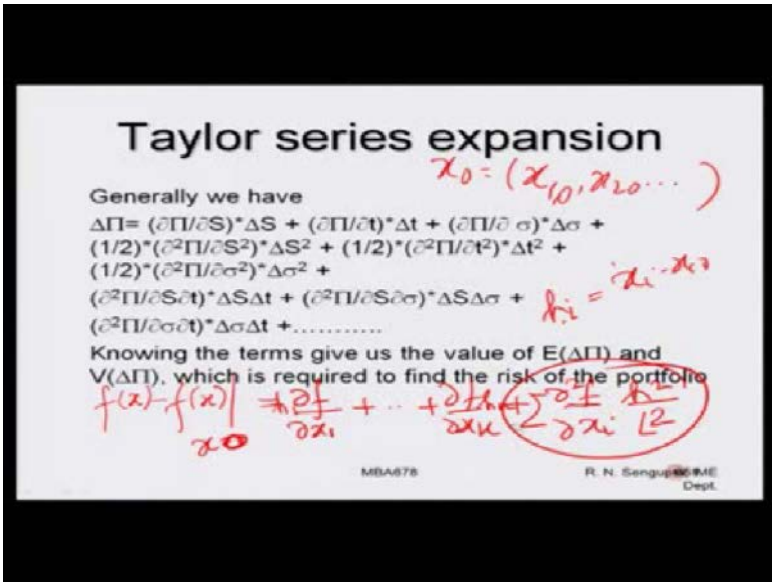
If you see this, this should remind you about very famous equation or expansion which we have done in class may be in class 11 or 12. This is known as a Taylor series expansion, but here the Taylor series expansion is the multivariate case. Now if you consider, so, I will come continue the discussion of finance later on, but first let us take the deviation and considering the multivariate deviation. At the first those univariate Taylor series expansion; then expand it to the multivariate one. So, if f x was the function and I want to find its change f x at naught; this would be given by the derivative of this function with respect to x naught with h, by factorial one where h is technically given by x minus x naught. Then you have basically f double prime x at x naught into h square by factorial 2 and continues still.

(Refer Slide Time: 03:56)



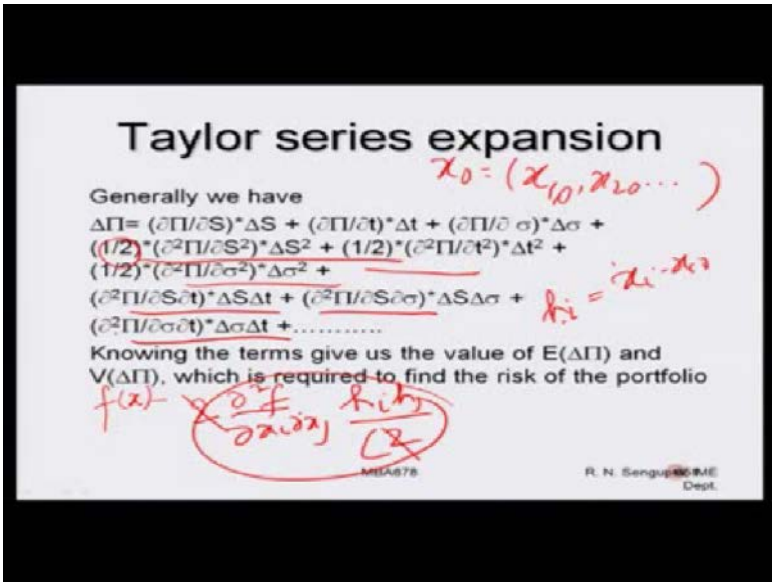
Now if I consider, so, this is the univariate case; remember. So, now to draw the multivariate count, so, let us delete this. Let me delete this also. So, it will be easier for me to scribble on this slide. Yes. Now for the multivariate case what you will have. Again I am drawing fx; fx is now a vector; not a scalar. This x naught, sorry x naught is also for the case where it is a vector. So, x naught is this. The first variable, 0 value; second variable, 0 value and so on and so forth.

