Quantitative Finance

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Module – 05
Lecture – 27

So, welcome back students. I am sure this quantity finance program, till now whatever lectures we have covered is quite exciting to you, and you are getting interest in this new course; specifically design for people who are really interested in quantitative finance, faculty members are interested. And I am sure with this little background, considering the twenty hours of lecture we have, and considering that we have to cover a lot. This will definitely give you good head start in trying to pick up different concept of different books, different url's data assets and so on and so forth. So, again let us come back to the topic which we were discussing. It will basically to do with the put call parity. So, depending on when you have an option, so we can call on put, and; obviously, put and call can be short and long.

So, you want to find out the parity at which, the prices can be determined; such that either the seller or the buyer does not make any extraordinary profit, because you should always remember; that the options are derivatives whatever are being studied here, are from the point of view of trying to minimize your over all risks. So, you are going for some financial transaction and your main aim to reduce the risk. So, that is why we are trying to utilize, different type of derivatives. Like we have dealt inquires detail about swaps. So, there can be interest rate, there can be currency rate, there can be combination of that. Then options can be call and put, forward futures are there, then we already considered the how you can, if you go back earlier, in the beginning on the course, you are considered in depth different type of optimization model, the safety first principle, maximizing r f and so on and so forth. So, I am just trying to recollect, so your main aim is always to reduce risk.

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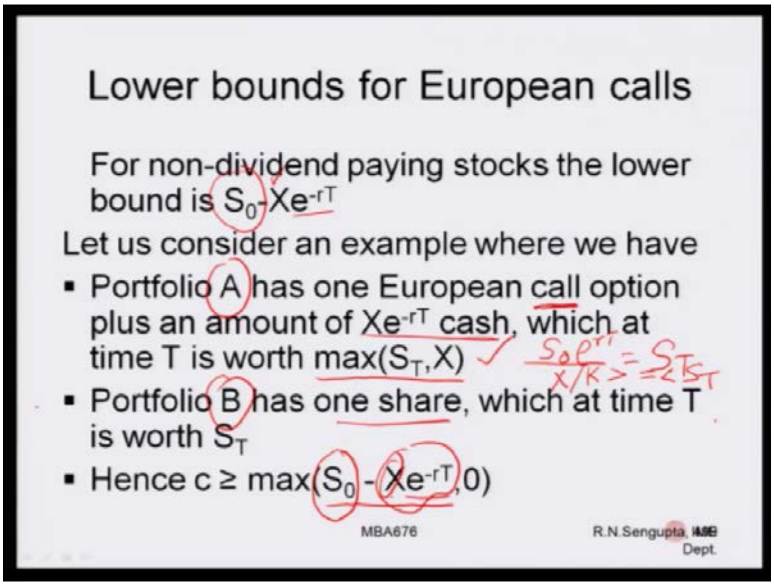

So, for non-dividend paying stock… Coming back to the slide we are going to find out what is the lower bound for the European calls, for non dividend paying stocks; that means, intermittent payments are not there. The lower bound is given by s 0 minus x e e to the power r t. So, even though it may be a repetition, but I would like to highlight again, that the value s 0 is basically the spot of the stock as of now with the suffix zero means; x means, x or k means the delivery price which will happen at time t is equal to T, and when you multiplied by the term e to the power minus r t you basically try to bring the price as of now. So, let us consider again I am again rehashing the same example portfolio a has one European call, option plus and amount of x e to the power minus r t. So, remember it is a call option, and you have this amount of cash. So, now, if you consider, at which at time t, this combination would be worth max of s t minus x. So, how does it happen? Because if you are a call you want to buy, buy back that particular stock or commodity whatever it is.

