Quantitative Finance

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Module – 06

Lecture – 32

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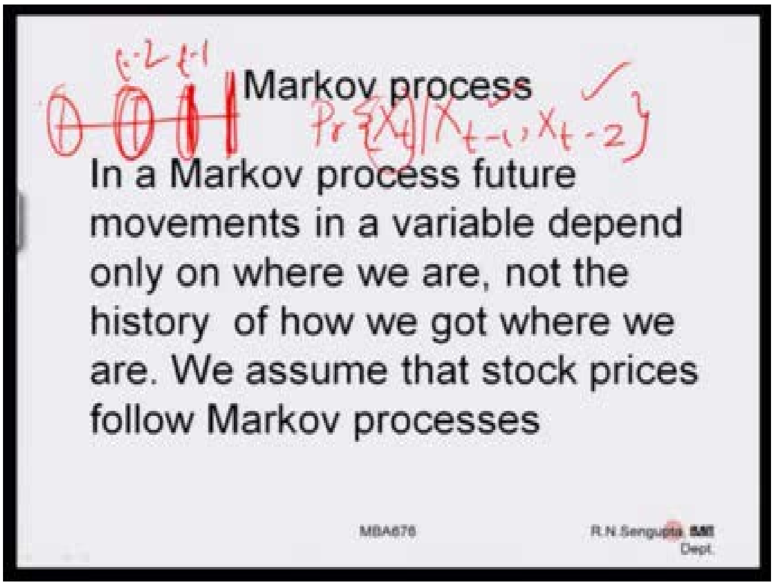

So, good morning, good evening everybody, hope you are fine and you are enjoying this course. So, as we were discussing that the last stages of the last class was basically the concept of stochastic process in a very simple manner, the Markov process and the Markov chain, and what are the implication for the stock prices. So, we are considering with repetition I am saying whatever the prices are as of today are only dependent on the last days prices or the last moment price. So, if I am basically trying to find out the prices at intervals of 1 day, then conceptually if I am I do remember that I have mentioned that the closing price would be taken as the price which basically predicts all the information which has been available throughout the day.

So, today's closing price would basically have all the information based on which you can predict tomorrow closing price. Then the tomorrow closing price would basically has would have all the information based on which you can predict for day after tomorrow price so on and so forth. So, this is only for first order; that means, 1 step backward. In case we are considering that today's and tomorrow's has all the effect on day after tomorrow then it will be 2 step process or second order and so on and so forth. So, in a Markov process movements in a variable depend only on where we are not the history of how we reach there. So, our main concern is what we are now and based on that we are try to product. If you consider the concept of probability theory very simply, it would be that the probability of X t that is of today would depend on X t minus 1. So, this is of first order. If you are saying that rather than this let me erase this. So, if I am saying this which means probability of X t or whatever the random variable is its value or price or whatever function we are trying to analyze, this would be dependent on the just immediate backward t minus 1 and back to back which is t minus 2, which is if I draw time scale this is basically t minus 1 t minus 2 then t would basically depend on the first and the second then t minus 1 would depend on t minus 2 and t minus 3. So, it will basically go 1 step at a time backwards.

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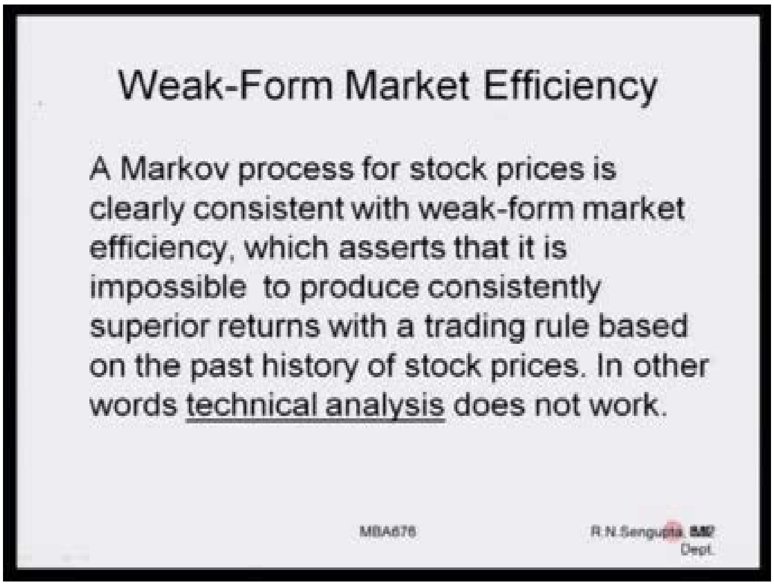