(Refer Slide Time: 04:57)



This would be equal to the rate of fx with respect to x 1; x 1 is the first variable plus all the other variables till x k and the next one would be; so, this is these are all the first derivatives; first derivatives with respect to h 1 and h k where h i is equal to x i minus x i 0. So, these are the h case and the next one would be basically, second derivative with respect to x i h square factorial 2 and they would be k number of times. So, h are different. In the first case when x 1 is 1; it would be x 1 minus x 1; not second case when it is x 2; it would be x 2 minus x naught and so on and so forth. Now here the overall terms are not finished, because if you see the second derivatives can be taken also in this respect. So, this would be when, the spot which I am just highlighting. So, for the time being, consider this is already there as I do not have space, hence I am writing. So, this would be, let me delete this also. So, this is already there; consider this return, but let me delete it.

(Refer Slide Time: 06:55)

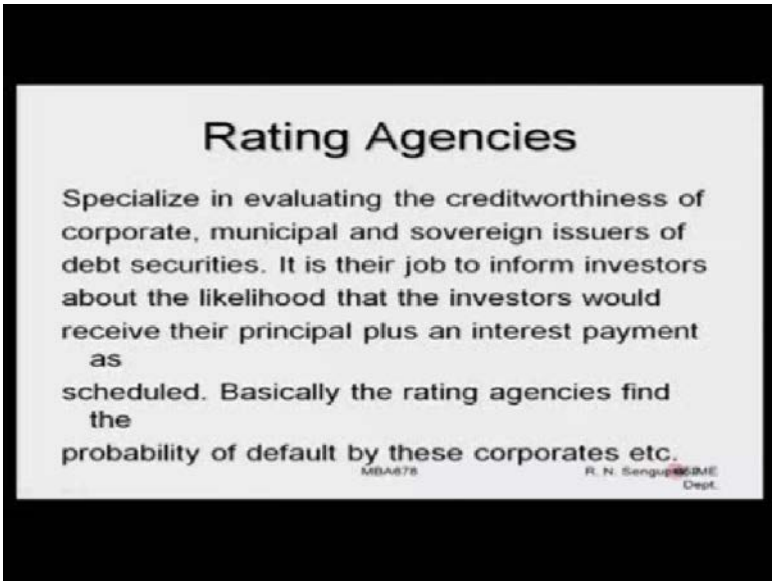


So, the second term would now continue is del 2 f del x i del x j h i h j by factorial 2. Now is x i and j are the terms say for example, x 1 x 2 or x 1 x 3 or x 1 x 4 and so on and so forth, and this h 1 i and h j are say for example, h 1 h 2 where h 1 would be x 1 minus x 1 naught and h 2 would be x 2 minus x 2 naught or else h 1 h 3 would be x 1 minus x 1 naught into x 3 minus x 3 naught and so on and so forth. So, if you consider these; these terms are this. So that means, if you take in either sense like x 1 x 3 or x 3 x 1 in this case they would be repeated twice. Hence in order to take into that action you have taken the value would be 2, but we are basically ignoring it, because if there is 2 here this 2 2 will cancel. Hence there is no 2 coming here, but if we consider only the first terms there is factorial 2; this is factorial 2s are this. So, if you do the simple multivariate Taylor expansion this is also which you have.

So, you can find out in any simple text book. This I thought, I will just make you clear. Let me delete it and then continue discussing. So, knowing the terms these are the full values or the expected value of the rate of change of portfolio and variance with portfolio which is required to find out the overall risk of the portfolio, which we are going to consider in one or two classes. Now we will just change; not change the overall discussion of quantitative finance, but would go into a different topic which is known about the rating agencies. Rating agencies; how they do the rating? What are different types of concepts which are doing by the rating agencies? So, it will be much less