Now, if the delivery price is x and s t is the actual price, the moment the price is higher; obviously, you will try to go into such an option that the person who is the buyer of the option, would excise that if and only if this condition holds. Now, what we have is that; s zero is increasing, it is increasing at this rate, and this is equal to s t. So, now, you have already decided enough price of x or k, which is based on this equation. Now, if this is greater less or equal to s t that will dictate whether the person will exercise or not exercise, depending on who is the buyer. That would the equation which is given is that will be worth in the maximum value of this, if and only a if it is a call option. So, for the put it will be just the opposite we will come to that again. Now, portfolio will again once share, which as time t is s t. So, now, you are basically has two positions; one is portfolio a, which as one European call plus a cash, and portfolio b has one share. So, in case if there is no profit and loss in going for portfolio as portfolio b, they should balance each other, if not there is no balance; obviously, somebody make a profit, or somebody makes a loss.

Hence, the values of c based on which the contract would be signed between the buyer and the seller. The buyer is paying that price that, would depend on two terms. So, this is the value, as of now the value of the so called spot, because this is the price of the spot, this is the price of the delivery price, counted by e to the power minus r t to bring it as of today. So, difference of them, if it is positive or negative. If it is negative; obviously, you would not, so the c value would always be greater than equal to max of these values, which is zero or negative, so it will be zero. In case it is positive; obviously, in that case, you would be greater, because in anticipation these value would positive which means, the net present value when brought back to this present value, net present value of this amount, when brought back to the present amount is less than s 0, because if that is the case, then the value of the c would be dictated accordingly.

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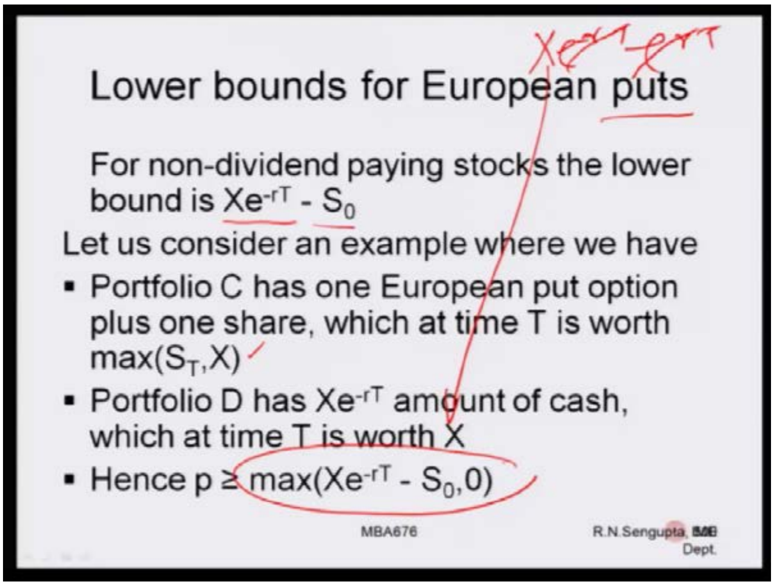

Now, when you go to a put option, just reverse the situation. So, here it was initially it was s zero minus x e to the power of minus f t r t, now it is basically minus has been replaced by plus here and a plus has been replaced by minus here. So, let us consider two different portfolios again, for the put option. Here portfolio c has one European put option. There it was a call option, put option plus one share, which as time t would be worth exactly this, is the same thing as we are doing. And portfolio d would have x e to the power minus r t amount of cash, in that case it was a share, now is a cash, which at time t would actually become x y, because if you have some cash of x to the power of plus r t, and this will increase, but increase of the term it is e to the power r t. So, these would cancel. So, the actual value would basically have is x. So, hence again, the same logic.

The different between the e to the power of e t minus r zero and zero, whatever is the maximum value is. Your value of would be that is for the put option would be calculated accordingly. So, with repetition I am again saying, whether a buyer or a seller that is immaterial, but just considered if you put you want to sell something later on. If it is call you want to basically buy something later on. So, your price is of the put and call that is small c and the small b, or the case of European position, and your time duration is capital T, would be dictated would max of the values. The second one which is basically given in this slide, and the first value has given in the previous slide.

