Quantitative Finance

Prof. Raghu Nandan Sengupta
Dept. Of Industrial and Management Engineering
Indian Institute of Technology, Kanpur

Module – 07

Lecture - 37

So, we have left the last class till the stage where we have given the robust optimization framework of one of the very simple problem, where it first trying to minimize the risk subject to some constraint. This term is greater than some r p star; some of the weights is 1 and so on and so forth. Now, we also showed you how the risk return frame work graph would look like on an individual basis.

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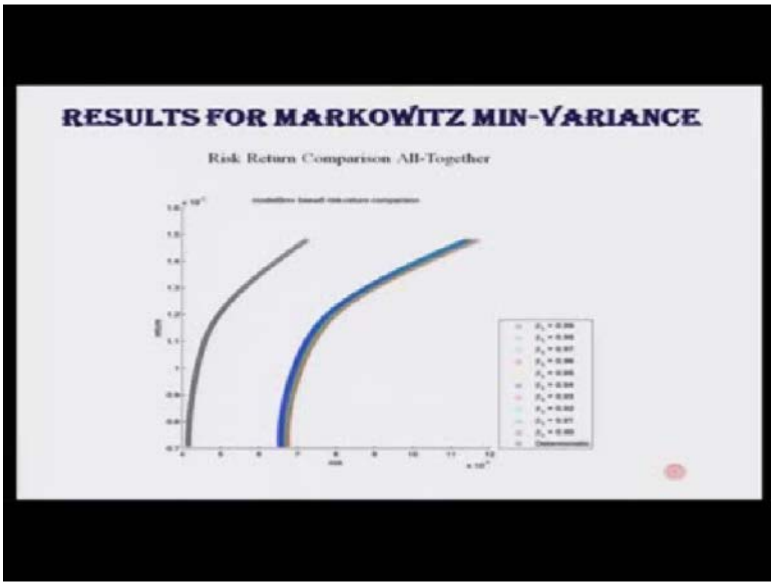

Now, once we compare them and overlap put one of that top of the other, considering different values of beta for the BSE one, you will see a very nice diagram is coming out and which is actually, what it actually, implies by the theoretical sense. The left most one, which you see is the deterministic one, which is the one, where we consider there is no concept of reliability; this one and this graph is here. Now, if you combine all the different values of reliability, considering this light blue one is for 90 percent; the light green one for 91 percent; bluish green one for 92 percent; so called violet color, light violet for 93 percent; a little bit dark blue for 94 percent; yellow for 95 percent; dark violet for the 96 percent; bluish green and different hue for the 97 percent; greenish one, this green and this green are different, for the 98 percent and the brown one for the 99 percent. If you merge all of them you will see that the top most envelop is for the level of beta, which is 90 percent and the lower most one is basically, for the 99 percent. Now, the diagram which I am going to draw will make a very good sense that what we mean by reliability. It may not be very much evident from the graph, but it will give you a lot of information of what the actual concept of robustness and reliability is.

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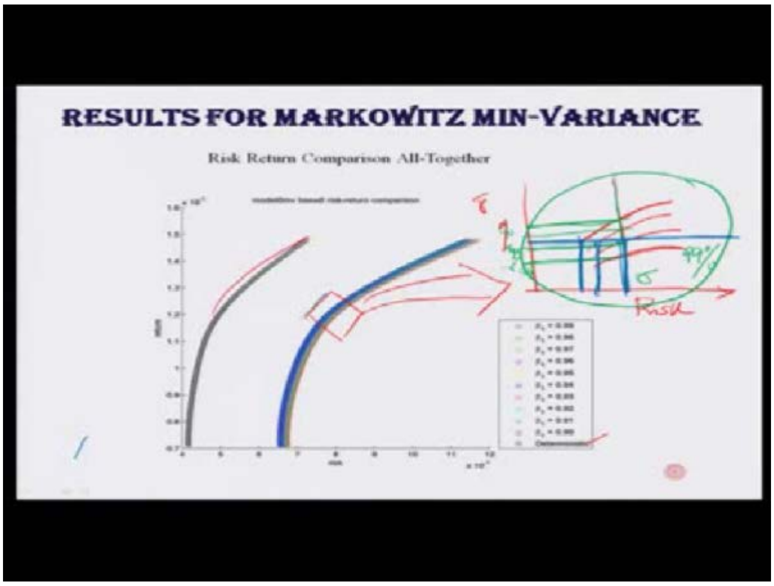

So, let us take this small portion. Look at it very carefully. If you see and if you basically, zoom in; that means, look at this carefully. The graphs are like this. If you look at it carefully, it is exactly like this, where the top most graph which we have is for the 90 percent and the lower most one which is basically, for the 99 percent. Now, you will be asking the question, whether that is true or not. Answer is yes; why? Let us consider as per this one; the risk is here, which is standard deviation or variance and the return is here. Now, for a higher level of reliability; that means, more concerned to or more reliable you want to be, more non deterministic level of high dependency you want to being taking the structure; less would be the output with respect to either the risk or the return. So, now you may be asking the question, why? So, let us draw a vertical graph first. In order for our concept to be true, let us change the color.

So, let us draw this vertical line and then, after the vertical line is drawn, let us draw these horizontal ones. Now, for the same level of sigma, for a value of 99 percent, which you have to be more reliable qualitatively, and in a very common sense wise, you will definitely say if you want to be very highly reliable for the same level of risk, your return would be low and exactly, this is what it is. Your return would be a value of r 99, which is at this level. Now, consider using then the reliability; that means, you are now, being not being that strict. It is 95. If you draw horizontal one, this is the level of return for a 90 percent 95 percent. For same level, you are not at all that level of high intolerance. You are little bit relaxed.

Now, this is 90 percent; that means, for a same level of risk your level of risk would change, depending on the reliability level and the concept was which you solve in the sense of theoretical sense and the practical sense match. Now, consider the picture from the other way. Consider again, little use in the same graph. If possible let me do it in a other color fashion. So, now, let us draw horizontal from here and so, in this horizontal one, all the return is fixed. Now, as you change the level of reliability; obviously, your level of risk would change; return is fixed now. Let us consider; for a level of 90 percent risk is here; here is risk, 99 percentage; that means, to maintain the same level of risk.

