Quantitative Finance

Prof. Raghu Nandan Sengupta

Department of Industrial and Management Engineering

Indian Institute of Technology, Kanpur

Module - 05

Lecture – 29

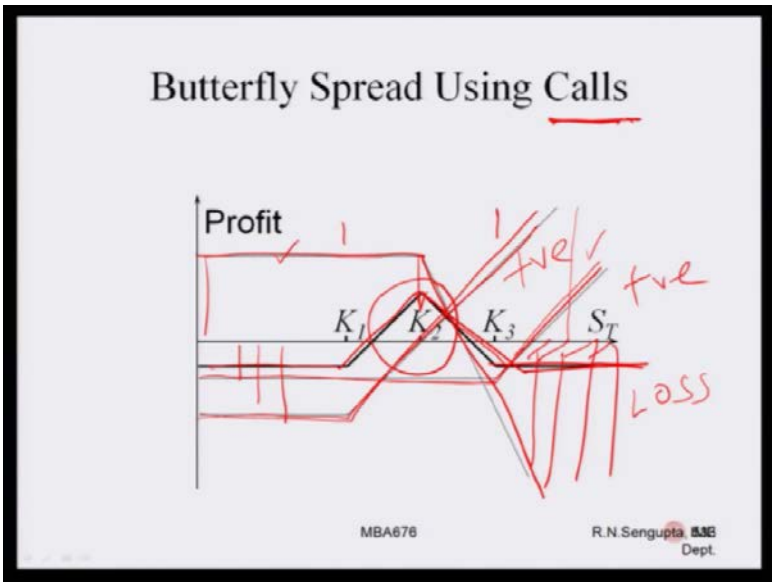
So, good morning, good evening, whatever it is for you, and I am sure you are enjoying this quantitative finance course, learning something, checking books, checking the net, doing lot of googling, reading, research paper, doing the problems, because there is a whole lot of issues which we have gone through, and I am sure if is spend your time, and definitely you will get all the concepts clear. And as I have been mentioning as you are aware there are different type of software which are available, which helps you to do all this problems in finance.

So, you have the MatLab, you have the r, you have the s p s s as so on and so forth. So, which I did not mention very explicitly, but considering that if you are really motivated you can find out all these information on the net. So, continue our discussion about different types of options, their combinations, whether the prices on a increasing trend, whether the prices on a decreasing trend, whether the price will be within some band so on and so forth. So, we did consider combinations of first; the combination of forward and option. So, that can be either a long or a short forward, then it could be different type of options put and call. Then later on we consider that if there was a bullish market bearish market. Bullish means price are on the increasing trend and the person thinks the price will increase. Bearish trend means the price will decrease, how the, those options can be combined a port and a call and so on and so forth.

(Refer Slide Time: 01:46)

So, now, we consider where we left is that will consider the butterfly spread; the investor is hoping that the large stock moments are unlikely; that means, they would basically fluctuate within the band. And whenever I say the large fell of fluctuation it will all do to the s t which is the spot price at time T, and will compare how s t is varying with respect to the different type of delivery price you have. You have k 1 k 2 k 3 k 4, depending on how many different types of options you have, but also remember that underlying assumption is that, they are time to maturity for all the options would be the of the same duration which is T. So, here in the butterfly spread, the investor is hoping that the in the large stock price moments are unlikely.

(Refer Slide Time: 02:29)