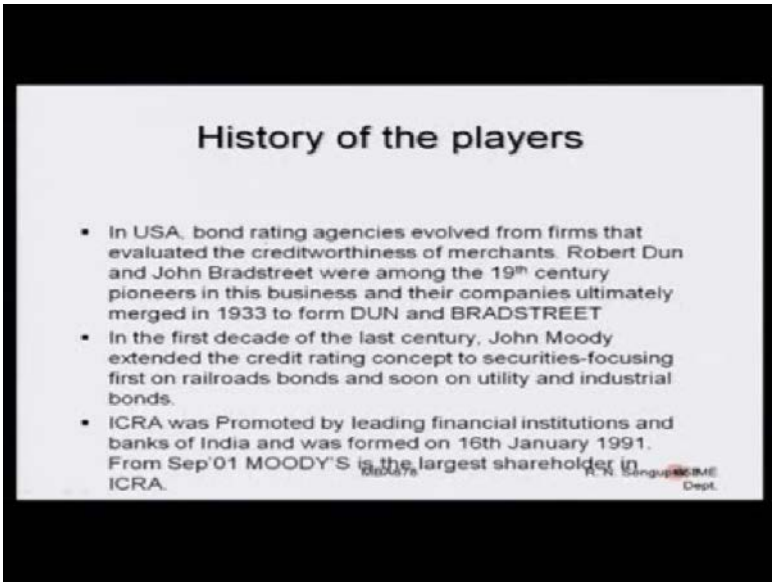
theoretical, but it will give you a good flavor that how the rating agencies are doing their work in very general sense.

(Refer Slide Time: 09:31)



So, what are the rating agencies we know? We have heard the name of Standard and Poor's. We have heard the name of Moody's. We have heard the name in India; we have the CRISIL in India. We have the ICRA and so on. So, these are the rating agencies about which we will very briefly study. So, they are, they specialize in evaluating the creditworthiness of corporate, municipals and sovereign issuers or some type of portfolios or financial portfolios which has been formed. It is their job to invest, inform the investors about the likelihood that the investors would receive the principal amount plus an interest net payment as scheduled depending on the overall change of the risk. Basically, the rating agencies find the probability of default by these corporates, etc. in order to inform the investors about the overall level of risk investors are facing.

(Refer Slide Time: 10:03)



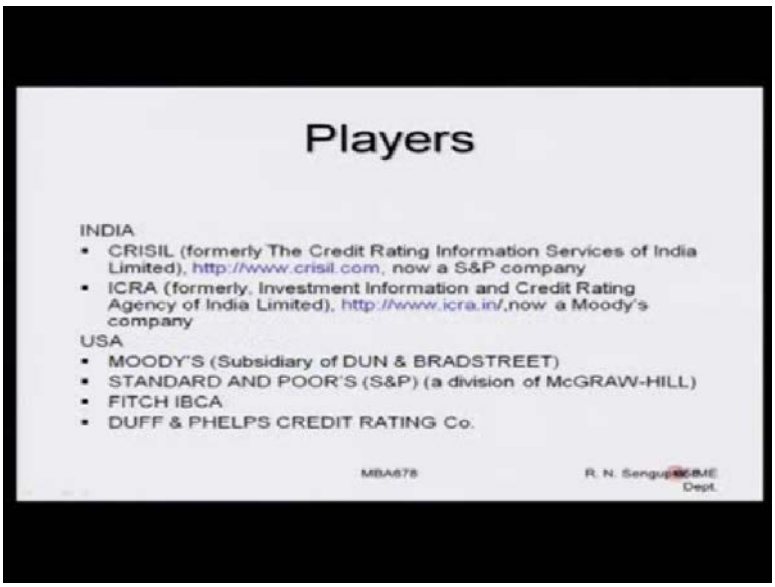
In US, bond rating agencies evolved from the firms that evaluated the creditworthiness of merchants and Robert dun and John Bradstreet were among the 19th century pioneers in this business and their companies ultimately, merged in 1993 1933 to form the company Dun and Bradstreet, which we have already heard and we know.

In generally, if you read the papers very history finance, its historical development if you check any good book you find all these details there. In the first decade of the last century, John Moody extended the credit rating concept to securities, focusing first on railroads bond and soon on utility and industrial bonds. ICRA was promoted by the leading financial institutions and banks of India and was formed in the 16th January 1991. From September 1 Moody's is the largest share holder in ICRA, and for CRISIL it is S and P.