So, Markov process for stock prices is clearly consistent with weak-form of market efficiency which asserts that it is impossible to produce consistently same level of superior return, because the reason is very simple, if all information is available with all the periods in the market; obviously, they would be demand and supply which would basically either move up or down.

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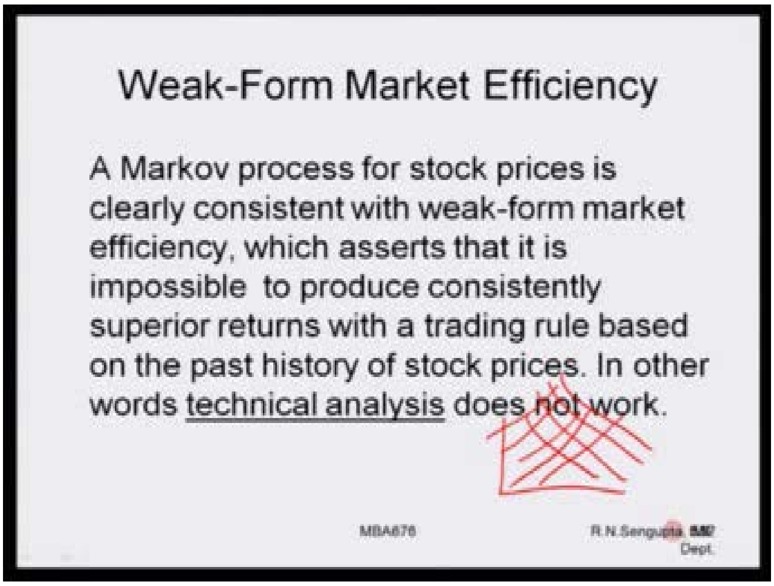

So, going back to the demand and supply curves. So, if the curves on the demands in the supply moves in parallel; obviously, they are shifting depending on the pull whether demand is more than the supply and supply is more than the demand or vice versa which is less based on that you can again find out immediately the price and quantity. Now the information would be very instantaneous considering it is a perfect information market, because if somebody has an extend the information you will immediately make a profit and that information of the profit would immediately emanate and go to all the ((Refer Time: 03:35)) of the market and everyone would start playing in that ways as the demand and supply would again come back to equilibrium.

So, as we were discussing basically consistent with weak form market efficiency; efficiency which asserts that is impossible to produce consistently superior returns with trading rule based on the past history. In other words we have not discussed this technical analysis that technical analysis unless would not hold true. So, obviously, but if you consider the fundamental analysis just for the information, it means that general concept of what the actual characteristic of the companies are whether we are able to understand its price to earnings ratio, we are able to understand its market share, we are able to understand its profit analysis account, the depreciation, the assets, the liabilities that would give us much better information and we are able to predict the prices of that particular company in a much better way.