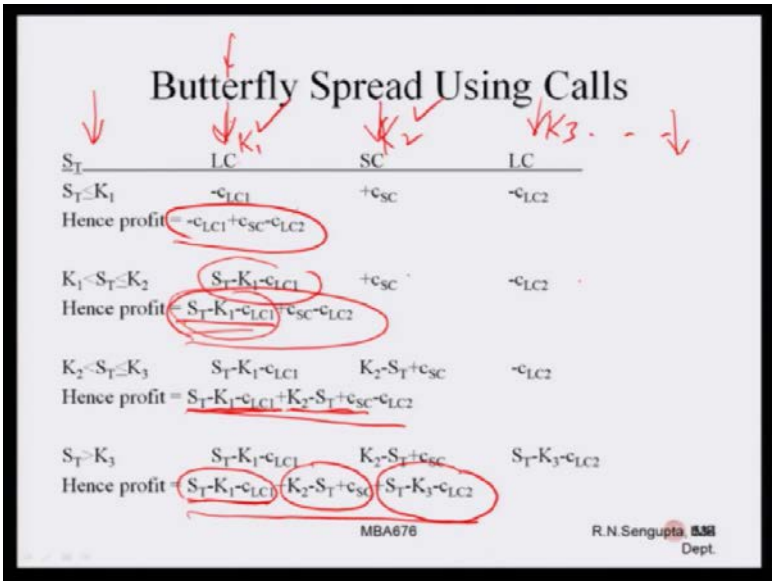
So, if you see the profit; the profit function would be this portion which is as marked. So, no large stock prices are unlikely. So, you have one option which is this, with the strike price of k 2, and another one is basically with k 3, and this is basically k 1. So, in case one; which is the strike price of k 2, you have receive some money and correspondingly and for the other two option which is strike price of k 1 and k 3, you have bought that such that you have given out some money. The money has gone out of your pocket. So, technically will try to draw this graph considering k 2 is the midpoint between the k 1 and k 3, even thought that need not be true, but thus being drawn in such a way that will able or a able to get a good feel that how this different type of options are combined considering put call and so on and so forth. Another thing remember we have assumed, and that is true, that the prices of this two options whether p c small p small c what I am talking about. Not capital 1 c of the capital p which you have discussed about for the American one. We have all considering the European call and put. So, this small p and small c are of different values.

So, the reason being that because if you have different type of delivery price, and considering the time to maturity the same, still you will have basically the prices at which they are being brought and sold are different. That is why this height which you have drawn this one, this one, and this one are all different. So, if you combine them you will see, that for any moment of the stock price s t being less than s k 1 being greater than k 3 or being between k 1 and k 2 the price fluctuation which you are seeing is within certain band. So, that means, the person thinks that any price fluctuation between k 1 and k 2 k 3, you will always make some profit, considering that you are able to hedge to the best possible extent. And any negative movement, negative means either it is below k 1, or the s t is basically over k 3 you have basically have formulate options in such a way that you make minimal loss, because if you consider this only; case one or case two for price is below k 1; obviously, this is the loss, but on other hand price is greater than k 1 and greater than k 3; obviously, there is a gain.

This is fine for you the k as find for you, but what you are more interested is that if it a loss the loss should be as minimal as possible. But on the other hand if you the other option, for prices less than k 2 you will make a profit, because you got money and the person would not exercising it, but the moment the prices or greater than k 1; obviously, person exercises, and you make a whole amount of loss. So, this loss is being compensated with the positive gain which you are doing; such that the overall lose or

gain whatever you have, is basically bounded between the small band width. So, what you have been able to do is that, combine the options is such a way that any mark fluctuation greater than k 3 less than k 1 you make minimal loss. And any variation within the bandwidth which you think is possible, you are able to make some profit in the sense you are able to minimize your overall loss, so this is what you see. And you have basically formulated the butterfly using calls.

(Refer Slide Time: 06:10)