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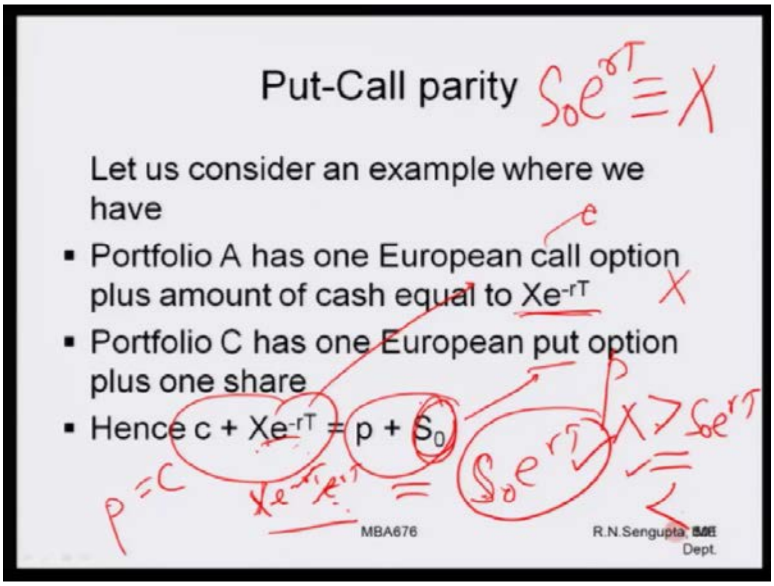

So, let us consider example to basically clarify the conceptual thing about put call parity. Parity means there is some balance. So, you consider a portfolio has a, has one European call option plus an amount of cash which is x e power minus r t. Because in that call of option would increase; obviously, you will exercise it at time t in the cash will increase by the value of e to the power r t. So, it becomes x into e to the power minus r t in to e to the power r t. So, the net value becomes x at time t, and portfolio c has one European put option plus one share. So, in that case if you basically balance both of them; that means, if the call option it is c, and for the put option it is p, whether this is the case here, and this is the case here, because if you see x e to the power minus r t is here. It means it will increase, but it will increase by this amount, and this will also increase, this will increase by this amount, which means this value and this value if they are same then the actual values of in the difference of the c and p would basically b zero. That means, they would be a parity or equalness value in the c and p value. Again I am repeating this will increase by this amount, this will increase by this amount, which is e to the power minus r t in with r t. So, these are cancelled. So, this is x. So, if this is equal to s zero e to the power r t is exactly, if you go back to the original concept, where you have a price of s zero. It increases by this value. This should be exactly equal to x such that the person would be willing to buy or sell that particular call or a put. Now if you come back, if this difference is zero so; obviously, it means the difference between c and p also zero. So, in that case there is the put call parity. In case if x is greater equal to less. in this equals case, you have p is equal to c. in case when this x is greater or less you will basically have put call disparity, or this uneven difference in the prices of p and c which is the put price and the call price for the option which somebody is buying.

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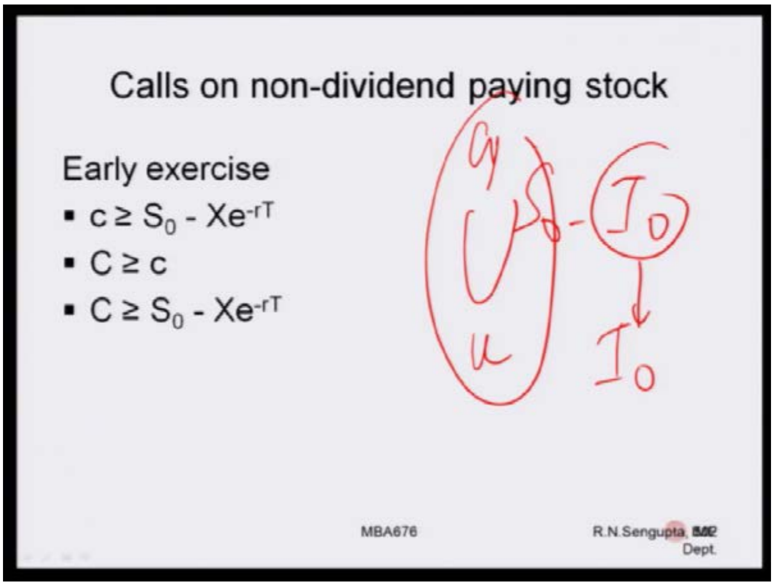