If you are more reliable then; obviously, your level of risk would be, for the same level of return, if you have to be more reliable, then your level of risk would increase. For a lower level of reliability, for the same level of return, your risk is low, which means that it actually, address the fact that for a higher level of reliability, to maintain the same level of risk, you have to basically have a higher level of returns and exactly the opposite should be true in the sense; for the same level of return to maintain different levels of reliability; you will see that the risk would also change accordingly; that means, for higher levels of reliability, risk is high. For lower levels of reliability, risk is low and exactly this is what is being portrayed in the diagram. If you zoom in again, I am saying the blue, green, violet, red and all these things would now make sense with respect to all the reliability which you draw.

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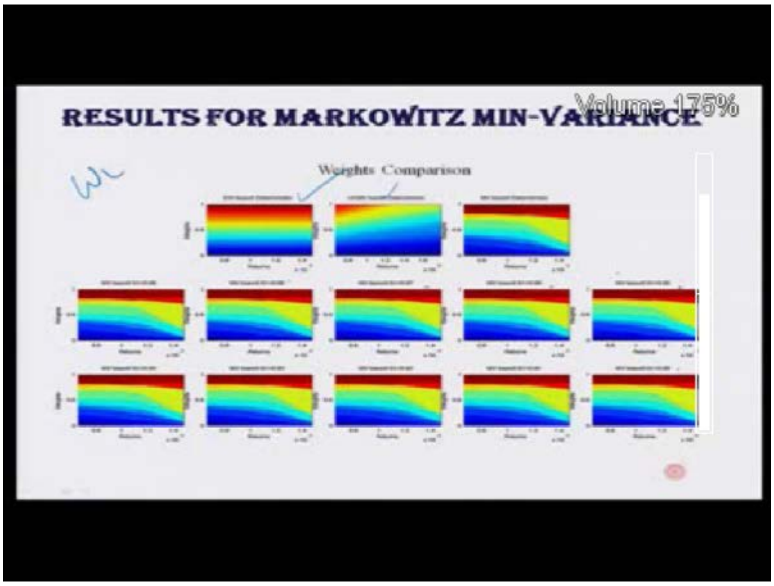

If you do the weight comparison; that means, the wi's which you have found out; then for the deterministic case, for the case of the same for this BSE, for the 99 percent, 98 percent, 97 percent, 95 percent, 90.6 percent so on and so forth 90; you will find different weights and using the weights, we have been able to draw the risk reliability rates that we turned, as shown in the last slide.

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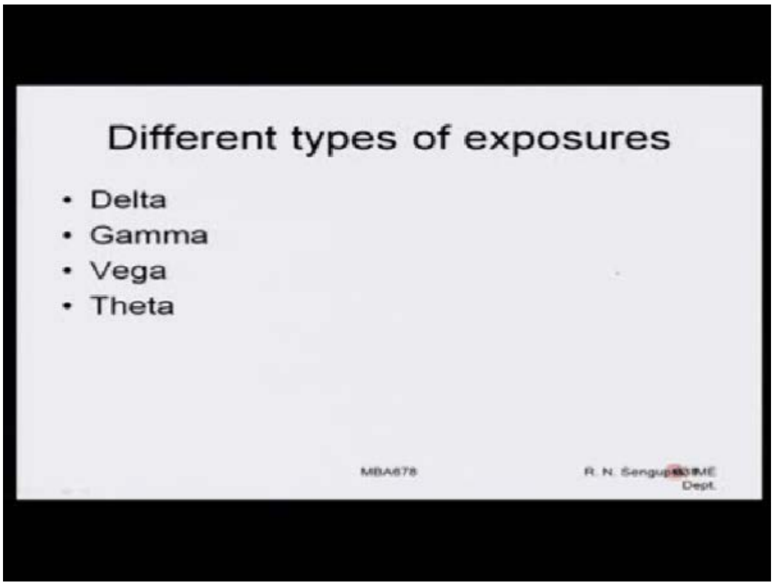

Now, I will pause here. I will basically pause in the sense that I will come back to the simple explanation of the solved problems later on also, but considering that we have few other important topics to cover, I will consider and continue the discussion using different type of risk exposures which we have. So, the risk exposures are in the mathematical sense, they are denoted by delta, gamma, vega and theta and higher levels of risk, depending on what is the level of sophistication you want in order to understand the concept of change of an a price with respect to the change of the portfolio and what is the level of risk it will sustain, depending on these changes.

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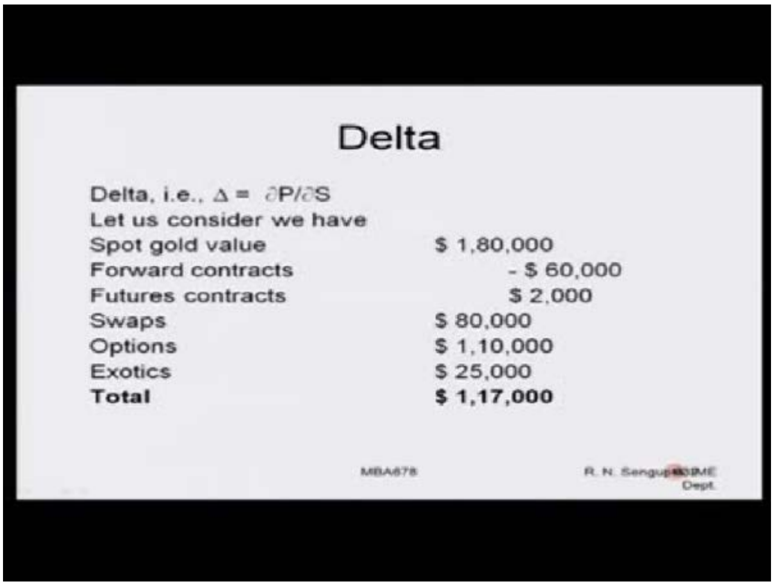

So, what is delta? So, consider the delta is in a very simple way, the rate of change of portfolio value with the rate of change of the stock price. That is the only thing. So, consider this is a simple way, putting the first derivative of the portfolio with the rate of change of the port of the stock price. So, let us consider the spot value of the gold is given; the forward contracts are given; future contracts are given; swaps, options and exotics are all there, and the total value is given as 117000 dollars.