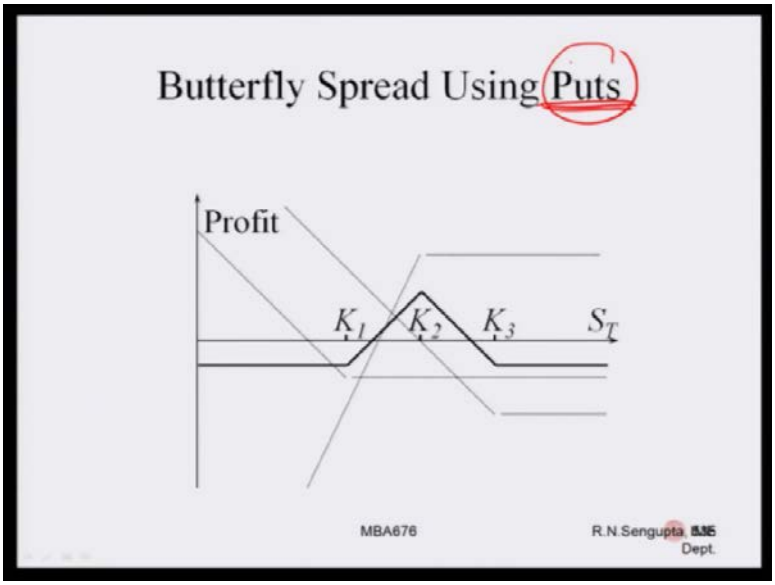


So, that was the diagram. Now let us basically consider the matrix. So, what you have is basically the first column as usual s t in different domain. The any of the long call, any of the short call, and you have basically short call. So, if you if you see, I have written it in this fashion or in the book (Refer time: 06:30) written in, sees in this fashion. The strike price for the long call the number one, which is the second column, would basically be k 1; this would be k 1. The delivery price for the short call is basically k 2, and for the next one is k 3 and so on and so forth. So, what we have done, is that, written in such a way that the increasing delivery price is basically written on first column second column. First column means this one, the first column, second column, third column, forth column so on and so forth, where technically zeroth column has the values which are basically domain of s t, whether s t is between s k 1 and k 2, whether s t is greater than k 2 and less than k 3 and so on and so forth. So, if you see, in this case the overall profit which is being given. I did not have any extra column that why I am wrote it down; hence the total profit with this; that means, you have basically paid two amounts.

So, these are the total amount of money which has gone from your pocket for the long call the first row, in the long call the second one. And the total amount of money which you have received is basically the short column, which is again c, but with the suffix s c which is for short call l c 1 and l c 2 is basically long call one and long call two. Now in the next case when k is between k 1 and k 2, again you see the first one is activated and the second and third are not; hence the total. Now the value of the option is given, by the values of minus c l to c continues as it is plus c s c which is the short call price is continuous, whoever the person is, has basically exercise in that way; hence would now consider this extra term also. Now as you move to the next higher level s t is between k 2 and k 3 which means what, the long call will be activated, the short call will be activated. Long call means the first long call, while that second long call would not be activated; hence the profit would now be the first portion is basically for the first long call. Second is basically the portion for the short call, and the long call to remains as it is.

Now, when you basically go to the third instant, that s t is greater than k 3. Now all three will be activated; hence the price would be or the total profit would be the sum of the first one; which is this one, second one which is this one, and the third one which is this one. Where first second third implies the long call one, the short call, and the long call term. So, the overall profit is given. So, whenever you have this overall profit, remember you can draw and retrace back and understand how the graphs look like that.

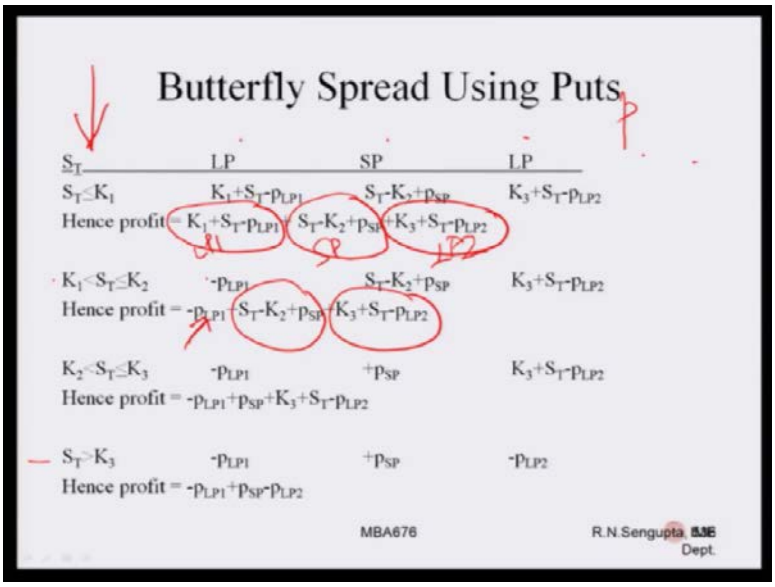
(Refer Slide Time: 09:20)