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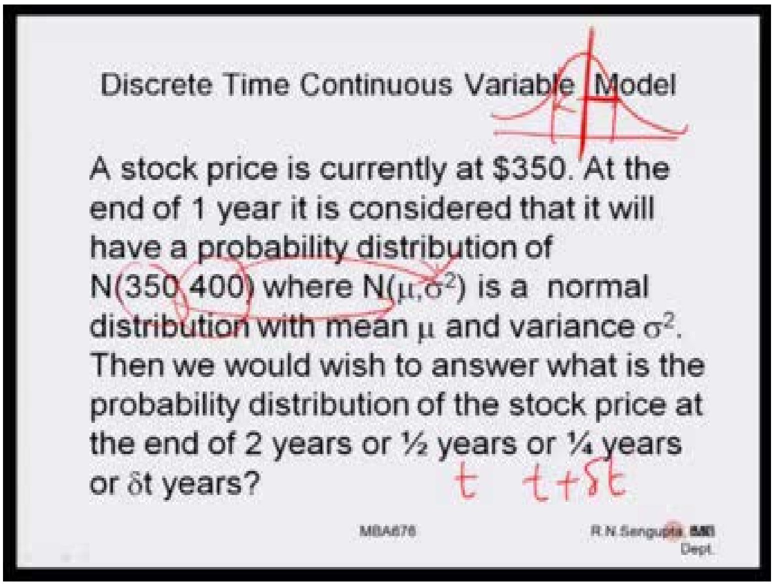

So, consider a stock price is currently at 350 at the end of 1 year it is considered that it will have a probability distribution given by this. So, this 350 is the mean value and this 400 would be basically the variance. So, if you see this is the nomenclature we are using. So, mean is the mean value. So, in the normal distribution this is the mean median mode whatever we have, and this is basically the 2 standard deviation. So, this is 1 standard deviation we are trying to find the square of that which is the variance. So, n mu comma sigma square is a normal distribution with mean mu and sigma square as the variance, then we would wish to answer what is the probability distribution of the stock price at the end of say for example 2 years, at the end of say for example half-year, at the end of one-fourth year, and so on and so forth.

So, what we are actually interested is that given the distribution at time t we want to find out what is the distribution at time t plus delta t; that means, instantaneously next movement of t the normal distribution would have certain mean, and certain deviation standard deviation, we want to basically find out given the information as it is today considering that simple concept of Markov process and stochastic processes really hold through for the stock prices.

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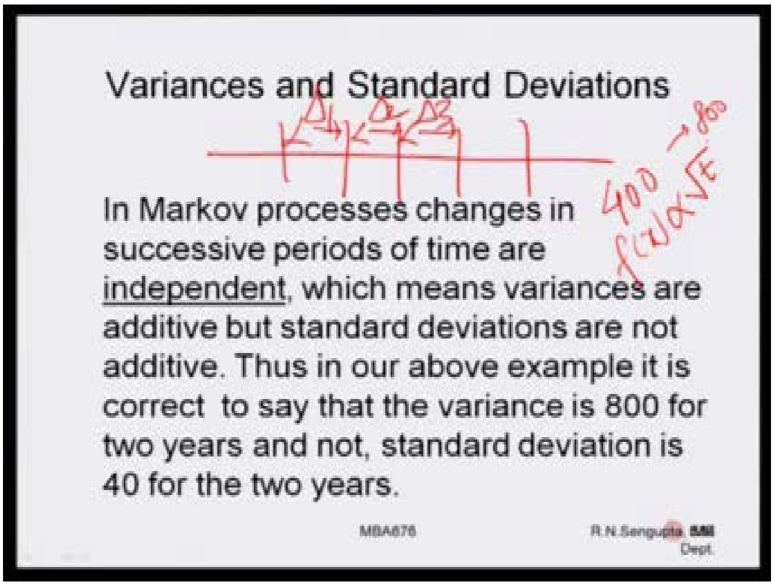

In Markov processes changes in successive periods are independent, this is the important remember which means that if we have the values, and you want to find the difference with in these values. So, let us mention as delta 1, delta 2, delta 3, and so on and so forth, this is the differences. Then the differences are IIds; that means, independently and identically distributed with a certain mean and certain standard deviation, which means the variances are additive, but standard deviations are not thus in our example it is correct to say that the variance is 800 for 2 years, and not standard deviation is 40 for the 2 years. So; that means, given the values of the standard deviations, the variances which we had... So, 400 would basically the variances and 2 years on the line it will additive, so that means it will become 800.