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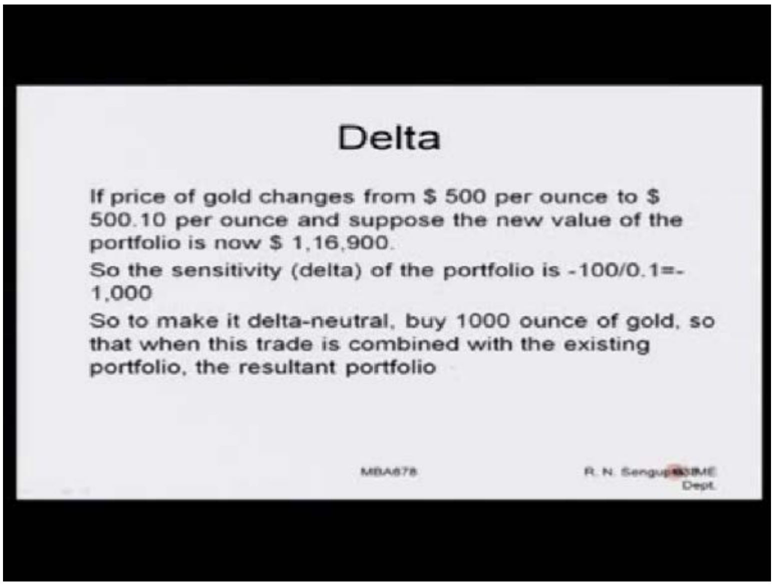

Now, if the price of gold changes from 500 per ounce to 500.10 per ounce and suppose, the new value of the portfolio is now, 116900. So, it means initially it was this. Now, it becomes basically, 116900; that means, 116900. Now, the gold price has changed 500 per ounce to 500 to 500.1. So, the sensitivity of the rate of change of the portfolio or the delta of the portfolio would be what? Rate of change of the portfolio value, which is what? It was initially, 117000. Now, it has decreased to 116900, which is minus 100, which is the numerator and the price of gold has increased from 500 to 500.1.

Hence, it will be divided by 0.1, since that delta value of the portfolio comes out to be 100. It means for rate of change of the stock price of one of that particular constituent of the total portfolio, the rate of change of the portfolio would be 1000, with a minus sign; obviously, because it is decreasing with respect to the increase in the gold price. So, make it delta neutral; that means, that there is no change in the overall value of the portfolio, considering the delta point of view with the rate of change. You buy 1000 ounce of gold so that when the trade is combined with the existing portfolio. Due to the change in the price of the gold, the exact value of the portfolio again, goes back to the same value, which means there is no rate of change of that particular portfolio with respect to change in the value of one of the inputs, which is in this case, gold.

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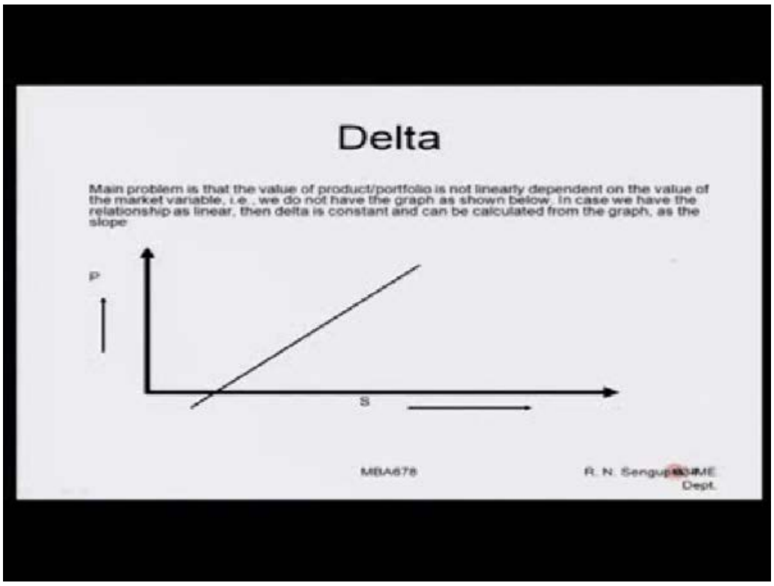

Main problem in the delta is that in the value of the product portfolio is not linearly dependent. So, whatever the calculation if we did by minus 100, divided by 0.1, is exactly a liner ratios, but they actually it is not that; not linearly dependent on the value of the market variable; that means, do not have the graph as shown below; that is not a straight line. In case we have a relationship as linear, then delta is constant and can be calculated from the graph as the slope. Technically, it is changing.

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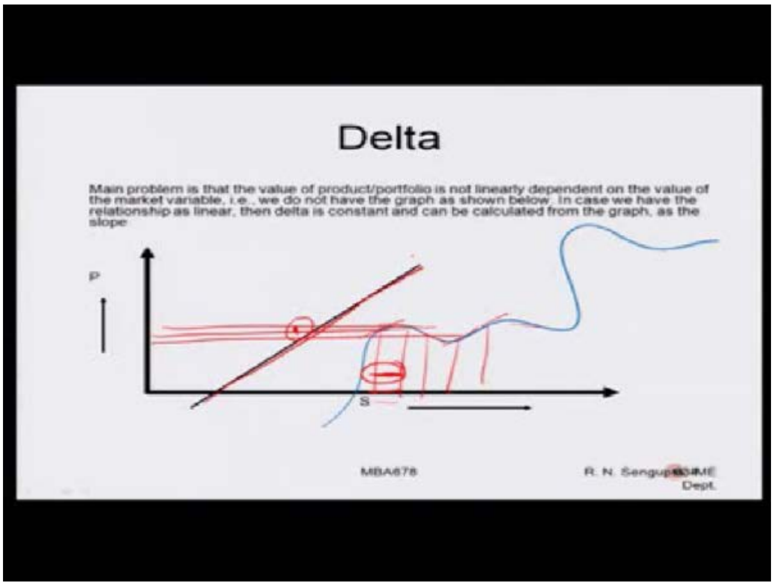

The value of the portfolio is changing like this. So, if I want to find out the delta of the portfolio and it is changing, then what I would do is that I will find out the rate of the change of the curve as the value moves, along with the rate of change of s. So, s has been here; then s has moved here; then s has moved here and so on and so forth. So, as it is moving, the value of the portfolio is also changing, either increasing or decreasing. So, for the change of the rate of the portfolio which is here for denominator, the value of the portfolio is now changing or decreasing by this value. So, it will be negative. So, this value, divided by this value, will give you the rate of change of the portfolio, which is delta for the first instant, but if as you see which keeps changing; that means, dy dx is changing in a very simple sense. Here it is being considered that the rate of change of that value of the portfolio, the rate of change of the stock price such that the delta would always be constant.