So, now, for the dividend case, follow the exactly same thing, but in this case remember, for a dividend, when this dividend case is being paid, remember this would come, where this i zero is the present value all the intermittent payments. We did recall the concept of zero interest rate or forward interest rate whichever a calculated we have calculated the concept, but some dividend is being paid dividend is being paid, on a continuous basis there is inventory cost or storage cost an inventory cost are a cost is continuously compounded and all this concepts, like the value of q value of capital u capital value of small u all these things can be considered in the same way as we have done earlier in this case also, to order to understand what is the relationship between small c and small p, which are the call price and the put price for the European options.

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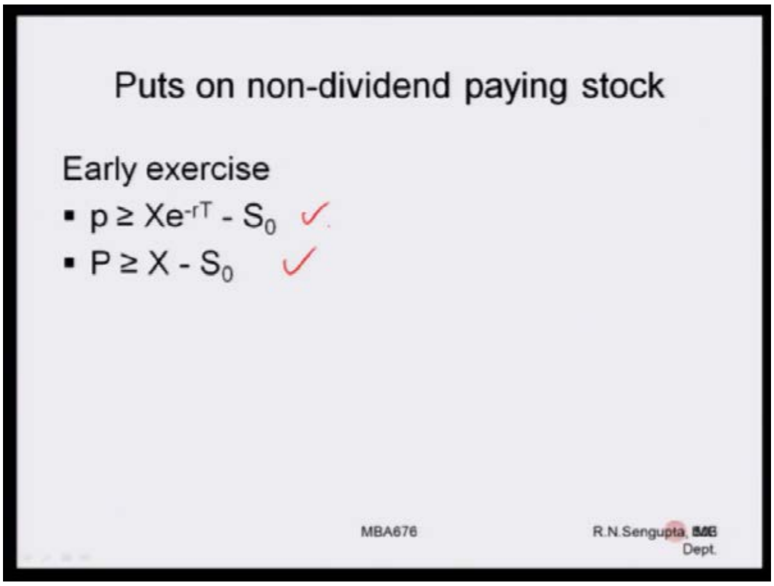

Again, here also, you will exercise early exercise would be happening the moment, the price difference of this. So, if there is equality, there is no difference in the prices; that means, people would going into an option, thinking that nobody is going to make a profit, nobody is going to make a loss; that means, put call parity would be maintained, in case not it will not maintain. I will strongly urge in order to understand this actual concept, please solve the problems which are there in (Refer time: 11:40) point one, point number two there are small problems which you have already solved, point number two is the read the concept of pull put call parity after you have completed the concept of forward, futures, swap, options, and the interested calculations; such that things would become much more easier for you to understand.

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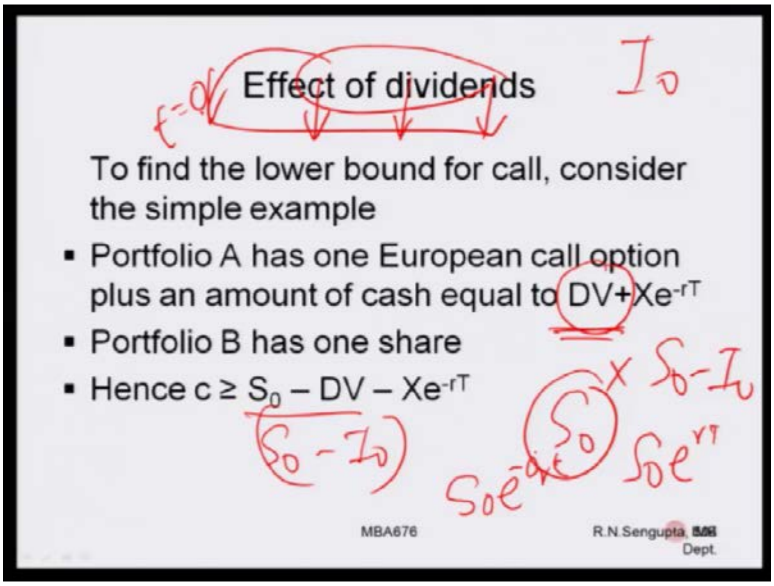