So, generally we will see later on that the functional relationship between the volatility and time would be would be very important, which means that volatility whatever the function, it is basically would be proportional directly proportional to the square root of the time difference between as now. And say for example, 2 minutes down the line, 3 minutes down the line, 10 years down the line whatever it is, but in the case of trying to find out the volatility, which is the variances for the stock market. We will follow the simple concept that the variance can be additive, which means in the long run the values of the standard deviation the value of the volatility would basically be a function of either the square root of time or time depending on how you trying to implement volatility that is important. So, whatever the function you are using to denote the volatility that would basically dictate, what is relationship of time, which is coming into the picture.

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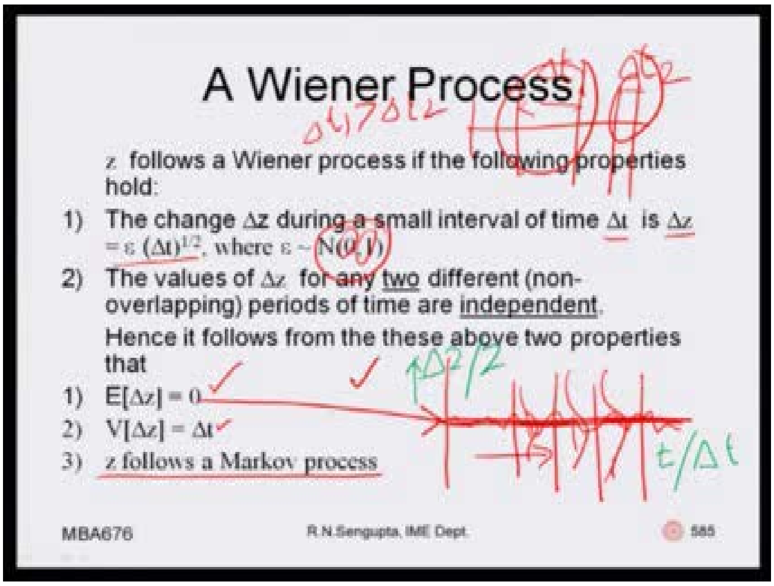

So, we will consider 2 very simple process based on which we will try to find out the implication, and what is the actual formula for model. So, first is the Wiener process, let us first pick the perfect background for the Wiener process, consider z is the random variable, and z would follow a Wiener process. If the following procedure of the assumptions hold, what are those first the change of delta z; that means, z are the values of random variables. So, between each z period z values, we will basically have the delta z which is the difference. So, say for example, z is taking the values of z 1, z 2, z 3, z 4, z 5. So, technically the differences would be z 2 minus z 1, z 3 minus z 2, z 4 minus z 3, and so on and so forth.

So, we would be considering that delta z as the difference in the values of the z. So, this is what it is say the change delta z during a small interval of time, delta t amount of time would basically be related to in this way that, if delta t is the time, delta z is the difference then the relationship between delta t and delta z would be given by this, that is delta z is equal to sum of epsilon multiplied by the square root of delta t. So, if you remember when we are talking about the volatility and mention, it can be dependent the square root of time, it can be dependent on time also.

So, this is what we are trying to basically bring into the picture, and also remember that the delta that epsilon which is has would be a normally distributed 1, which is the standard normal deviate with the mean value of 0, and a variance or a standard deviation of 1, the values of delta z for any 2 different non-overlapping periods would be independent. So, if you if you remember the values of random variable, the differences of them, if they are basically IId from time to time. Then obviously, they would have a distribution. So, this is what it saying that the delta z for 2 different overlapping periods they are would be IId, they would basically have a type of distribution, and the relationship between time delta time when I am mentioning time is basically delta time, and z which is delta z is related by a standard normal deviate.