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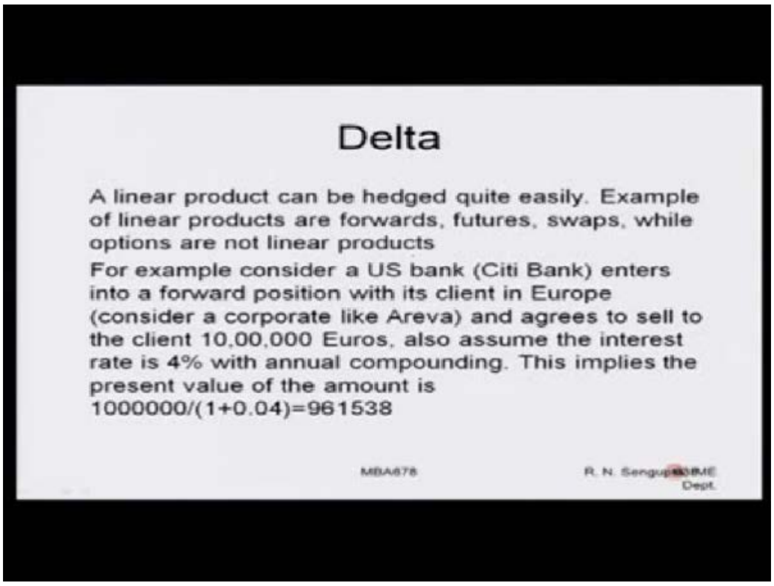

A linear product can be hedged quite easily, because we know the linear relationship, hence it will be easier for us to hedge. Examples for linear products are forward future swaps, while options are non linear products. If we remember the option price, where the options values would be exercised, depending on whether the max of some value is 0 or non zero, or whether somebody was making a profit or loss. So, if you consider that point of view then; obviously, it is a non-linear function. For example, consider US bank; Citi bank enters into a forward position with its client in Europe, considering a corporate like Areva, and agrees to sell to the client, one million Euros; 1000000 Euros. Also assume the interest rate is 4 percent with annual compounding. This implies of the present value of the amount is given so; that means, it agrees to sell the client one million at that point of time.

Now, what I want to do is that I want to find out the present value. I will immediately divide it by 1 plus that percentage interest rate. In the case if you remember, it was a e to the power of minus rt. So, this is continuous compounding. Here, we are not considering continuous compounding. There is no problem in that. If the actual interest rate is given with annual compounding; that means, being compounded one time; we can easily find it as quarterly compounding, continuous compounding, using this formula which we have already discussed in the interest scenario cases. This is just a simple example to denote the concept of delta and there, you find out the amount of the value of the portfolio. So, change of the value of the portfolio with respect to the change of the price of the input, which is s and then, you can find out the deltas.

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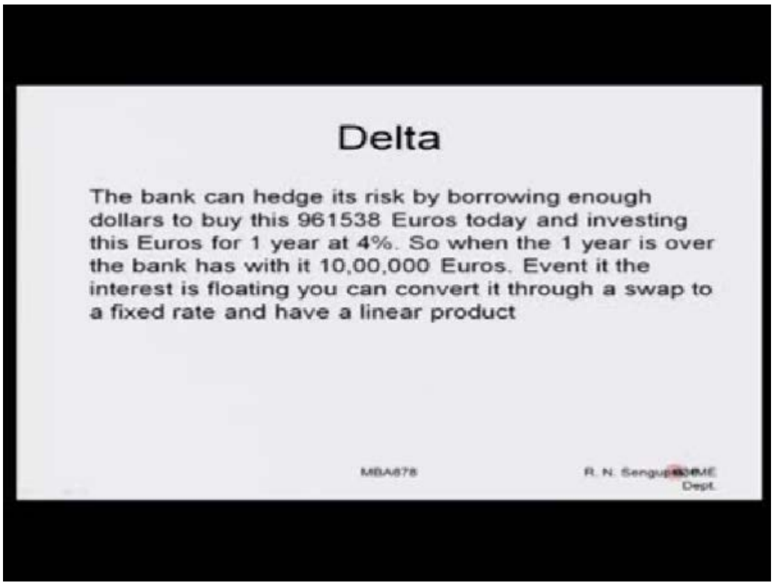

The bank can hedge its risk as we are considering by borrowing enough dollars to buy, to buy this 961538 Euros today and investing this. This is the present value which you calculated and investing these Euros for a one year at 4 percent interest rate. So, when the one year is over, the bank has, with its one million Euros, as calculated at the rate of interest for 4 percent. Even if the interest rate is floating, you can convert it through a swap to a fixed rate and hence, a linear product can be easily formulated. Linear means linear rate of change of the prices.

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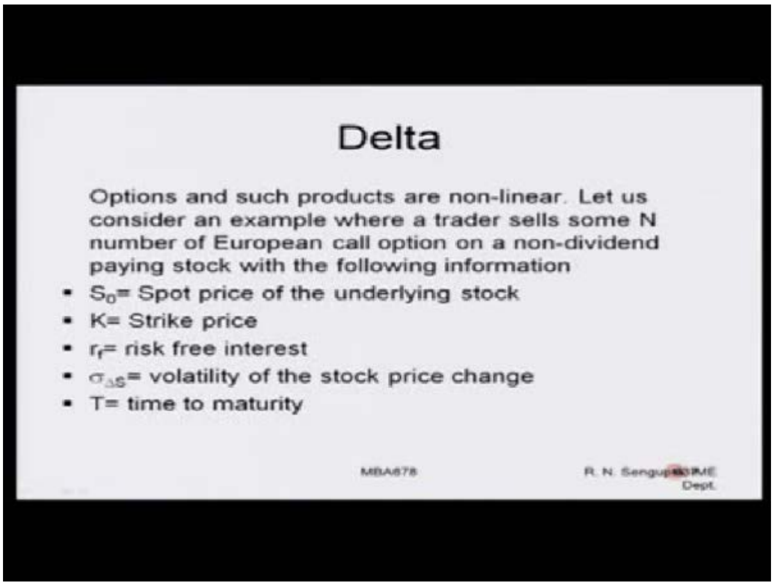

Options and such products are non-linear. Let us consider an example, where a trader sells some n number of European call options on a non-dividend paying stock with the following information. What are the information; s 0 is spot price; k is the strike price; rf is the risk free interest rate; sigma delta s is the volatility of the stock price rate of change; t is the time to maturity. So, the relationship in the non-linear case it would be like this.