Now, effect for the dividend.; so, as I mentioned in the i zero. Here i zero is being replace by d v which is dividend. In the same case, this is the same case s zero minus i zero. So, actually s zero is now not only s zero; that s zero has subtracted by the value of i zero which is the net value of the intermittent payment which has already made. This is one intermittent payment on second third. So, all these things basically calculated at time t is equal to zero that is basically i zero which is d v. So, in the similar case portfolio a has European call option plus an amount of cash equal to now, is d v is the basically the value of dividend value which will be getting. So, in technically, initially if you remember we had considered s zero. Now, you would not consider this you will consider s zero minus i zero. Now if I ask you the question what happens for the continous dividend yield, it will be s zero e to the power r t and another term would also come is basically s zero e to the power q t minus. Minus means, because it is r minus q as in the case of continuous yield.

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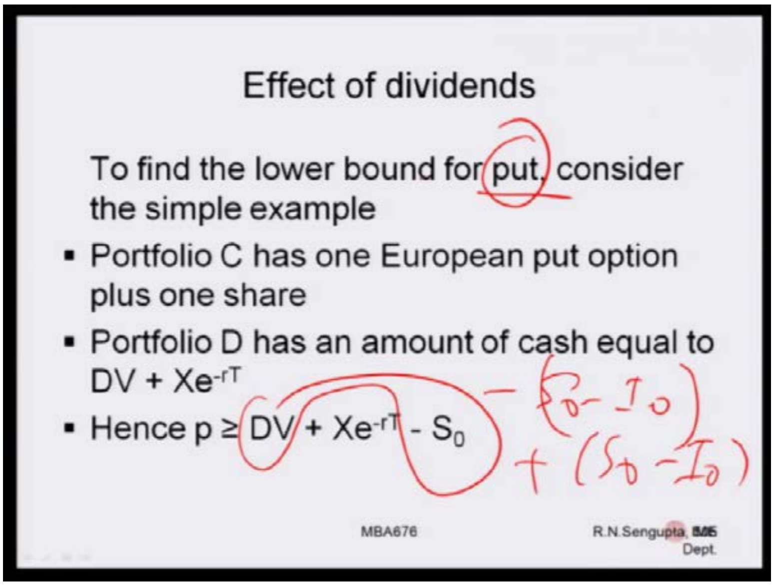

So, similarly for the put option, again consider the same thing. This is the value which you had s zero minus i zero. In one case it will be minus, and in the case it will be plus, depending on whether it is a put or a column. So, only to understand again whether is a put or a called, depending on the whether you want to sell or buy later on, your prices would be dictated accordingly and you can find out put call parity and these cases. So, if I go back, let me go back one step back. So, in this case, when you trying to find out put call parity this s zero is no more s zero. It will basically be s zero minus i zero, depending on how the dividends are being calculated.

