Quantitative Finance

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Module - 06

Lecture - 31

So, welcome back. We were discussing after considering, spending a lot of time in different type of a combinational options. We have just started in the last slide, in the last class we just disused very briefly about the Binomial tree. So, what is the main focus is, to understand how the stock prices in general, I am not going to the details that how the prices fluctuate and how different concepts in mathematics and statistics can be utilized in order to find out the overall pricing our option in a very simple manner.

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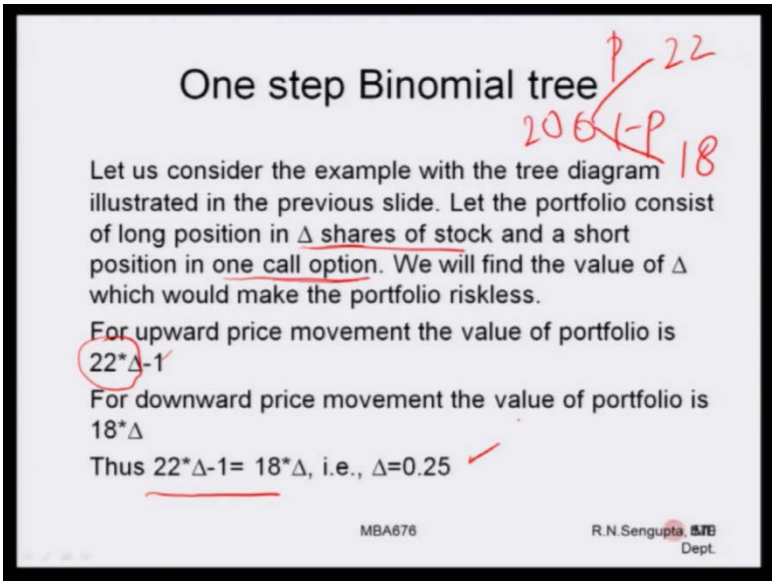

So, let us consider a Binomial tree and we consider the tree diagram as shown. So, what the diagram? Rather than going back to the slide, I will just draw it. So, you get price 20 here, it increases to 22, it falls to 18 and this p is 1 minus; p is the probability. Again, I am mentioning p is the probability not the price of the output option. So, let us consider the example with the tree diagram illustrate in a previous slide. Let the portfolio consists of what? What are these? A long position in delta number of shares of stocks and a short position in one call option. So, we will find the value of this delta number which will

make the portfolio riskless. What do you mean that? restless that will become a within 1 or 2 minutes. Now, for the upward price movement the value of the portfolio would be what? It would be, if you had delta number of shares so, the actual value would be 22 into delta. So, this is the number of shares which you had multiplied with the price, and where this 1 is coming from? 1 is basically the one call option which you have. So, you are basically having a call option. It means what? Call means you call back; that means, the total amount of money would be going out of pocket and the price of this share where which you had initially would be positive. Hence, we are basically utilizing the fact that you are 22 into delta. The total quanta of the spot number and this minus 1 is basically the one call option. Now, for the downward price movement the value of the portfolio would now be b. Because if the prices, if you think or not downward then obviously you would not exercise it.

So, overall value now the remains as 18 because that is the price, multiplied by the number of stocks. Which is delta? Why this minus 1 is not coming here? Is that because you would not be exercising that call option. So, if you consider the overall all value of that option and if you consider that you are neither going to make a loss now, going to make a profit because, is going to be such a position that you are going to even out. So, the even out position would be what? You in one instant, is the price increases would 22 into delta minus the 1. The minus 1 is coming because the call option is being exercised and in the downward trade it would be 18 in delta. So, if they are all of the same value, what it is? The overall expected value would be same, you equate the expected value and you find the delta is to be 0.25. Which means, that is the price is 20 as of 2. If you are thinking prices would be increasing with probability p to 22 and is going to decrease by probability 1 minus p to 18 then, the total amount of stock which we would buy in order to basically make an even out position would be 0 point 25, which is one fourth for the stock.