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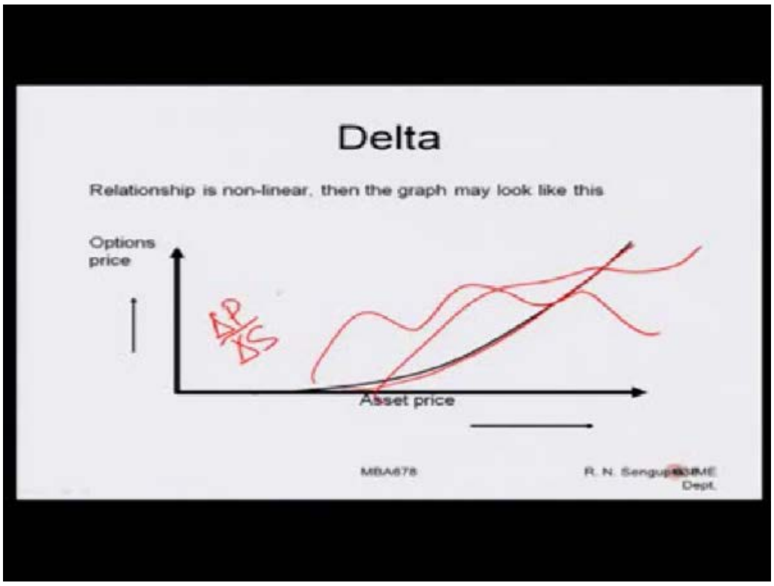

I am just drawing an arbitrary curve. It could be like this also. It can be like this also. Whatever it is, you can find out the rate of change of the cross portfolio value with the input price and that will be given by the delta value. It can be either linear or it can be rate of change can be changing, depending on how the change of the portfolio happens with the rate of change of the spot price.

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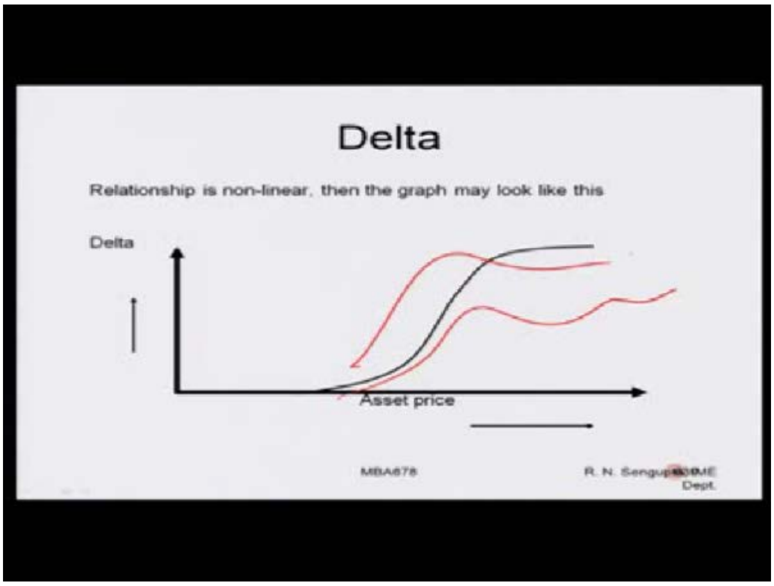

In a really relationship in a non-linear case, is this one or this one, whatever you consider it is; whatever it is, you have to basically find out the best fit curve and find out dy dx of this curve.

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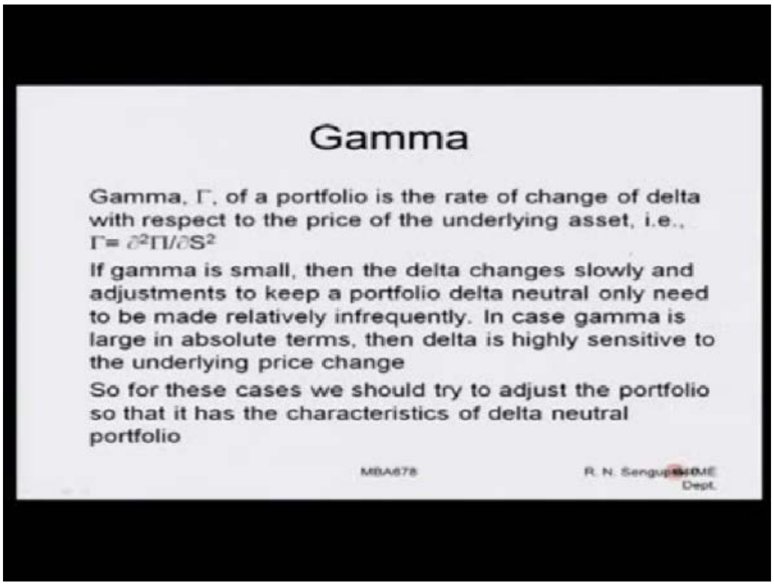

Now, we have considered delta. What is gamma? Gamma is simply the second derivative. So, if delta was basically, the first derivative, gamma is the second derivative which means gamma is also the first derivative of delta. So, if say for example, delta is given by del p by del s, gamma will be given by del 2 p by del s 2 or it would be given by del delta. Then the second delta is for the delta we have just considered, divided by delta s. So, gamma of the portfolio is the rate of change of delta with respect to the price of the underlying asset as noted down here.

