Quantitative Finance

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Module – 06

Lecture - 33

Okay. So, continuing our discussion of the Wiener processes and we did mention that in the Wiener processes what are the assumptions and the flagrant. I did also mention that General Wiener Process will be utilized and I did mention that what are the terms which can be included. So, let us continue the discussion with General Wiener Process.

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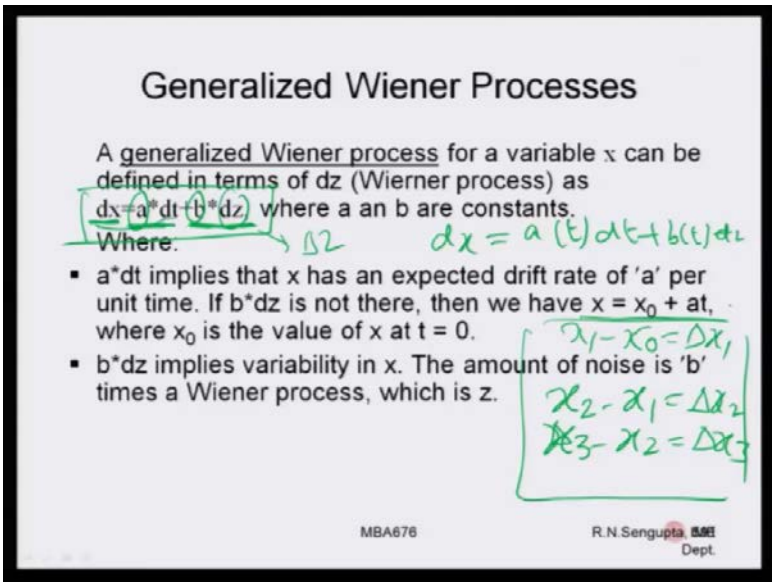

So, generalized Wiener Processes for a variable x, that means z or x whatever; it is a different variable. I am trying to utilize of the different symbol. But, conceptually here is a same can be defined in terms out d z which is the general very simple wiener processes. So, now here a Generalize Wiener Processes will have the terms d z was basically delta z which you already consider which is the simple Wiener process. Now, that would have basically 2 terms: the rate of change of d x would basically be consisting of 2 parts; 1 is a constant multiplied by the simple Wiener processes plus a constant multiplied by the rate of change of time. Now, just before I proceed these values of a,

these values of b which we are taking in this case there are a simply constant. It could have been, that the rather than a and b we could have had a which basically a function of time into d t plus b which is a function of time into d z.

So, we are not going to make our life complicated so, we will you continue on discussion. Where, d x which is a Generalized Wiener Processes is now functions of 2 terms; 1 is delta t multiplied by constant and another is the simple wiener process multiplied by the constant. So, now what does these 2 terms on the right hand side mean? In the first terms which is a into delta t implies that x is an expected grief rate of a.

Now, why is it I am saying and expected drift rate. Now, if you take d t down so, what you will have? You will d x d t is equal to a. So, what we are trying to find out is the rate of change of x function with respect to time is basically constant a that is why it is known as the drift rate; expected drift rate in the long run. So, the expected drift rate of a per unit time that d t is basically per unit time. If b into d z is not there then we would basically have the equation which is x equal to x 0 plus 80. So, x 0 is basically starting value. So, if i want to find out d x and considering that the first value of the difference in the first instant is delta x 1, it will be a basically be x 1 minus x 0 is equal to delta x 1 and the basically. So, if you want to find out the second 1, it will be basically x 2 minus x 1 is equal to delta x 2. If you want to find out that third 1, it will be x 3 minus x 2 is equal to delta x 3. So, we will have x equal to x 0 plus s 8 where, x 0 is the value of x at time t is equal to the 0 which would be known to us; as of today we know the value.

By the second term which is being is it implies the variability. So, now when I want to find out the expected value on the variability of x or delta x it would basically have 2 terms. If I take the expected value, it would be given with some function of a where, a is basically the expected drift rate; while the amount of noise would basically be given by b such that, using the amount of noise b which is the external forces we can find the external effect, we can find out what is the variability of the whole Generalized Wiener Process.