Now, you are basically formulating puts. So, they can we different puts, and long put short put, and different combinations of that, different strike prices. So, we have

considered the strike price is r as usual k 1 k 2 k 3. Again, we are assuming inherently that the prices are moving within this bracket; which is k 1 and k 3. And the prices for any untoward moment of price is greater than k 3 less than k 1. The prices moment have been made such, in such a way using put options, that the overall profit in regions of less than k 1 and greater than k 3; obviously, or minimum that there is no much fluctuation. So, whatever fluctuation is happening in between the band width of k 1 and k 3. Again consider k 2 has been drawn in such a way that is the such a way enough k 1 and k 3. Also remember the time to maturity are same, and they are of the same type of European option.

(Refer Slide Time: 10:20)



If you follow the same fundamental the principle; the zeroth column of the first column whatever you denote, is the domain space of s t. In the first instant s t is less than k 1 is the second instant which is this one. It is basically s t is between k 1 and k 2. In the third instant s t is between k 2 and k 3. And the final instant you have basically last row is, s t is greater than in k 3. And again in the similar way the first column, the second column, and the third column are the different type of puts. In the first column you have the long put one. In the second one you have basically short put, only short. There is only one short put, so this is short put, and the last column is basically again long put two. So, you can have basically short put two, long put three and so on and so forth. you can differently do different type of combinations. So, again in the same way; in the first instant when s t is less than k 1 well all these three are activated.

So, what I am circling now. The first, the second, and the third are the respective profit which is happening from the long put one; this is long put one, this is the short put, and this is the long put two. Remember also p is the price of the put, and the suffix l p 1 the basically long put one, s p is the short put one, l p is the short put two. If you go to the next row there you will see that for the first instant which is long put one, it would not be exercised, then the price which has gone out of your pocket, is the negative one. The next two columns with the short put and the long put two have been exercise; hence the profits are denoted as it is, and this is the one for the first instant. Similarly, as you go step by step in the case then f b is between k 2 and 3. The long put one and short put are not exercised, the long put two is exercised when the last instant when s t is greater than k 3. None of them will be exercised; hence the overall profit would be the combination of. I am not mentioning the sign, but it will be p which is the price of that put option, suffix l p 1 is long put one.

Then, you basically p with the suffix s p which is with the short put, and the last one is p again with the suffix l p two which is for the long put two. Whether minus sign will come or a plus sign will come, would depend on whether you have bought that op options or sold that options, similarly as we have considered in the calls also. So, whatever the combinations are for the call and put. In the first instant, we have only consider the call, and the second instant you have only consider the put. Now your question would be whether we can combine different calls and puts. the answer yes, depending on whatever you think are the scenarios, whether the prices are on increasing trend, prices are on decreasing trend, prices would be fluctuating between one level k 1 and k 2, prices would be fluctuation between two different levels k 1 and k 2 in once instant, or say for example, k 4 and k 5 in other instant. So, you basically have different types of puts and calls, depending on what you thing is the best option, in order to minimize your overall risk or the loss.

So, remember the another thing which I am mentioning time and again, the underline fact because that the options are being combined in such a way, that you want to hedge or minimize your overall risk.

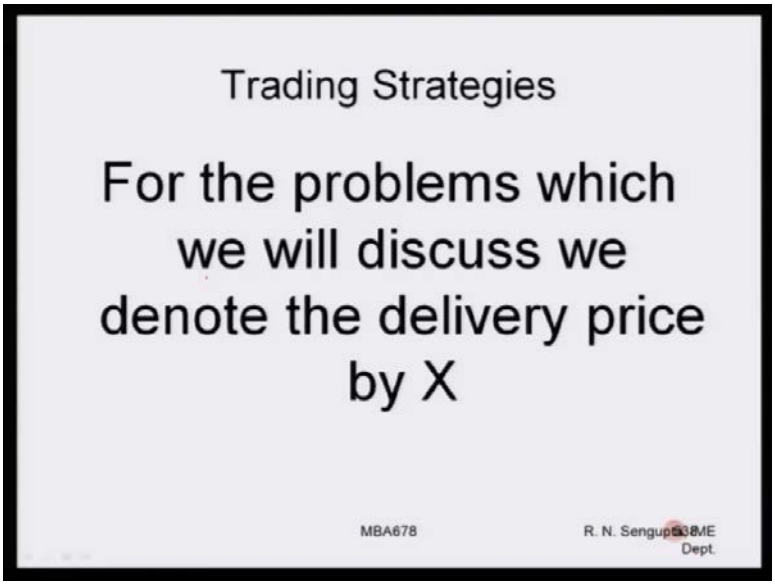
(Refer Slide Time: 13:35)