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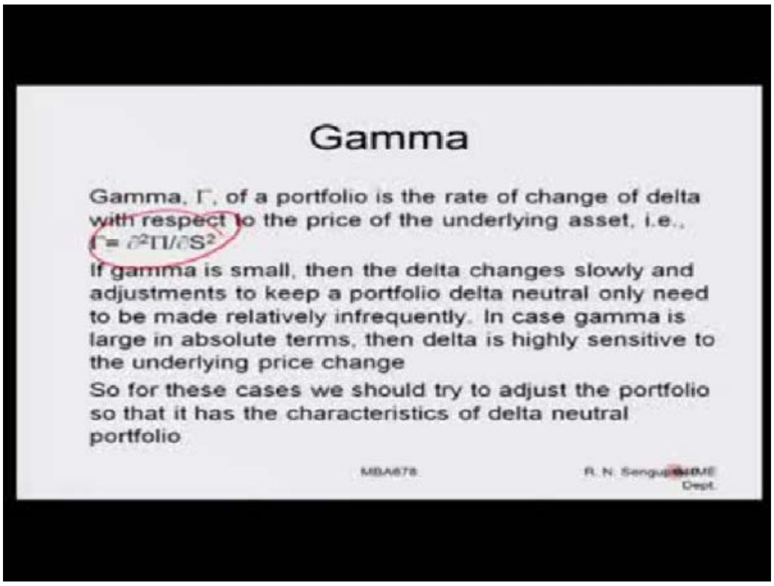

If gamma is small then the delta changes slowly, and I will just going to keep about fully delta neutral only need to be made relatively in frequently, but if gamma is very big, which means the rate of change is happening very fast such that your risk gamma delta neutral concept, which you are trying to bring into your portfolio such that such readjustment has to be done, has to be done very fast. In case gamma is large in absolute terms, this delta is highly sensitive to the underlying price change. So, for these cases, we should try to adjust the portfolio such that it can has the characteristics of the delta neutral portfolio as already discussed very briefly.

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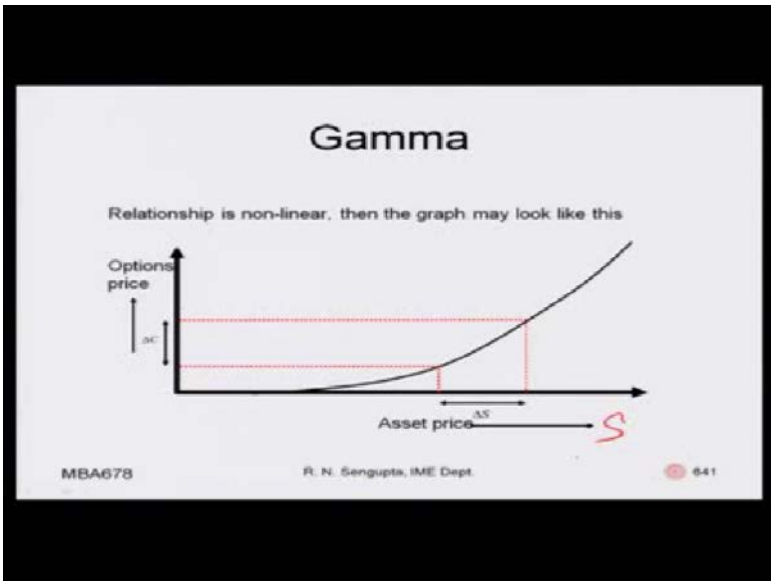

Relationship in a non-linear graph; then the graph may look like this. For the gamma case, this is delta s of the change of the spot price. So, this is basically s which you are measuring and along the y axis, you have the option price. So, delta c is the option price. The rate of change of this graph would be given by the non-linear characteristic of the gamma curve.

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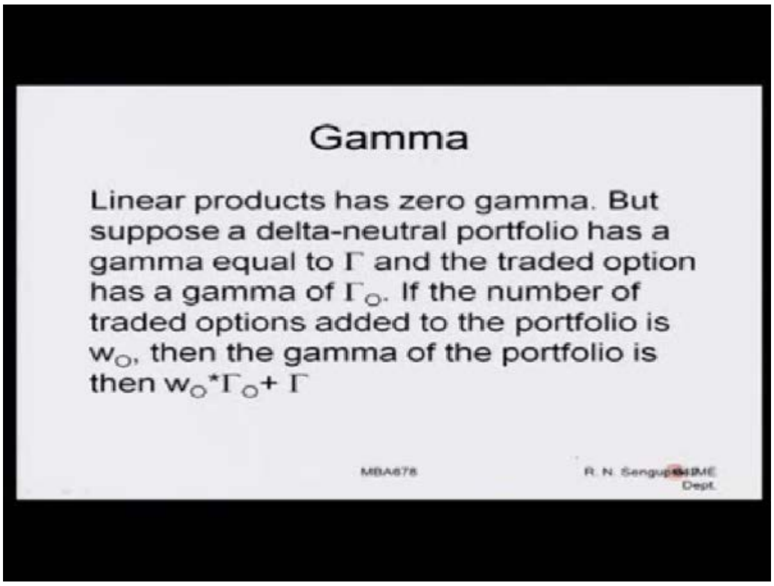

Linear products has zero delta, but suppose a delta neutral portfolio has a gamma equal to delta and gamma equal to gamma and the traded option has a value of in a gamma of gamma naught, if the number of traded option added to the portfolio is w naught, then the gamma of the portfolio is given by w naught into gamma naught, plus gamma, because this gamma value which you have, is the delta neutral portfolio has a value, the gamma value is given by this and if you are trading in n w 0 number of such this stocks or options that w 0 is the total quantum of that considering the total price, then multiplying by gamma would basically, give you the overall gamma of the portfolio such that if you have three different such weights and you have three different gammas.

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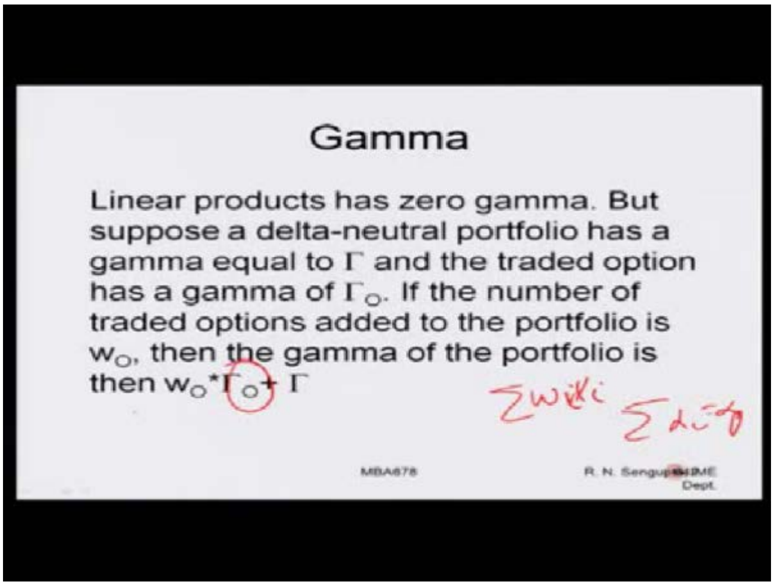

In the same way as you did here or as you did here; first is equal to alpha p. In the same way, you can find out the overall gamma of the portfolio by multiplying the weights into the rate of change of that particular gamma on a for each and every stock which you have, which means this value which we have is basically, partial derivative of the rate of change of the value of gamma of the portfolio, corresponding to the stock which you have. So, say for example, there are ten stocks; you will basically have a partial derivative of all these ten stocks and then, try to analyze whether you want to add some stock or whether you want to take out some stocks in order to make your overall portfolio as riskless as possible.