Now, if you notice I return now this equation somewhere, where given x 0 plus 80, I can basically go interactively and find out this value. So, that will come out later. On that should give you a hint that given the starting value how you can stimulate. Now, you may be thinking that so stimulation could give us whole lot of information. The answer is

yes because if you able to simultaneous infinite set of times than we will be able to find out that in which direction the prices would be moment that would give you a intuitive feel that how the whole process works.

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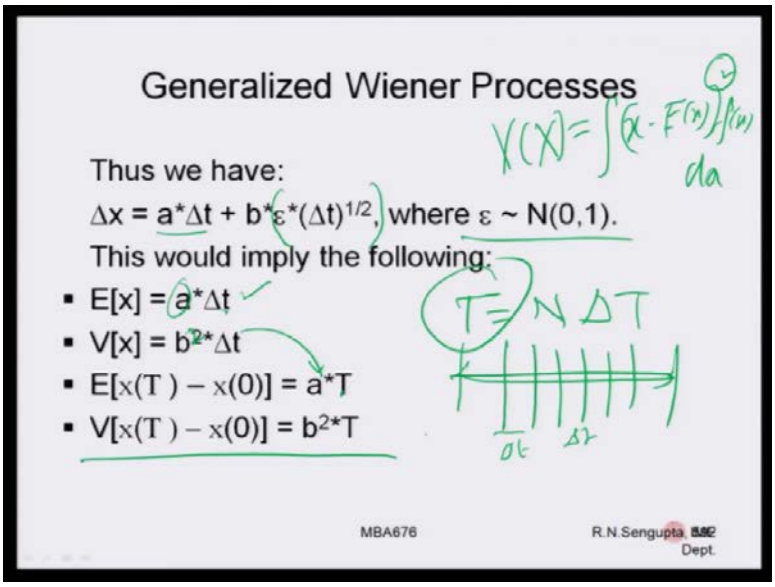

Thus, we would have that if you consider the Generalized Wiener Processes. So, initially you would differenced had a into delta t than in the second terms you had b into delta z. So, now this is delta z which is being replaced by epsilon not 2 square at all delta t where, epsilon 0 1. So, if you if you continuous this and if you try to find out the expected values, the expected value of x would be a into delta t where, this is the discrete per unit time. Varies would be given b square into delta t obviously, 2 b a squared because if you remember the formula initially which we are d1 basically x minus ex delta f of x d x. So, this is the squared term which is basically be implied by b squared.

The difference in the expected value of the expected value difference what we have is basically x t minus x 0 would be given again by this. But, remember 1 thing this was delta t and this is t and also remember this was true which is the number of search which you have small deltas and if you count n number of them that would give you the overall time length. Again I am drawing. These are the deltas, these are the deltas and there are n number of them; add of all of them you get the capital T. And again, the variance of the different of x capital T and x 0 would be given by the same formula which is b squared. But, here rather than delta it would be capital T, again capital T is n into delta t.

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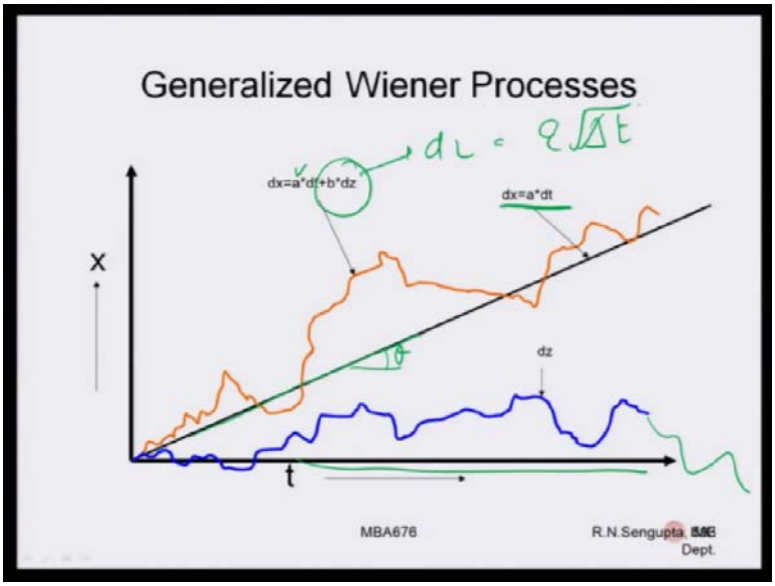