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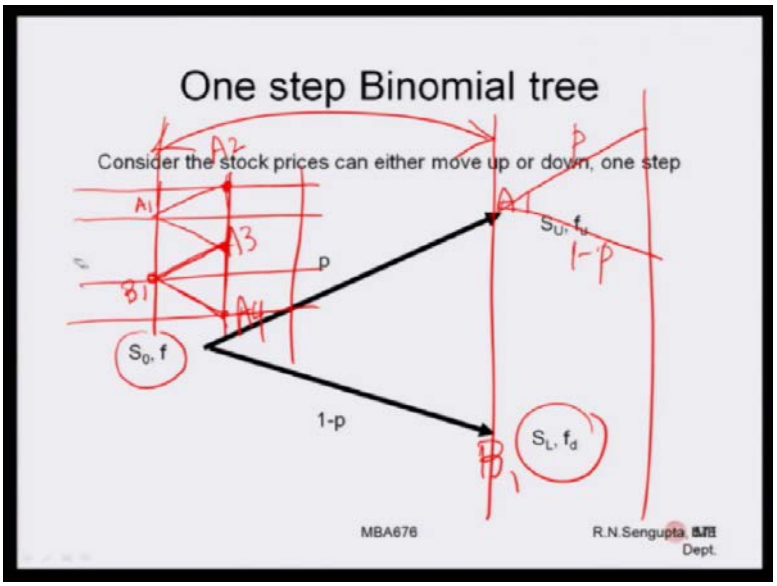

We consider now, let us consider symbols in much your explicit details considering the stock person can either move up and down in 1 step. So, now the question is that what is that one step? So, let me draw a diagram explicitly, consider these are grids and the grids have also horizontal lines and consider this point is somewhere here. So, now if it is a full step process, it will be either move up or go down. So, this point which you had, which is a 1, is this a one point which you had b 1 is this, b 1. Which means this diagram which you have is for this plane. Next, what we will see again? In the next we will see this price would increase and decrease increase with probability p, if remain same decrease with probability, it remains the same. So, this upward train would happen in such a way that will be this to this which is a 2, decrease to this or v 1 can increase to this decrease to this.

So, what you have? That depends on 1 step simple Binomial tree you have been able to formulate that in the 2 second stage. The prices are of 3 value consider them as a 2, a 3 and a 4. In this case b 1 which was the price which is basically given the year would increase by probability p would decrease where probability by 1 minus p. Similarly, a 1 would increase by probability p 2, a 2 and decrease by probability 1 minus p a 3, if we continue going in this way basically will fan out and you will be able to find out step by step. What are the corresponding problem in the prices now? If you will be thinking that this is Binomial tree would can we make a multinomial, one can we make at the 3 step process, answer is very simply, Yes. How we do that? So, consider that let me erase this.

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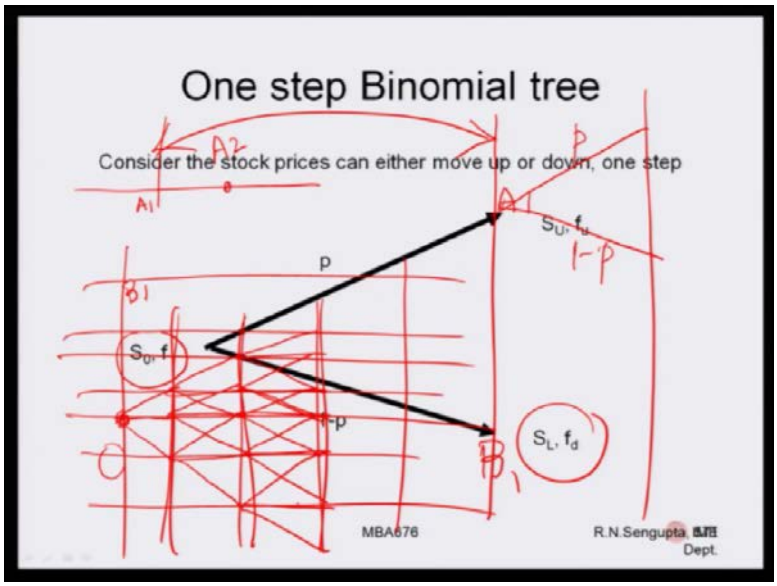