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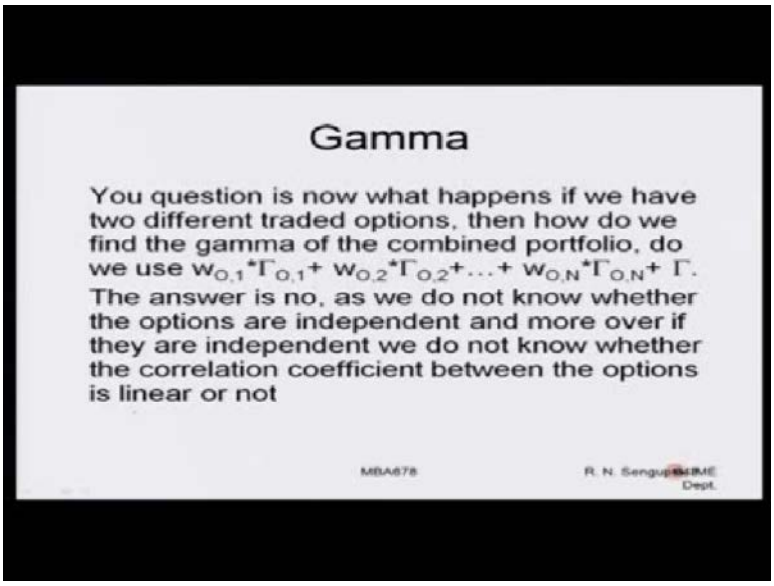

Your question is now, what happens if we have two different traded options as I mentioned. So, basically this is exactly the case we discussed; w naught 1 gamma naught; w not 2 gamma 2 naught 2 so on and so forth. So, considering all of the rate of changes which is happening for each and every stock plus the gamma value of the original value; that will give you the rate of change of the overall portfolio, which you are trying to formulate.

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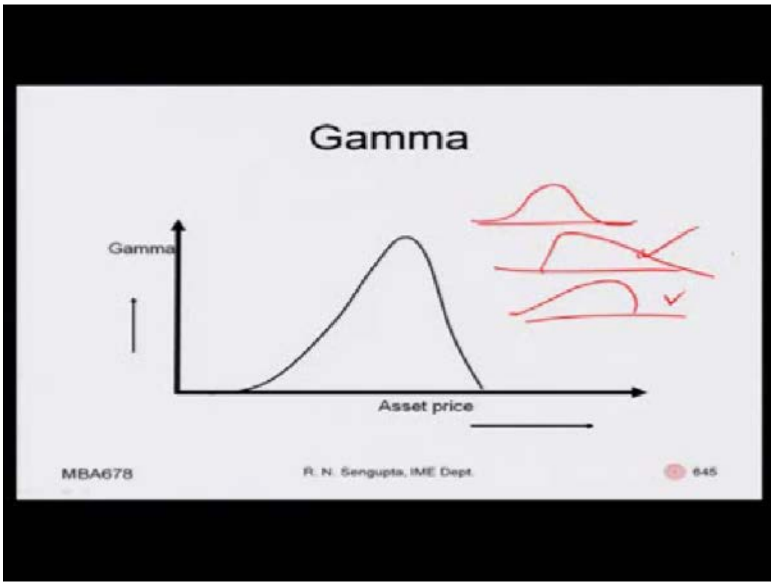

So, gamma is given in a very simple graph. This is not normal. I have tried to make it normal in order to understand with the normal distribution, but in general, this is not. So, the asset prices are given along the x axis and the gamma values are given on the y axis. Similarly, if it is skewed; so, in this case initially, it was normal distribution. Now, you have basically left skewed or right skewed depending on whatever distribution you are considering, but generally the distributions of skewed to the right of the skewed to the left, would depend on what type of returns you are taking. If you are considering only the positive returns or only the negative returns, your graphs would be corresponding to the fact, whether you are using the portfolio's loss or portfolio's gains, as the main variable, based on which you are trying to draw the diagram.

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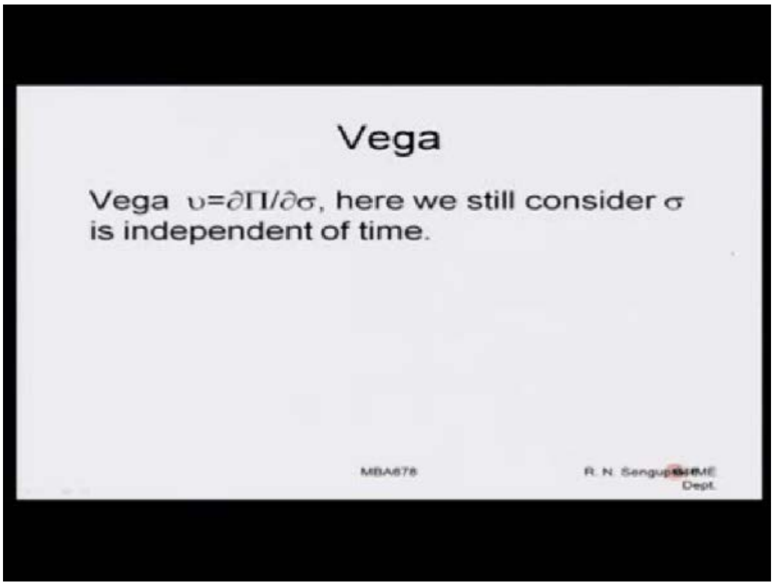

Now, let us come to the concept of the vega. So, initially we considered gamma; then you considered, we have considered delta; then you have considered gamma. So, delta was rate of change of that with respect to spot price; then is the second derivative with respect to those spot price. Now, we come to consider vega. Vega is basically, simply is change of the portfolio with respect to the standard deviation. So, here is the first derivative. So, initially it was the spot price and it is basically, standard deviation of the rate of the change.