Now, I am trying to basically draw given this. This is if you remember, this is Wiener Processes simple Wiener Processes where the expected value was 0. So, this is the Wiener Processes which is going like this. So, the average values always 0 and if I consider the Generalized Wiener Processes with the drift and the white noise than you can see this would be given by the rate of change of the drift. So, if you find out the time of theta that would have functional value which you related to a, a is here. So, this is what is being given t x equal to a plus delta t and the variable t would be give by this term. Where, t z have basically express as epsilon in delta t. So, you to utilizing this you can formulate very simply the Generalize Wiener Processes.

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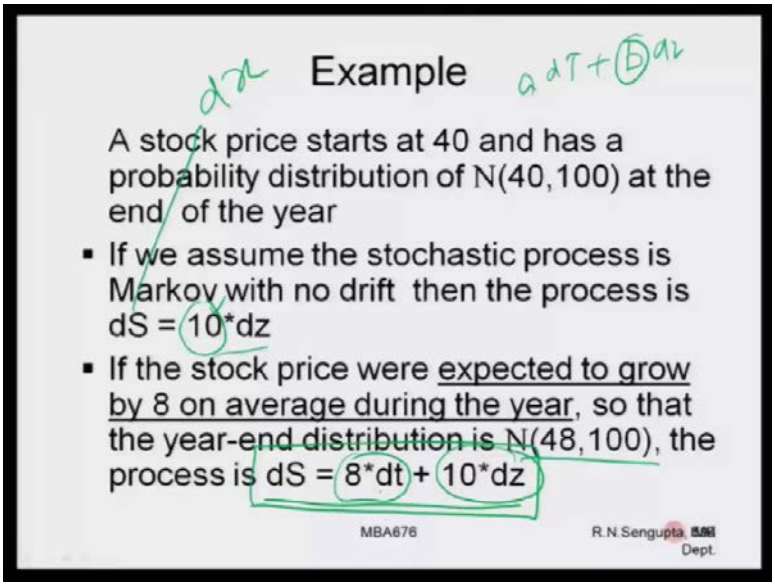

A stock price starts at 40 and has a probability distribution given by normal distribution of 40 as mean value is 100 at the fixed and the variance at the end of the year. If you are assuming the stock processes, the mark over process with no drift that means, a is not 0 then the processes would be given by s which is the sport price or the stock price is basically x which we are discussing. So, this is rather than d x we could happen d x also. So, this is now you are trying to utilize the symbol in order to make are understanding simple.

So, d s is equal to this is 10 would basically the factor which you already seen. So, if you see the formula, what you had is a into d t plus the b in to; say for example d z. So, this is the value of d which is 10. So, d s equal to 10 into d z. The stock price was expected to grow by 8 and average during the year. So, that the year end distribution would be normal distribution with 40 it does the mean and 100 basically at the variance. Now, you are seeing that why various is not changing. Because, the value of, I will come to that just was let me finish it. So, that the year end distribution is given by n 48 is the mean value 100 is the variance the process would now be d a x, is basically d s or d x whatever you are considering. This is the term which is the first terms which is a into delta t and this is the second term is which is basically be into delta z. So, now, depending on now the process is fluctuating, you can simply do like this. This I am giving in a radiating vary qualitative field.

If you see the equation, the left hand side is d s. So, you will ask yourself what is the question. What is d s? So, d s if you remember is the difference in the stock price. So, now the next question would be how do we find it? The answer is very simple. You know s 0 which is the stock price of today. So, note it down. So, let us note down s 0. Now, your next question is that what is 8? 8, if you remember is the average rate, which if you remember we have been discussing the rate of change of interest rate which is basically the 8 that so called r f.