So, consider have a 3 step process. So, this is the red diagram which I am drawing again. Consider you are here, this 3 step process one step up, one step constant and one step down. Again one step up, one step constant and one step down. So, up, constant and down, up, constant, and down and where equal up, constant, and down, up, constant and down, up, constant and down, up, constant and down, up, constant and down. So, in this case, what you are seen? That each step by step as you move where able to red it or all the different about instance form a binding by one as you go in to 3 step process 4 step process. So for, we are not going to discuss in detail, but once you understand the Binomial one things would be easier to understand of the problems can be solved.

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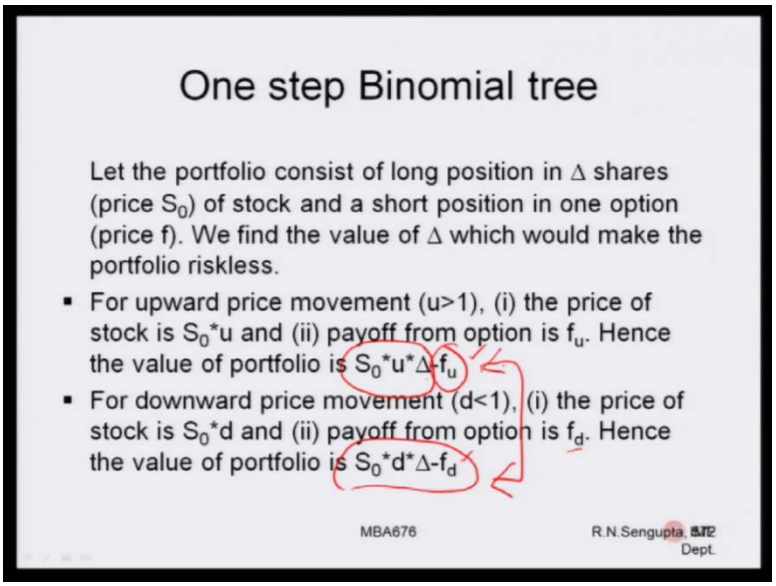

Let the portfolio consist of long position in a one-step Binomial one process along position is delta shares, we have already found out that delta would be point 25 considering the p is increase and decrease now, we trying to rather than using the prices of 18, 20, 22 were using a very simple concept of variables or symbols you would not use any numeric values. So, considering a long position and delta shares at a stock price of s zero initially and we have a short position in one option price of f . So, option I am trying to denote by not by, because y in order to not make things in confusing because we are using this symbols p, for the probability, hence we are not going to use the p and c as we use earlier for the put and call option we find the value of the delta which we make the portfolio discuss, we seen it in a very simple numeral example. Later in the last slide, but will again continue in order to make it much more general for upward price movement where probability is u is greater than one, but the price of the stock is now it has increase.

So, it; obviously, increase some probability and it will zero into u where use greater than one, we have from this option is f zero hence, the value of the option would be access and the value of the actual portfolio would be as zero is the price initially e u is the actual percentage. You will increase the stock and delta is the number of shares this is the overall an increasing share price and this is basically the p of from the option; that means, the option has been exercise for the downward movement. The price of the stock is was initially as zero it will now fall down these less than one and p of the from the

option is basically f, d I am trying considering (08 point 17) excess not excess in initial case excess and non excess we considering in a way simple way that if somebody exercises then the value of the profit and loss would be only the off run price paid, as if it is being exercise; obviously, off run price plus where there the person makes a loss or profit would come the action. So, you should remember it very explicitly in one case if it is not exercise the profit and loss. Is only the upward price payment and if it is exercise it would be upward price payment plus the profit and loss which is mean that is why it is denoted by a zero and f d. So, u means upward trend and you missed down ward trend for a 2 step process or a Binomial tree. So, the payoff from option is basically f d hence the overall value of the portfolio is given. So, you want to find out the value of the delta equal both of them and find out delta considering that is zero, is stock prices today, u and d are the percentage increase and decrease and f suffix u and f suffix d and basically we have happening for the case of upward in the price and lower down lower trend in the prices of the particular stock.