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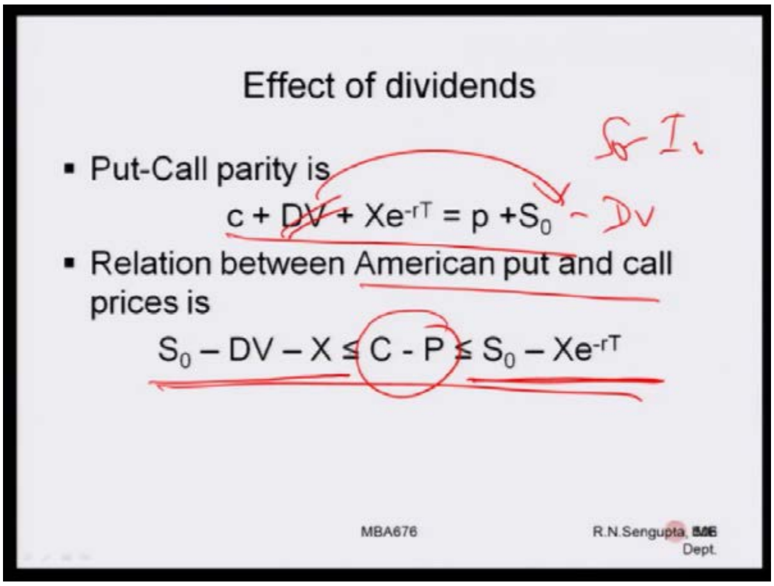

So, here you see, this value, because you bring it here, it will become minus d v. So, this is s zero minus i zero, and in the case you have continuously it will be basically e to the power minus r q t. So, relationship between the American put and the call would basically be the same thing. So, the difference between this price would be between the bound of the no dividend and dividend being paid, because you will basically exercise that if and only if the prices are in between, or else somebody will make a extraordinary profit or somebody will make a extraordinary loss.

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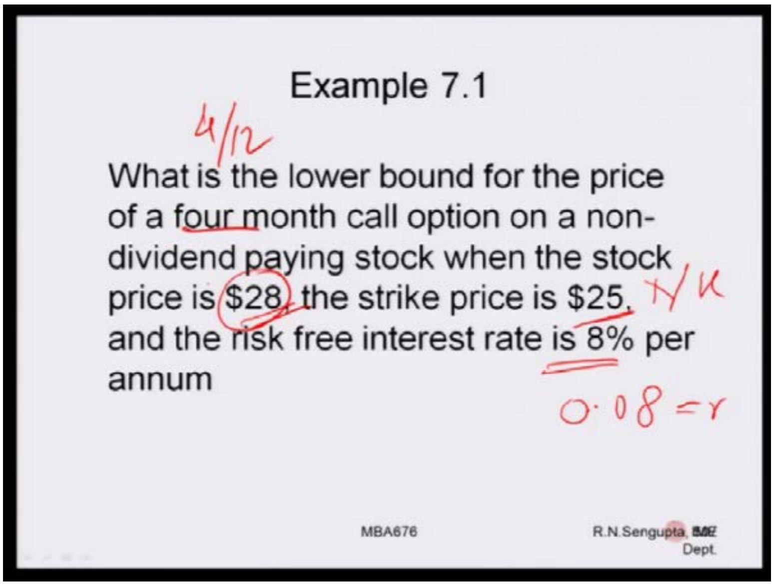

So, you can, to solve this simple example. So, what is the lower bound for price of a four month call option. So, remember four is a basically 4 by 12 in the time scale of the year, and an non dividend paying stock when stock price is s zero, and you have stack price is 25 which is k or x, and you interest rate is given by 0.08. So, this is basically r, and this is s zero, and you can find out the corresponding value of put or a call parities, there you can find out the prices.

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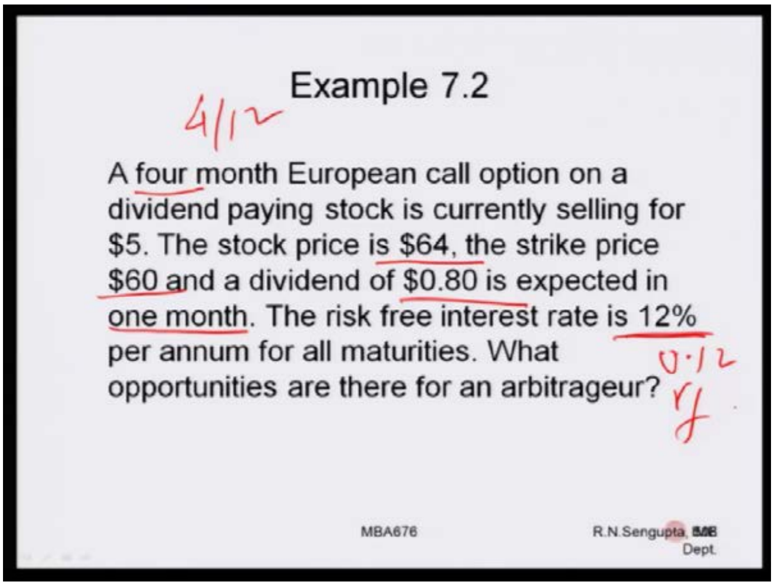