If the stock was exactly equal to the risk free interest rate .They could be instances where the stock was better than risk interest rate then that interest rate will be greater than 8. And what is d t? d t is basically simply again the difference in the time. So, time as of now t is equal to 0; at the next moment t would be 1; next to next that would be t is equal to and so on and so for. So, when I am measuring time t is 1 2 3 4 correspondingly s which was s 0 at time t is equal to 0 will keep changing to s 1, s 2, s 3, s 4. So, the moment I mention that, it would be immediately become clear to you that d s or delta s would be now this s 1 minus s 0 which is the first difference; s 2 minus s 1 which is the second difference; s 3 minus s 2 which is the third difference and so on and so for.

Correspondingly, what is the difference in time? It will be 1 minus 0. 1 is not the numeric 1, it is the time unit. 1 minus 0 is the first difference on the time, 2 minus 1 is the second difference of time, 3 minus 2 is the third difference of time and so on and so for. So, we have been able to understand the first term on the left hand side and the first on the right hand side. Now, what you remains is to understand the second term on the right hand side which is 10 into delta z. So, now 10 was basically given as the white noise which will also calculate from the external sources as we did for the 8 which use the rate of change of the stock price from the historical data. Now, we will ask what for the d z. We already know it is a simple linear process. What is the equation? It is epsilon into square root of delta t. So, now will ask yourself, where do you find out delta t. We have just found out delta t for the first unit was 1 minus 0, 2 minus 1 is second unit of time difference, 3 minus 2 is third unit time difference. So, you are already have that. Now, the next question to answer is and the last step is that what are the inputs? What is the epsilon? Epsilon means already know is normal distribution.

So, given the picture of the whole equation delta s are the difference the stock price is and you know what is s 0 when you start. On the right hand side you know 8 as from the

historical data, you know 10 what from the historical data and you also know what is the time difference; 1 minus 0, 2 minus 1, 3 minus 2 so on and so for. You also know an normal distribution with the 0 mean and 1 variance on a standard deviation and you also know delta t, square root of delta t which will use into the linear processes. So, what you need to do is that you need to do very simulated very simply. How you do that? Consider, the time different is now, say for example delta t is 1. So, based on that fact that delta t is 1 you randomly generate 1 stand on normal deviate from the normal distribution. So, on the right hand side you have 10 into that normal standard deviate value which you are generated the first instant multiplied by the time difference square root of that, which was delta t square root. So, it would defiantly you will have it.

So, if the time difference was 1, it will be 10 into say for example, standard normal deviate is coming out to be say for example, 0.292 it will be 10 into 0.292 into square root of 1. So, this is the second term on the right hand side. What is the first term on the right hand side? You already have a 8, 8 would be multiplied by 1. And what was on the left hand side? It was basically s 1 minus s 0. You already know s 0. So, take s 0 on to the right hand side. So, everything on the right hand side is now known, now you need to calculate in find out s 1. So, once s 1 is find out again you will basically generate the data for the standard normal deviate, get the data for the time difference and basically try to generate s 2. So, you will go doing like this. But, remember one fact this is just a sample set of the realize value. If you do the experiment again it is highly likely that all the values you will generate would be, which is fine. So, what you are trying to do there.

So, if you keep generating all of them in infinite number of time like, if you keep tasking in coin infinite number of times, the average value which you get for a head, average value get for a tail or if you keep rolling on a rise for a in finite number of times; the probability of getting number 1; probability of getting number 2 or so on and so forth the one-sixth is the same way as you keep generating that you will get the average values which is what we are just discussed in the last 2 slides. That means, the average values, expected value of in the bracket z t minus z 0 is 0 or expected value or the average value of x t minus x 0 is 0. So, if you are able to do it infinite number of times you find out the values which was stated are actually what they are going to come in the simulation run.