Now will consider the trading strategies involving combination of forward and options in order to reduce the risk. So, we already have done so in similar fashion. So, we will see that how the overall risk have been minimized. So, by the word risk I always mean always been some sort of one trading value; Rupees, Dollars, Yens, Pounds, Australian Dollars, Canadian Dollars, Australian Pounds, whatever it is, you will always consider that you are trying to minimize. And when you are trying to minimize remember all the one thing, that as you have done for the out swaps; that means, when you are doing the case that you are in domestic country, and you are trying to basically find out the swap for the foreign country, and you consider r and r star. Where r is basically the risk free interest rate for the domestic country, and r star is basically risk free rate for the foreign currency. And you consider the domestic currency with respect to foreign currency or vice versa depending on wherever you are.

You will basically consider the same concept, in a way such that the options in the futures have been done such a way, that you are over all unit based on which you are trying measure the loss would be only one currency. There may be instances where we will see later on, that the risk can be termed as variance, risk can be termed as standard deviation, risk can be termed as value at risk, risk can be termed as conditional value at risk whatever it is, or it can be beta also, or it can be the ratio of expected value to the standard deviation. Now it can be ratio of expected value to the beta. So, if you remember we have done different type of problem solving using the ratios, where you rank them to the highest to the lowest of ratio of the excess return of the risk free interest

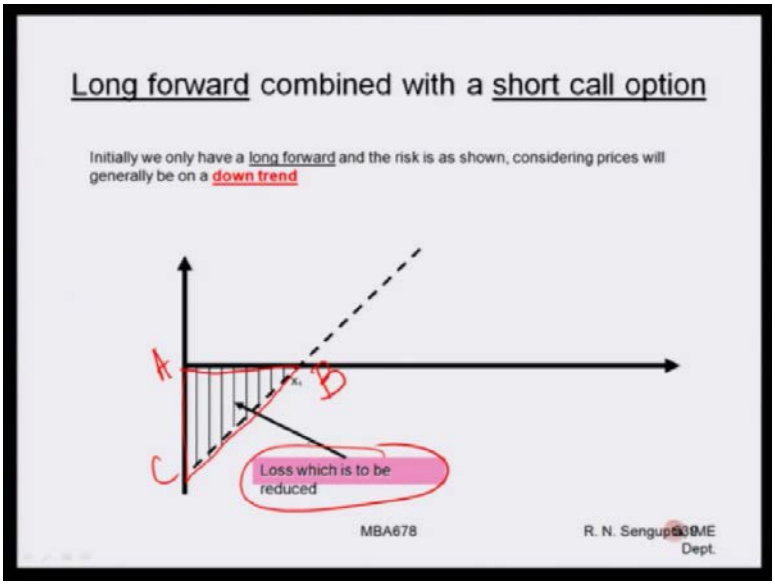
rate divided by standard deviation. Or the access return of the returns over the risk free interest rate divided by the beta. So, we consider different concept of risk. So, they are basically some sort of loss in a very simple sense.

(Refer Slide Time: 15:27)



For problems which will discuss we will denote the delivery price always by x. It can be k or x, but will continue with concept of x as the delivery price.