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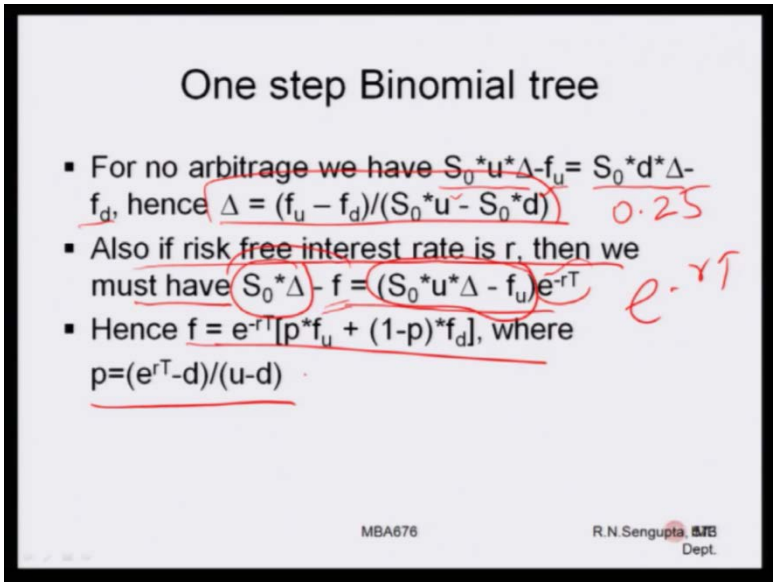

So, now, restless means they are no arbiters no bodies going to make an extra profit nobody is going to make an extra loss. So, once you equate this values increase in the value of the stock this is the option in decrease in the value others stock. It is the option hence we are once we equate this is the equation. This is the v a same value how we found out point to five. So, also if risk free interest rate is r then we can find out using. So, now, remember one thing this upward trend and the down ward trend which you have

noted down here these are being utilize in such a way that risk free interest is going to come a picture also the risk free interest is r then we must have. So, this would basically be the increase in the price we are not multiplying with the increase and decrease this is the value of basically, the forward which was there as of time t is zero once we equate that and this is what this was the price which would happen later on because guess zero into u into delta is the price increase and this is the value of the option this would be basically multiplied by the e to the form minus r t to bring that the prices of today. So, hence f is given you can find out the value which means that ties the total value of the option which would be as s add time t is equal to zero says that the overall profit and loss or the overall edging is done, in such a way these basically needed profit known of loss for both the person is bought that option or sold that option.

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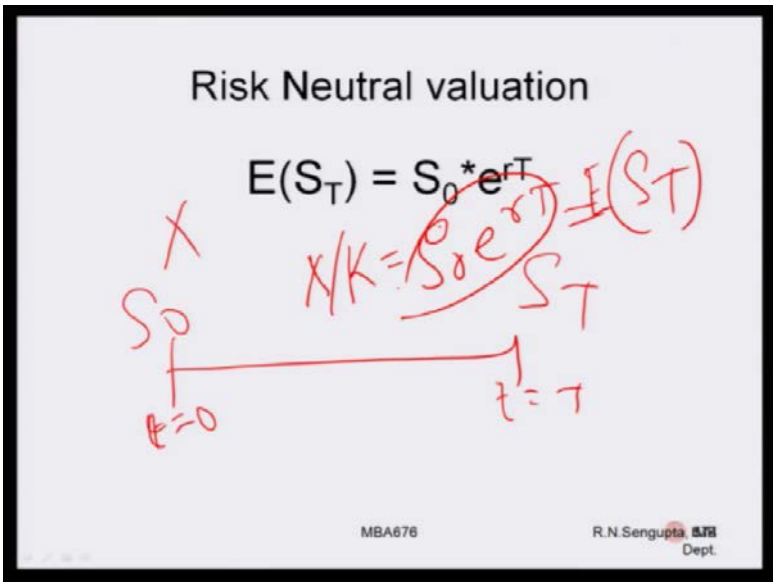