In the case it is dividend paying one. So, here the stock price is 64, stock price s zero this dividend s zero calculated is being paid one month, the four month period 4, 4 by 12, interest rate is given 0.12 which is r f, and you find it out accordingly.

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So, now, considering the concepts, we will continue a discussion, where we will slowly expand or discussion in the sense. Till now you are considered of forward and standalone basis futures are standard on basis, options whether a put or a call; put long, put short, call short, call long on a standalone basis; that means, you are only dealing in only one option, but now consider what we start combining them. Combining in what sense; say for example, we have one put call one and one put and one call plus a forward, or say for example, if you have two forwards plus one call, or consider we have two different forward the different prices, or say for example, consider you have one European call option two American put option. If you have different combination, the main focus would be what do you think about the price fluctuation? Is it going on a increasing trend? Is it going on a decreasing trend? Is it going to fluctuate between two value? Or is it basically hover around in some value? So, based on that you will basically make a plan; such that you can buy sell, put call, go for a forward, go for a futures in such a way that you will mitigate your overall loss; that means, your main aim is again I am stressing is to minimize your overall loss or the risk.

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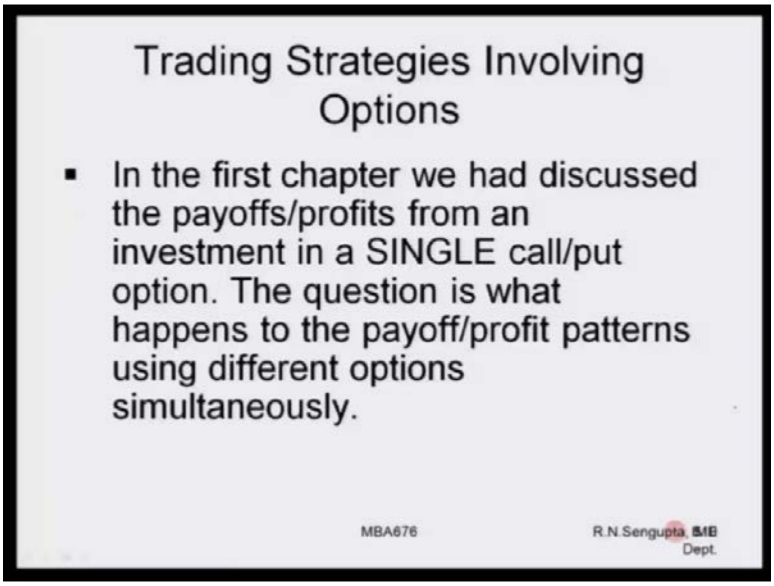

So, in the first chapter we had discussed the payoffs or the profit from a investment in a single call and a put option only, this is again I am repeating. The question is that what happens the payoff profit patterns using different options now; different combinations. So, let us go one by one, in a very simple method, will consider both the diagram plus the profit matrix. Profit matrix is that as s t changes, and as you two different, three different, four different delivery prices, where different type of options, different call and put, it will combine the accordingly. First draw the matrix, then from the matrix that you will basically find out the diagram. Or else if you think it is easier, you can first draw the diagram, and then basically derive the matrix. Whichever you do the answer would definitely come out to be same, but if to understand in conceptually that as s t varies, how does it is effect the overall profit and loss for the combined option or a pool of option, or portfolio option which you are trying to formulate.