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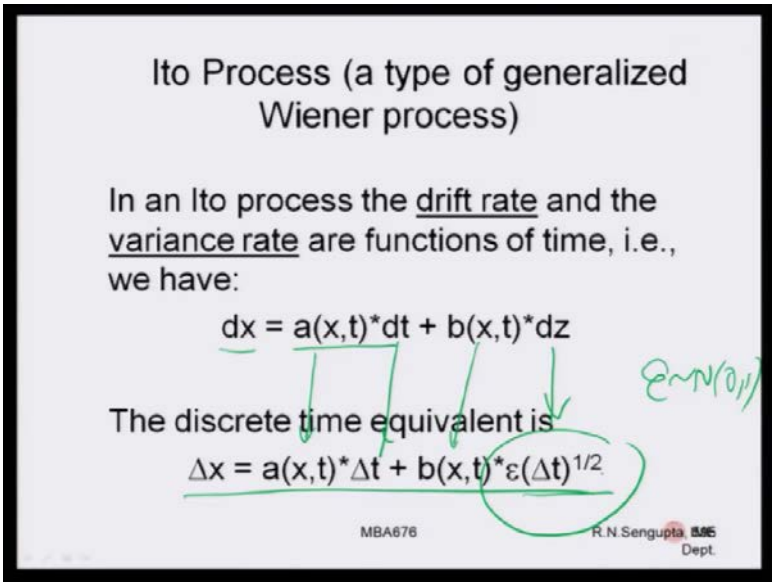

Now, let us basically proceed to the Ito cinema is a type of Generalized Wiener process. In Ito's cinema the process the drift rate and the variance rate are now functions of time. So, if you remember in 3 or 4 slide before I scratched and told you that a can be a function of time, b can be a function of time which is the average rate can depend on time and the white noise also depend on time. So, this is what it was Ito's cinema is going to consider. Here, now you will have d x, which is the different between x. Initially it was a into d t now it is a into x into dt. So, x is basically is where you are.

So, the drift rate will depend on 2 things; where you are and what is the time different based on which from where you are come. And the second term was what? b into d is a these a remains as it is as the simple Wiener Processes. But, now b is basically depended on 2 function which is basically x where you are and the time difference from various started various standing. So, the discrete time equivalent would be again given by this, which is delta x is equal to a function of x in to t into delta t. So, this is the equation and b x into which is function of x into remains as it is and the linear process now again expressed as an epsilon which is normal distributed with n 0 1, which is normal distribution of 0 mean and 1 variance are a standard deviation and delta t square root. So, again you will basically generate this data according to the same concept but, remembering that now a and b are not constant but dependent on 2 things time and place where the overall variable is being measured.

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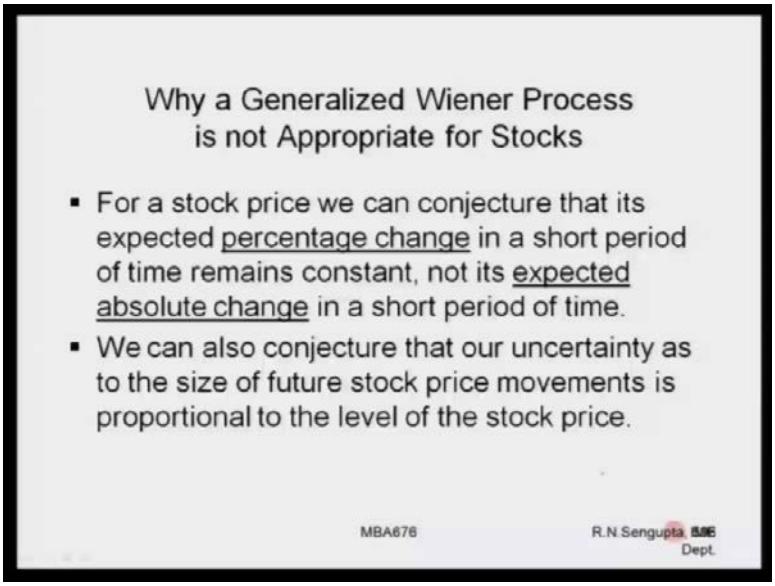

Why a Generalized Wiener Process is not appropriate for stock. For a stock price we cannot conjecture that it expected person which change in a short period of time remains constant, not its expected absolute change in a short period of time. So, what we are saying is that depending on the value that and the rate and the validity would change. Or the rate of change on validity change happens in such way that they are not exact replicas of the Generalized Wiener Processes or the concept of Ito's cinema but, the consensual frame work based on which we are trying to understand would remain the same. So, this would give you much better understanding of the stock market or the stock price is fluctuate. We can also conjecture that are uncertainty as the size of the future stock price moment is proportional into the level of stock prices. So, where we are today would basically have an effect on the voluntary later on. So, if the stock price is 10 and another instant for the same stock is 100 then the variability then the expected value v would also depend on the values of 10 and 100 from where we have started. Leave aside the time. Time is also effected. But, now it is being dictated at what value we started there. So, if the values are very low, if the values are very high it will have a positive or a negative effect on the overall expected value and variance.