So, now, we will considered very simply the risk neutral value is in. So, if you intuit everything that this is the time scale t is going to zero t is going to t when we have basically as zero as the price and s t is the price and time price is decided we should decide in such a way that any increase and decrease in the price and option has been compensated by the risk free interested in what sense if I want for a call option or I am want for a put option; obviously, might counter movement would be what it either I go to the bank and deposit money or go to the bank and withdraw money depending on which position I am not play as the time expire. So, if I am not going to make any profit and loss you should definitely mean the amount of money which you have deposit in the

bank and when I draw should exactly match the price of the particular stock or particular position I mean or else, it would mean that if I am going to sell it and initially the money which I have use was basically by the option, it should happen in such a way that once I close my position in the option I get some money utilize that money and go to the bank and return to the bank, those overall principle amount was the interested we should be such a way that neither that bank amount of money which you have drawn and basically I deposited and when I close my position for the bank and when, I close my position the option they should exactly be balance says that there is no arbitrates of the no extra profit made by any other party which going either for a put option or call option.

So, in this case it should mean that technically if you remember x or value which you had was this s this is s. So, these should exactly match the value expected value of the price says that any fluctuation on the price which is their if we try to basically simulated infinite number of time in the expected value s t should exactly match this says that risk neutral value is that says that is exactly matching the concept of the arbitrage no arbitrage position which process I want to take says that neither a person is going to long position neither a person is going to a short position is going to make an extraordinary profit.

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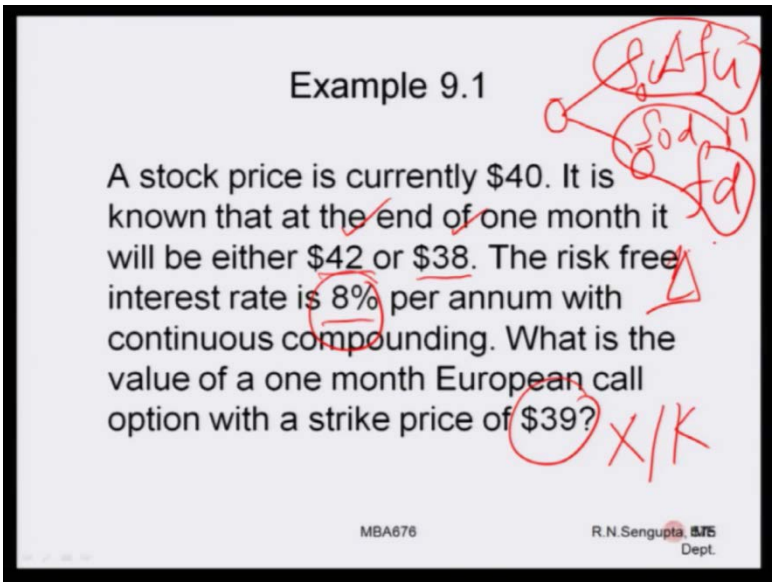

So, consider a stock price is currently at forty it is known that at the end of one month it will be either forty 2 or forty. So, these are the increase in the price or decrease in the

price the risk free interested is given as the eight percent with which is continuous compounding what is the value of one month call option with a strike price of thirty nine. So, this is a thirty nine basically that is one k and you can find out the value of the call options or that is basically risk free interested been taken on the condition now eight persons are that this valuations which is be done for the both call and the put option should research that is the neutral for the both the party you use this concept prices increasing prices decreasing in a option when fu is the value of the option f, d is the value of the option plus and minus depending in a in on a put price increase would s zero into u price decreasing s into d you utilize combination of this is the portfolio combination of this portfolio says that they equated says that both an upward and down ward and price fluctuation is such that neither makes a profit non makes a loss. So, they should be equated the moment you equated you can find out the value of delta. So, this is the delta value here are also then the number of stocks which you find you find out that the value of delta that will give you the risk neutral position based on which neither a person is going to make a profit or loss.