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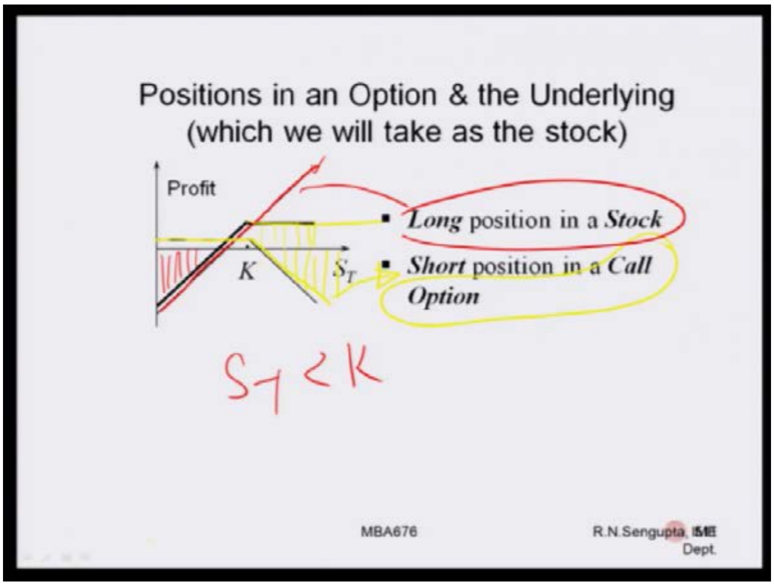

Now consider this for the first three, you have two things one is the long position in a stock, that long means you are going to, the concept is again recapitulating; long and short, short you want to sell, long you want to buy. So, you are basically going for a buying position in a stock, and you are going on a selling position on a call option based on the stock. So, you are basically taking two different modes. So, if it is a long position on the stock. So, I will mark it, this is the red one and if it is a short position in a call option. So, this is yellow I am using, hope you can see. So, this is the call, and this one which I had marked in red is the long. So, now, if you combine them, the combined position looks like the one which is marked in the bold. So, now, think what is the advantage, why have you done it? Now, if you consider the prices of an increase of s t over k.

So, you will see that, if you are holding on for a long position, brought the stock at the some point of time down the future, and you are price agreed upon was k, and the prices increase two value s t which is greater than k. so; obviously, would made a profit, because you bought it at the price k immediately showing in the market, and made some extraordinary profit, but thing what would you have happen if the prices was low. Still you have to basically buy at k; that means, you would have made a loss, if the s t was less than k on to the left hand side. Now, consider you are going for a option, and you have sold that option; such that the price agreed upon you have sold, price agreed upon for this is again k. Now what is happening is that, if the price is less s t is less than k, as you gone in option, the buyer of the option would not exercise, which means that you are not going to make the loss which is being shown here. You are mitigated that, but; obviously, if you are mitigated that, you should also think what would happen if the prices were greater.

If prices were greater; obviously, you were thinking you will make a profit, but the person who has bought that option you will definitely exercise that. If he or she other party exercise that, the overall loss would now be this portion, which means now you are saying that going to face a loss, but if you combine both of them it will even out how. Any increase which is happening here is positive. Any increase which is happening on the lower side is negative. If you combine them the overall combined long position on a stock plus short position on a call option would be parallel to the x axis, and the overall loss or profit would basically depend on this portion. So, now, this need not be just above, it can be little below or towards zero also, that does not matter, but what you have you done is that, you are combined this long position combined the short position in such a way that for any negative price moment of s t less than k. You have been able to mitigate and less than the overall impact, but on the other hand, any positive moment of s t greater than k, there also you have been able to mitigate the overall loss to a maximum possible extent, because your main motivation is not to make a profit in order, that when you have actual dealings with options that futures, or stocks, your main concern is basically to minimize the risk.

So, with this we will end our class today, and will continue discussing different type of position of combining different options, different forwards, different features in a long and short position. and as you continue doing the examples in options futures and forwards, you will see that trying to formulate different combinations and options and forwards with different stock price, different time duration would definitely give you a good feel that how this problems can be solved.

Thank you .