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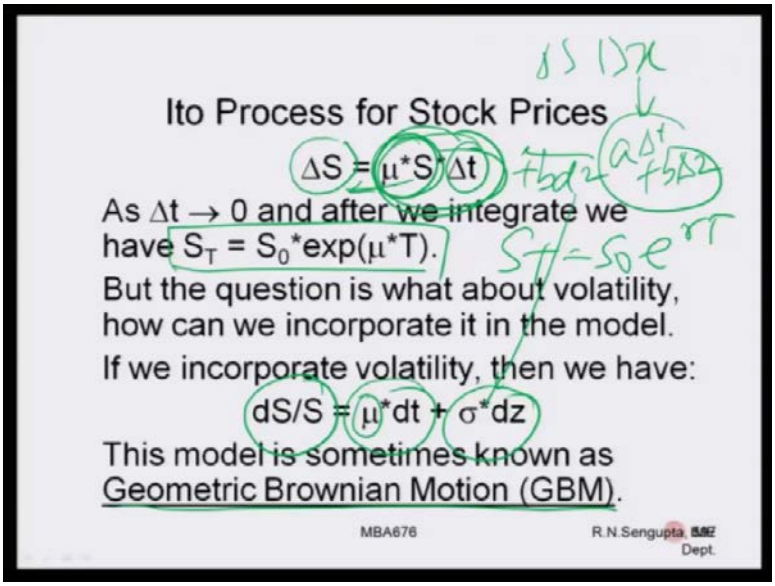

So, now it was process for a very simple stock market. So, if you remember that delta S or delta x, if you consider delta x; it had 2 terms a into delta t plus p into delta z. Now, if you basically expand this with respect to this delta x with the change as the stock price this is exactly what you have. So, this is basically a and this is delta t what you have and the second term is not being consider for the time begin. So, as delta t change to 0 and after integrative we can easily prove this is true. Which is exactly what we have done in few slide before and we have discussing time and again. Which means the smart price s t is equal to s 0 into e to the power r t. So, we have been mentioning r as the risk interest rate which was fine for our calculation. So, what should be remember that the s t which is the specific spot price of a particular stock would depend not on the risk free interest rate but, would depend on the mean value of the rate change of that particular stock prices. So, now we are trying to replace r suffix f with the corresponding rate of change or the continuous compounding interest rate or that particular stock.

So, it can be positive or negative depending on whether the price are increasing or decreasing. But, the question is that what as happened to the validity? As I mention, we have only considered the first part, the second part has been missed. So, if you continue considering the second part. So, here s was there, now bring s downward. So, what you will have? Delta s by s which is here, d s are delta as the time different what you are considering. Now, here it will be as positive is mu into delta t, which is here where mu is the rate of change of the mean value increased which is basically a. And the next term

was what? It was b into delta z. So, this is what we are considering. b is the, if you considering there was basically white noise. So, this is what is standard deviation on the stock into simple Wiener Process. Now, if you consider this whole equation is a partial difference and equation but, on the left hand side we have the ratio of rate of change of the stock price divide with the stock price as of now s is equal to 2 terms.

First is the mean rate which is mu into delta t, delta t is the time and mu is the rate of change of the functional form of the stock prices which is happening on a daily or a unit time basis. And the second terms is basically consists of two things which is the white noise which is now being replaced by the sigma which is the volatility or the standard deviation multiplied by the z which is simple linear process. This model is sometimes known as the Geometric Brownian motion. Brownian motion, if you remember Robert brown; who was basically starting the pollen gains and as light fell he saw them vibrating very randomly. So, based on that the concept of Geometric Brownian Motion came. So, now if you if you remember and if you can recollect we have been able to break down the overall Ito's cinema , Wiener's Process whatever the mathematical proof has been into 2 simple concept. One is the average rate which is mu into delta t and is the volatility which is the sigma into t z which is the Wiener Processes. So, now again the question; how we will calculate. We will try to answer that in the next class.

Thank you.