(Refer Slide Time: 10:56)

Credit rating norms have been successfully developed by the two most celebrated rating companies; one is the Moody's; one is the Standard and Poor's.

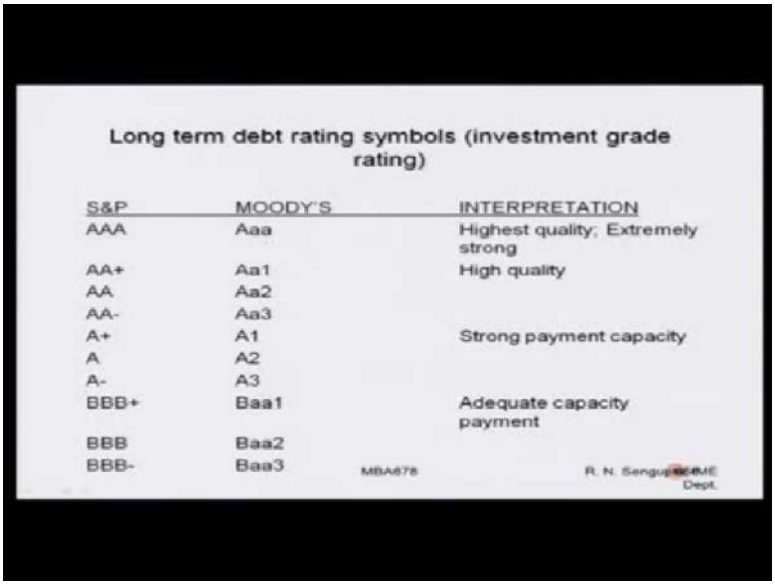
(Refer Slide Time: 11:05)



In India we have the CRISIL, formerly the credit rating information services of India and now, S and P Company. ICRA, formerly investment information and credit rating agency, is now a Moody's company. In US you have the DUFF and PHELPS, FITCH IBCA, Standard and Poor's, Moody's. Now Moody's is the subsidiary of Dun and

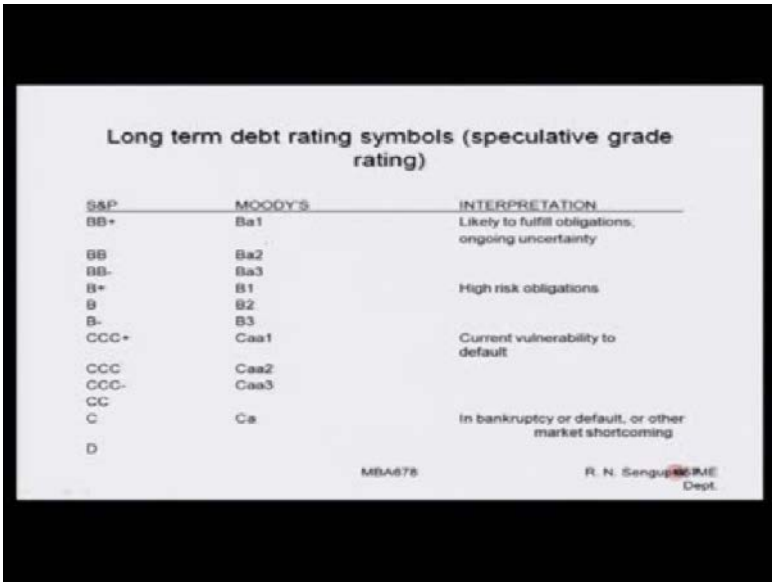
Bradstreet and Standard and Poor's is the revision of McGraw-Hill. McGraw-Hill is the publication big company which you all know.

(Refer Slide Time: 11:32)



So, the symbols, first let us go through the symbols and then I will come to the actual details of how the analysis is done in the mathematical sense. So, if there are different ratings which are given, if it is triple a, double a plus, double a, double a minus on the first column and in that, so, they are basically corresponding to the S and P, and the corresponding symbols which are being used for the Moody's are also triple a; a, capital a and then two small a es, in place of three capital a es. Then you have double a plus; it is basically aa 1 then aa 2 and so on and so forth, but in general the characteristics which they portray are almost the same exactly the same. If you interpret in the quantitative sense the first block would be the highest quality extremely strong and they less likely default. Then you have the high quality, whether default level has slightly increased. Then you have the strong payment capability where the default is still there and is slowly increasing.