So, hence if you see the actual standard normal deviate it would mean what its variance would be 1, so obviously its variance is 1, its mean value is 0. So, if we have different values of delta t given by the concept, that it is related to epsilon and delta t then in the long run the expected value of z is 0, and the variance of delta z would be delta t. So, here we will say z follows a Markov process. So, if you look at the values and basically trying to find out delta z here, then it would be going like this.

Now, if you take a snapshot here then this long run the mean value of z, which is expected value of del z which is this 1 is 0, which is fine, and if you look at the variance. So, variance would be at different point of time. So, if you are trying to find out the variance, which is the normal distribution considering, this is normal, then it will look like this. Now watch here, even though it is little bit clutter, if you see the mean value; mean value would always be 0, but if you consider the variance is they would basically keep increasing further it go down along the line.

Say for example, if you find out the differences further down the line means that how big is the differences, where you are trying to basically measure the variance of delta z. So, say for example, if your time period are this, and in one of the case the time period at this in this difference if delta t; obviously, is more and in this case the delta t is less considering delta t 1 and delta t 2, where delta t 1 is greater than delta t 2. Then in this region the variance of delta z would be much more than the variance of the delta of delta z in this time period or delta t 2.

So, again I am repeating given delta z are the differences of the random variables, considering z has some particular distribution. So, you are makes question would be what is the distribution, that is what I am going to come and I did mentioned delta z would have a IId considering, they are non overlapping. In that case the distribution of delta t would be given by a normal distribution, but remember it would have a mean value of 0 considering the fact that epsilon has 0 mean, and one as standard deviation, but its variance. Now dependent on the time factor and we will see later on also that the standard deviation as I was talking about the volatility is dependent on time depending on how you are trying to express volatility.

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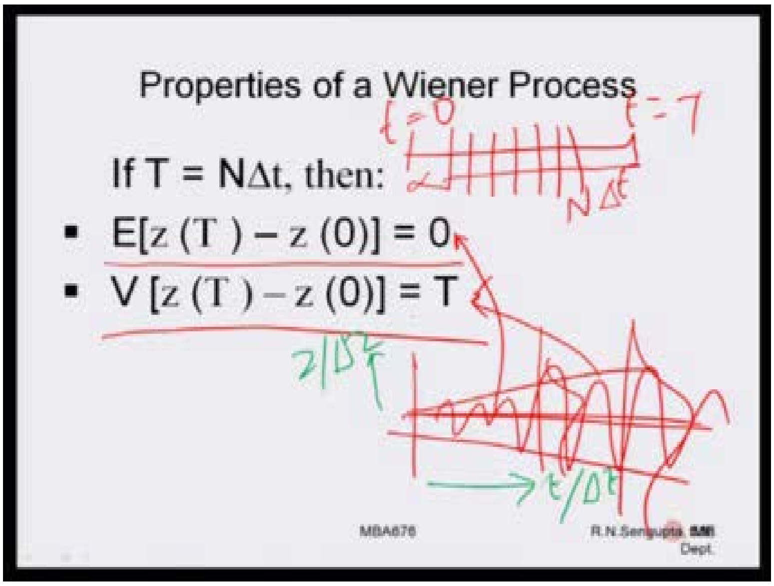

So, now if we are total time difference is the capital T. So, you have started here t is equal to 0, t is equal to capital T, and you have been able to breakdown into small delta t's, and consider the number of such divisions are n which means the total time difference would be n into delta t. Now in that case the expected value at time t is equal to 0 was 0, it will continued remain that which means the difference in the expected value of z, and 0 would be 0 which is the check this 1, and if we consider the variance, variance is exploding, because it is dependent on time t.

So, if you consider the variance is variance difference between when if started off the z process at time is equal to 0, and time t is equal to capital T would basically big t, which means the whole process would be expanding, in such a way that is basically moves out when the mean value remains as it is considering this 2, and the variance expands even the fact that it is dependent on time t.