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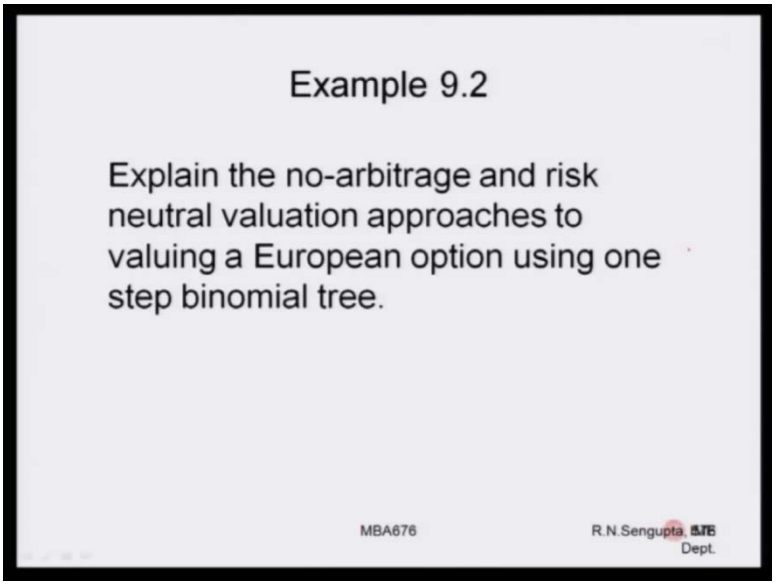

So, they are try to understand what is no arbitrage this neutral valuation approach is been done and valuation a European option, but remember on the we are always going to consider the depend option based on the fact that are time to make fix for us, it should remember in the same way when you compare the forward and the future consider one of them where the time to maturity was fixed and based on that we basically on the prices in

same way will consider the European option and once the prices are found out will try to compare with the similar type of American option which is this says the price fluctuation which you are trying to get on the European option would give you some knowledge about the price fluctuation for the American option also.

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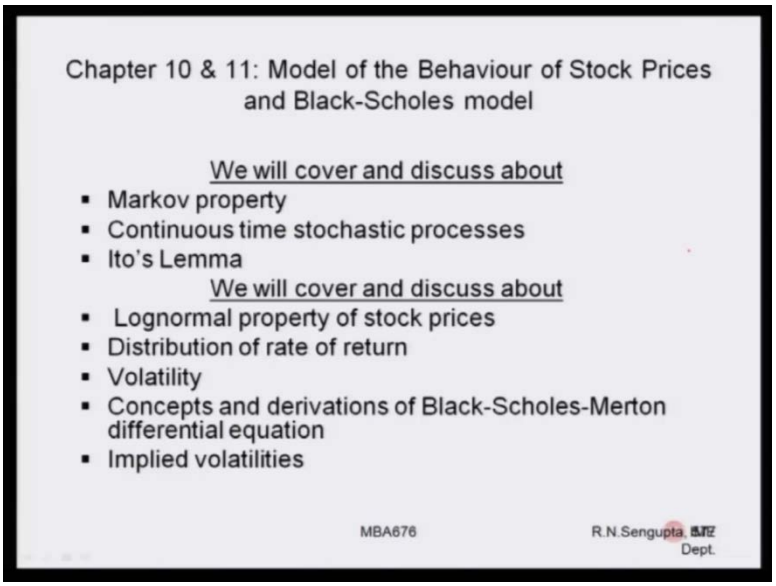

So, now, will call go to the next topic will cover and discuss about Markova. So, this this topic will try to basically keep it very simple will consider the Markova property continuous time stochastic processes it's lemma may not process based on which the natural models are done will terms will cover and discuss very simply the lognormal property of stock prices distribution of rate of returns the concept of volatility and the concept of derivation the black Scholes Merton in a very simple manner and basically then go into once we can consider the overall loop of closing the option you solved very few prop simple problems in the domain of portfolio and this is then going to the different of a risk valuations and how this actually this options and derivatives are use in a big way in order to medicate the overall risk.