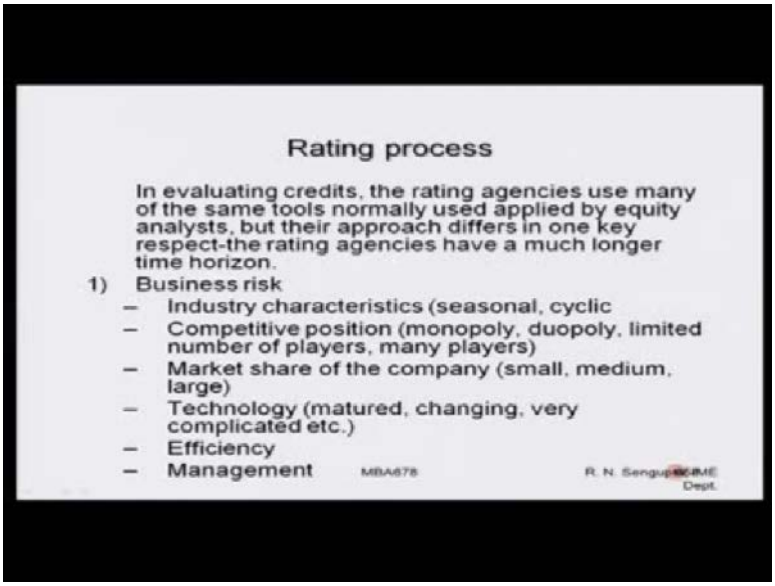
(Refer Slide Time: 12:32)



Then you have adequate payment capacity where now the concept of default has really increased and if you consider again, continuing that you have the S and P and the Moody's, again in the first and second column given by bb plus or correspondingly, it is ba 1; then bb which is bba 2 so on and so forth.

Then it goes to ccc plus for S and P; it goes to caa 1 for Moody's and the last one, which we have and the S and P is triple c, triple c minus, double d and the corresponding things are caa 2, caa 3 and ca. We see the interpretation. Basically, now here the level of confidence is decreasing; level of risk is increasing. So, that means likely to fulfill obligation and ongoing uncertainty is increasing. Then it becomes highly risk obligations are there. Hence the level of default has really increased. There is current vulnerability to default hence the probability of failure has increased and the last case; there is bankruptcy, which means everything which has been invested in that type of companies may, with a high probability, almost 100 percent probability, will fail.

(Refer Slide Time: 13:35)



So, the rating process; how it is done? I will give you very qualitative field without going into the detail, because all these things are under the purview of accounting concept. So, if you understand the accounting concept very clearly then understanding these concepts in the sense that how the rating agency utilize, goes using the fundamental analysis of the characteristic of the company.

If you remember, I had mentioned that fundamental analysis is one of the principal characteristics to understand the actual fundamental principle, based on which how the company is performing and the fundamental concept would actually, give you the information of how the stock market is dealing. I did mention also that there is other areas of study which is basically, studying of charts which is known as technical analysis which we would not consider. So, our discussion was going on in the area of credit rating. So, if you remember we were just in few minutes back, we were mentioning about different types of risks, corresponding to different levels of ratings. So, it can be double a, triple a, double c, triple c, whatever it was. So, now, what is important to know is that we did mention in one of the classes before frequently, and will definitely mention in more details.

What is needed is basically, how do those risk project or they change; change from set 1 to set 2. Set 1 to set 2 means that how do they change from, say for example, from double a to double b or double b to triple c and so on and so forth. What is important to

note that given the distribution or given the probability at any one stage, you want to find out what is the conditional probability that will change from stage 1 still stage 2. So, this is a big branch of studies in the area of, say for example, credit ratings. You would not be going into details, but will still cover the concept of hypothesis in a very simple way and how the transition probabilities are done in the later classes. So, with this, I will try to end this class today, and continue the discussions of credit rating and other things in later on.

Thank you.