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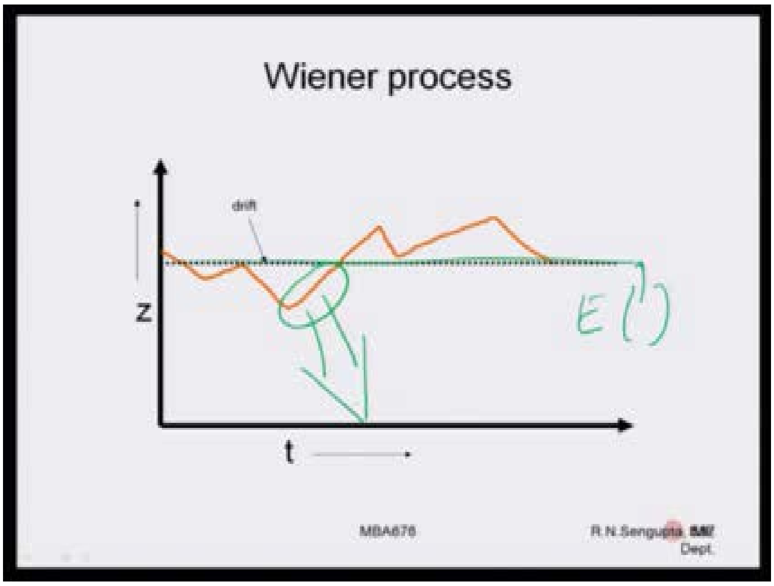

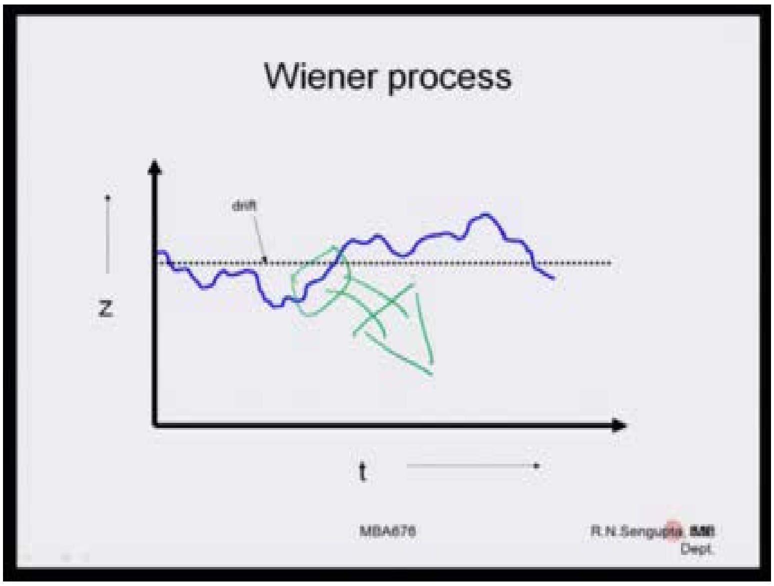

So, here I am basically consider the wiener process I am trying to basically have a microscope and zoom down in more in order to understand how it looks. So, if you see the drift, you have t, t not delta t, t along the values. I think I did not mention one thing properly yes, let me come back to the slide. Here you measuring t or some concept of t along the X axis, and here you measuring delta z or z. So, this is sorry, I just missed it. So, remember that along the y axis will be measuring delta z, and z some function of z and along the X axis, you will be measuring t delta t or some function of time. Similarly here also t or delta t along this z delta z, again come back to this, where we left. So, area of the drift and you have measuring t along the X axis and z along the y axis. So, this is basically the drift. So, this is technically the average value which you have, if you find the expected value; the expected value would be coming out, the straight dotted line which is parallel to the X axis.

Now, what I am doing is that, I am slowly, now zooming in trying to basically take a micro view trying to understand more in details, how the overall drift process considering the wiener process fluctuates. So, if you follow the diagram just shown the orange 1, and the diagram which is being shown in the blue 1, that is the same thing only I have used a different color, in order to make things like easy for all of us to understand. So, if you see the orange 1, it is very smooth, there is no jacket lines. So, now if I look at this, and expand it, how does it, look it would look like this; that means, there are much more variation as you basically zooming and take a micro look.