(Refer Slide Time: 15:35)



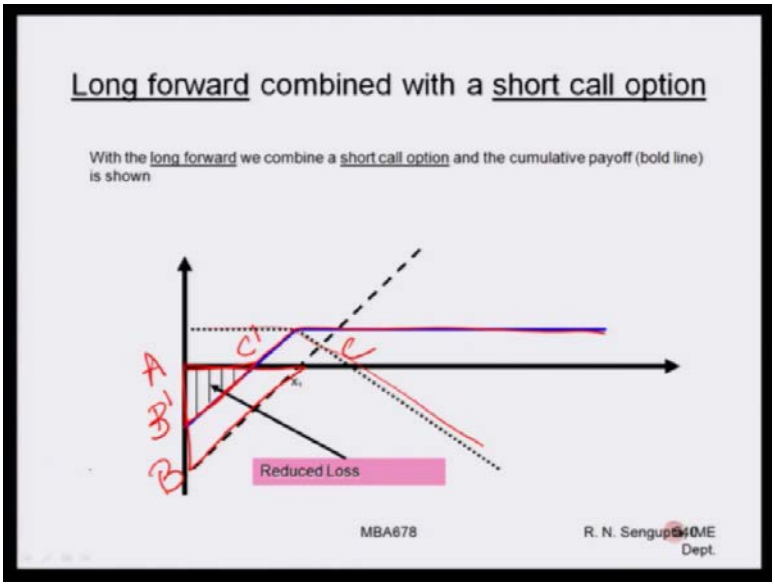
So, now, consider. We have already done that, but now I give you a very good pictorial notation, that how we can reduce this risk, because if you can understand this, show the diagrams, it will definitely make much sense. I have purposefully not given the matrices.

The reason is that not matrices would not have made the pictures clear, but what is more important is to understand and visually see that how the risk is being reduced. The moment you understand that, then making the histogram or other matrices would be much easier for you. And then when you have the matrices you can again revert back to the diagram and have a good look. So, rather than going in the details of matrices, I am only giving you or good understanding about the pictorial concept. Once you understand the pictorial concept, try to do and solve the problems from (Refer time: 16:25) whichever other book you are trying to follow, and use first make the matrices.

And once you have the matrices then you basically unable to do it in a pictorial sense. Another thing which I would have definitely urge the student to do that, because computers being available readily, use excel sheet basically make this matrices. Once you have the excel sheet, in the first column remember always plot s t in the different domains, and in the second, third, fourth, fifth column whatever you have, draw the different type of call and put which you have; long call, short call, long put, short put whatever it is. And once you do that last column would be the cumulative profit and loss. Once you have that, you can basically draw the first column which is variation s t the last column which is cumulative profit that will give you a good conceptual frame work, that how from the matrix you can basically draw the diagram in order to understand that how the different type of options can be combined. Keeping in mind that different type of fluctuations of the s t prices, whether is on the lower trend, or in a upper trend, whether it is basically being bounded between two values. So, you can understand the overall sequence of how the options can be made in such a way that the overall risk is minimize.

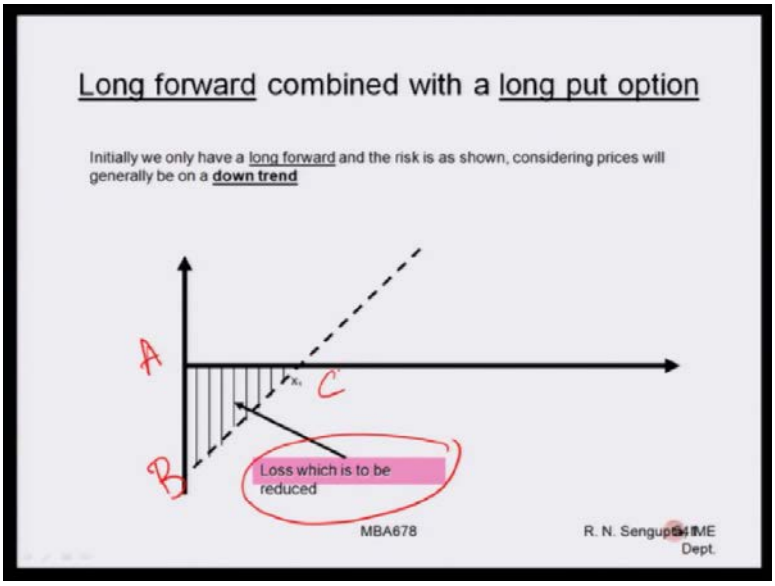
So, again coming back to the pictorial concept. Initially we have a long forward and the risk is as shown. So, what I am trying to make you understand is that, where there is case and how you combine in order to reduce the risk. So, let us go step by step. So, considering the prices generally would be downward trend. In this diagram the forward which you have, the overall hash lines which you have, this portion, which is the diagram and the triangle a b c. So, this is the overall loss. So, now, you will be asking yourself. Well, obviously, there is a profit also in the case the s t prices in the cases next one; that is true, but what we are more concern in the loss. So, our loss is basically this pink one highlighted, this is the loss which is to be reduced, how do you reduce it. Let us go the next step. This was the long forward position. Now you are basically trying to combine with the short call option.