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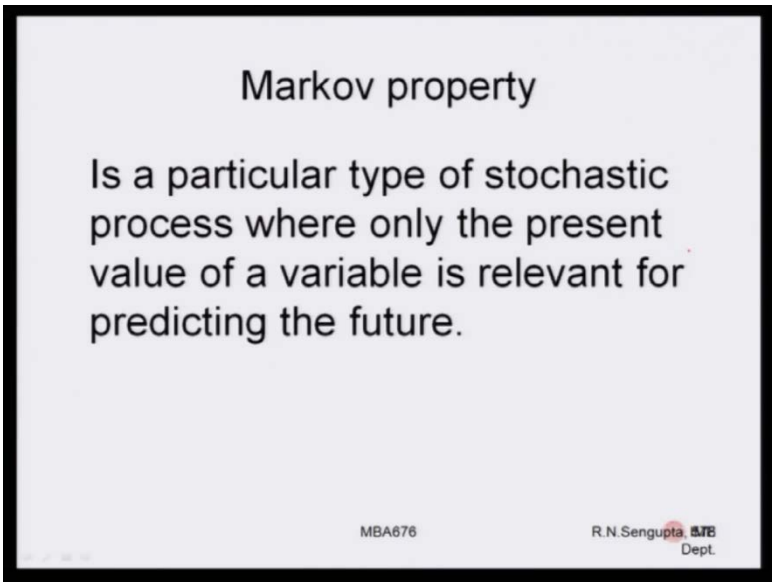

So, Markova property that is in a basically type of stochastic process Markova chain Markova property whatever discussing is a process where only the present value of the variable is relevant for predicting the future. So, what we are trying to do is that whatever the price whatever the value of that particular variable is as today is basically have an implication what would be the price d of to narrow or one step down the line or 2 step down the line. So, if we are says that price of putting is going to affect the price tomorrow only then is basically simple, on step process we are assigning the price of today is plus the price of tomorrow will affect the price of day after tomorrow. So, this is 2 step process in the sense the day after tomorrow prices is being effected by t minus one and t minus 2 where t minus one is of today t minus one is for tomorrow and t value is basically for the day after tomorrow price so; obviously, you can basically formulate different type of the prices depending on the order of the step of the process, but will very simply consider the one step process.

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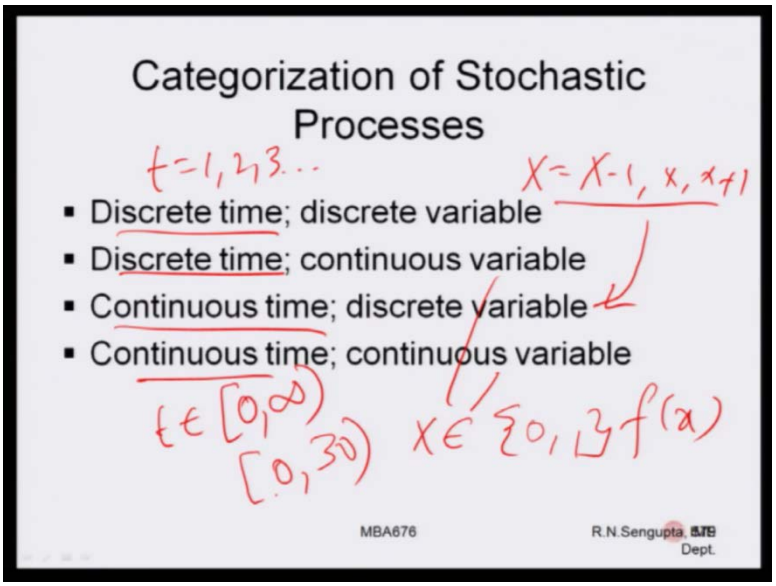