So, let us again take this small view and again expand it. So, if you see as you go more into the micro level from step to step more closer, you find out the overall variations in the actual stock prices would be happen like this. So, as that it will give you much better information, how it is fluctuating and the equation which we dealt would basically give you a much better theoretical knowledge, how the stock markets in a very simple manner can be model using the simple first order stochastic process or the Markov process which we have just discussed.

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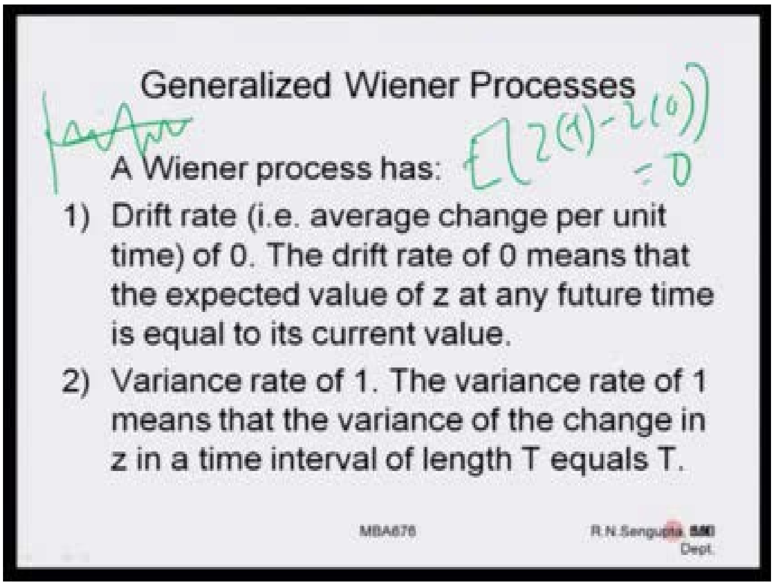

So, now a general initially we have consider the Wiener process, where at delta z was equal to epsilon into the delta t factor, whatever the function what and we consider epsilon was with normal distribution 0 mean, and variance of 1, and based on that we found out that what was the expected value of delta z, what was the variance of delta z, what was the expected value of z capital T minus z 0, where z capital T was the value of z at time t is equal to capital T, and z 0 was the value of the z at time t is equal to 0, then we also found that what is the relationship of the variance difference; that means, variance of z capital T and z 0, which means the differences of the variances we also found out what was the relationship.

Now, let us go 1 step forward, and let us analyze the generalized Wiener process. Now if you remember the formula, it had only the epsilon and t. So, there was no other constant factor or say for example, epsilon was a normal distribution with a fixed mean and a fixed variance. So obviously, many questions would come up in your mind, what in place of Epsilon; we had another normal distribution, where its mean value and standard deviation was also changing with respect to time. So, how would they happen? Now say for example, rather than have epsilon into the function form of delta t, you also have a plus addition of value which is no more constant, but it is changing with respect to time or it is a constant, which remains fixed with respect to time.

So, if you have different models, how would we basically generalized, this is what is the underlying fact based on which we are trying to analyze the generalized Wiener process; a generalized Wiener process would have a drift rate, that is the average change per unit time which would be 0, that drift rate of 0 means that the expected value of z at any future, time is equal to its current value, that is what when you are saying this, this is what we mean that has it is progressing z, the value of the difference of the z value as we proceed from time to time. In the long run would basically be 0; that means, there are fluctuations the fluctuation would basically even out, and it would be along the straight line, which is the X axis, the variance rate of 1, the variance rate of 1; means that the variance of the change in z in a time interval of t equals t, which means that if you remember the functional form, which we have discussed it means that expected value would be 0, it is not dependent on time, but the concept of variance of volatility would now be dictated by time. So, we will now close here and continue on discussions of the Wiener processes ((Refer Time: 19:45)), and how it leads us to the concept ((Refer Time: 19:48)) model.

Thank you.