(Refer Slide Time: 18:40)



So, now, combine with the short call option. We have already done, but this is the much more explicit view of the diagrams. So, initial loss was this one. Now you combine with short call option. The combine position is that dark blue one, and see you are overall loss has been reduced. So, initially it was a b c now it is a b 1 c 1, and you can understand overall area a b c is much more than a b 1 c 1; hence you can definitely say that the loss has been reduced. So, now, it gives you good pictorial way that how the overall loss has been reduced, using a long forward on a short call option.

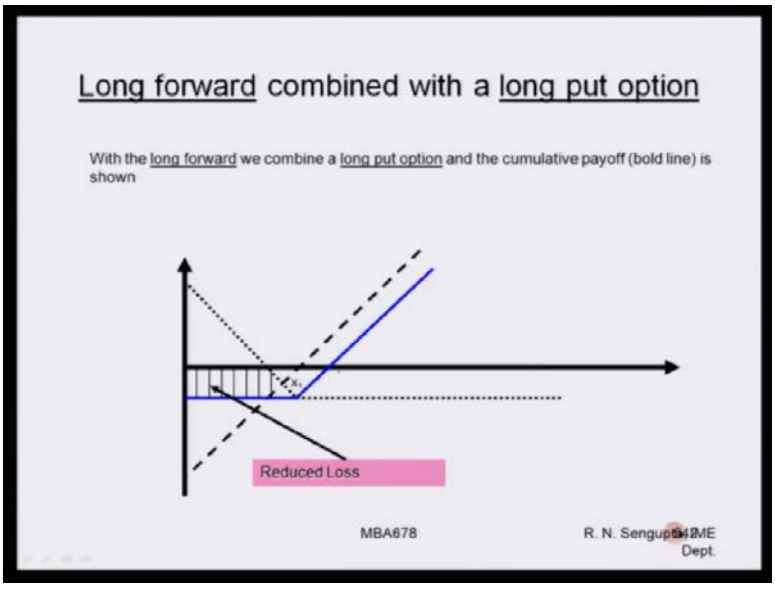
(Refer Slide Time: 19:29)



Now, you will basically again go back to the same example, but basically take a different strategy. Now we are a long forward position, again this is the loss, and the loss

is marked by a b and c. if s t is greater than a x 1; obviously, the this is the no loss. For any values of s t less than x 1 we have a loss. So, initially we only have a long forward and the risk is as shown, considering prices will generally go on a downward trend. So, in both the examples is on a downward trend.

(Refer Slide Time: 19:57)



Now, with this long forward we combine a long option, and the cumulative payoff is basically the bold blue line which is shown, which like this. Initially it was a b c, now when you combine it, what you have is basically a b 1 and this again c. So, initially this was the overall loss. Now this is the loss. So, generally I am trying to. Let us make it much more explicit. So, I will denote it by c 1. So, initially the loss was a b c now the loss is a b 1 c 1; such that you have been able to combine a long forward, combining with the long put option, and the cumulative payoff as being show and the blue bold line. Remember one thing the both the examples are long forward as been combine with the option, but with the notion that price are on a downward trend. So, we will consider later on that if the prices increase how they can be combined.

Thank you.