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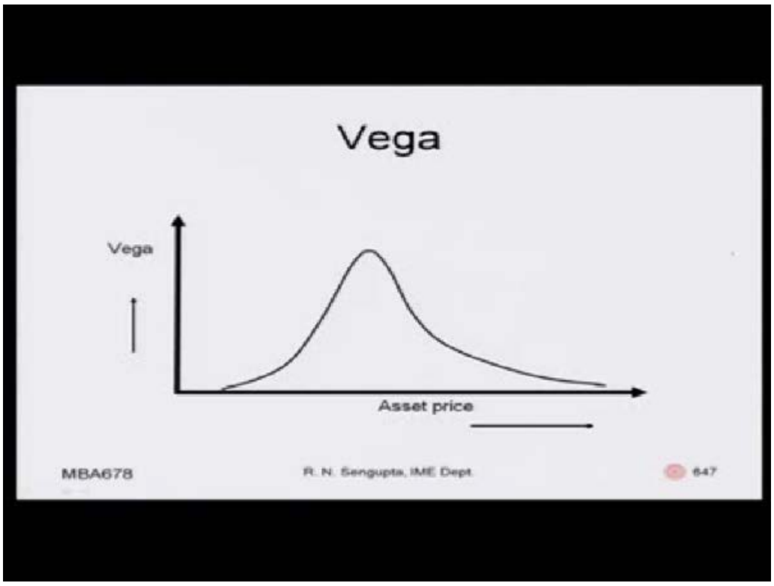

So, again we consider vega. Asset price is shown along the x axis, and the y axis we have the vega. This is just a simple theoretical construct or a hypothetical diagram such that depending on whether you are trying to formulate your total portfolio, considering only options or only stocks or combination of them, you can basically have a left skewed or a right skewed or a non-linear one, and that this graph which you have in front of you gives you some feel that how does the value of the portfolio change with the standard deviation; how the value of the portfolio changes with the spot price; how the rate of change of that particular second derivative of the change of the portfolio change with the second derivative of second value of rate of change of that s; all we have considered in a very simple term. One was basically, delta; then gamma; then vega.

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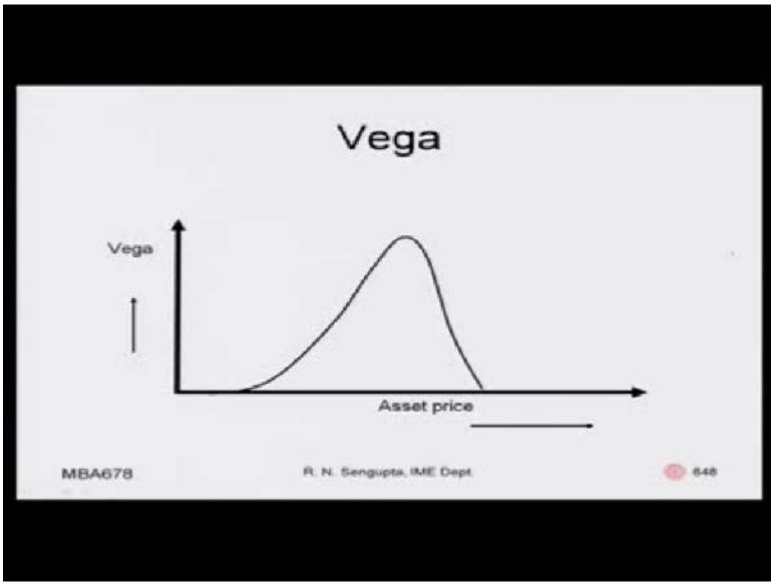

So, this is the left skewed one, depending on how you are being able to draw it.

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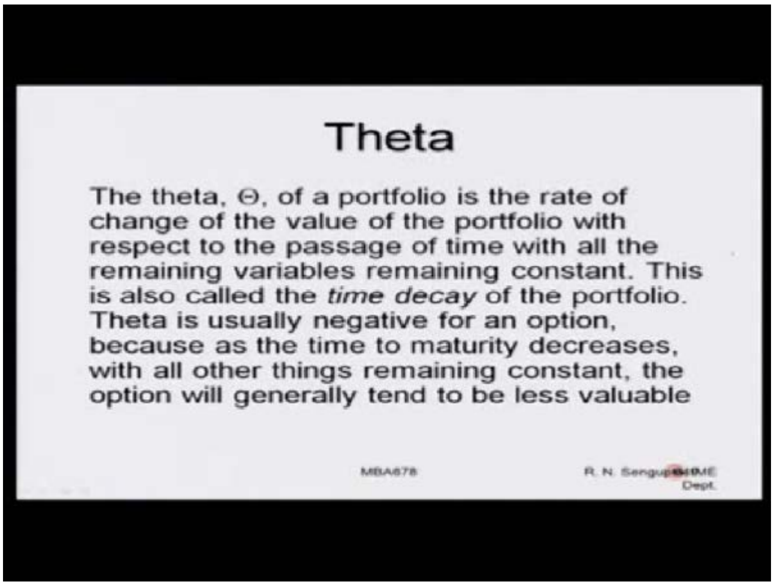

Now, consider the theta. So, we have considered delta; we have considered gamma; we have considered vega. Now, let us come to theta. The theta of the portfolio is the rate of change of the value of the portfolio with respect to the passage of time with all variables, remaining constant. So, the first case of delta was the rate of change of with respect to s; then the second one was basically, rate of change of the second derivative two times; then third case of the vega would the rate of change of the standard deviation. Now, we are considering the rate of change of the portfolio with respect to time.

So, this is also called the time decay of the portfolio. Theta is usually negative for an option, because as the time to maturity decreases with all the other things remaining constant, the option will generally tend to be less value. Hence, the value of the overall portfolio will decrease. So, now, what you are trying to do; if you remember clearly, that in the case of the Black-Scholes model, we had considered some drift rate; we have considered some value of external noise. So, in this drift rate was basically, dependent on spot price and time. This was also dependent on spot price and time. The same way, we are considering in portfolio. So, portfolio is now dependent on spot price, is dependent on secondary derivative spot price, is also dependent on sigma; it is also dependent on time. So, we are slowly trying to bring all the variables which are important in order to understand how the portfolio value and its risk change.

Thank you.