So, will consider these combinations of the movement of the stock prices in two different. One is basically discrete time discrete variable process then you will of the discrete time continuous variable process and will have the continuous time discrete variable process and the last one is the continuous time continuous variable process. So, remember that time factor is the time factor which we are considering and the variable would it would be basically the x of the y variable we are trying to consider. So, generally if we have the discrete time then t can be basically one 2 3 4 and. So, and. So, for and discrete variable would be say for example, x minus one x plus one and. So, and. So, for similar that discrete comes here while in the continuous case x would be basically in some domain of say for example, zero one or say for example, will have a certain formality function f of x we are now going to the details in the discrete time as mentioned it one 2 3 4 and. So, and. So, the inner continuous case it will be t is basically between a times skill of zero to infinity or say for example, zero to thirty seconds. So, and. So, for deepening on how the problems are mean and illustrate

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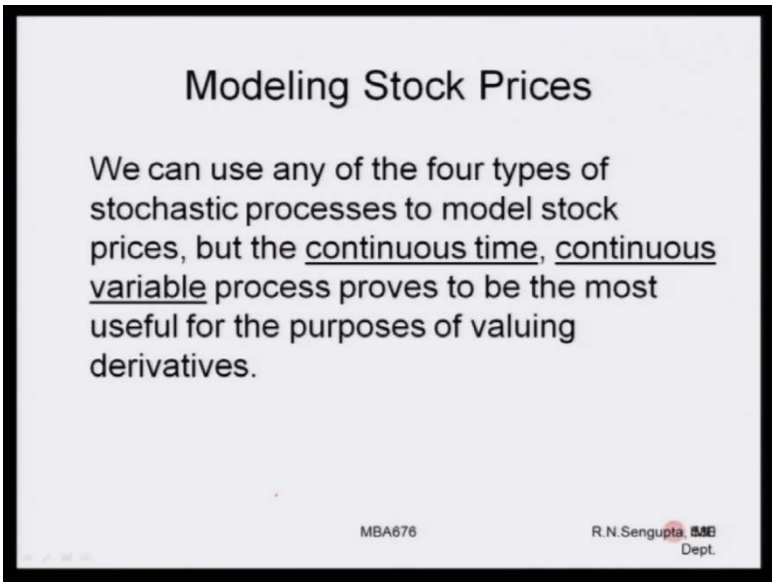

We can use any of the 4 types of stochastic process to model the stock prices, but the continuous time continuous variable process proves to a useful one and easy for us to understand in order basically model the stock prices.

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In a Markova process future movements in a variable depend only when where we are not the history how we reach they are we assume that the stock prices follow simple Markova processes in the sense as I mentioned it is one step process or 4 step process we considered very simply this is the first order or one step process in the sense whatever

the prices these off to today as all the information all the prices which is have happened in a past and all things are subsume in the prices of yesterday it means if it is a 12th of July 2015 we consider all the information of the stock prices are available on 11th July 2015 again if you go wants to backwards whatever the prices for the 11th July 2015 basically subsume in the prices of 10th of July 2013 in case we considered that are 12th of July 2015 all the information is subsume on 2 days prices eleventh of July 2015 and 10 th of July 2015 say basically this give a seven odd of process similarly we can for a third arrow process 4th of the process.

So, will consider very simply is the first order process with reputation I am saying you are also considered that the variable if we considered; that means, that x variable is only in one dimension may not going to considered 2 dimensions process or a 3 dimensional process because stock prices is price only along the y axis or time would be on the x axis. So, having said that I I will basically now and in this class and against in the next day again re hash or re capture whatever understand in the Markova process I would be repeating that because in order to make you understand in a much better way and I will large the students at least to understand and the read the book of channels here and whatever other references i given and once you read this as in a very simple manner again I started the Markova process what how the Markova process use it was lemma and inner process and how we are able to derive in a very simple conception way the black model thank you very much will consider on this parts in the next class.