Quantitative Finance

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Module – 05

Lecture - 26

So, continue our discussion the swap between A and B where it was for B was disadvantages by 1 point 2 in the fixed rate and disadvantages 0 point 7 in the floating rate. So, we want to basically convert, they are respective position from floating to a fixed and fixed to a floating.

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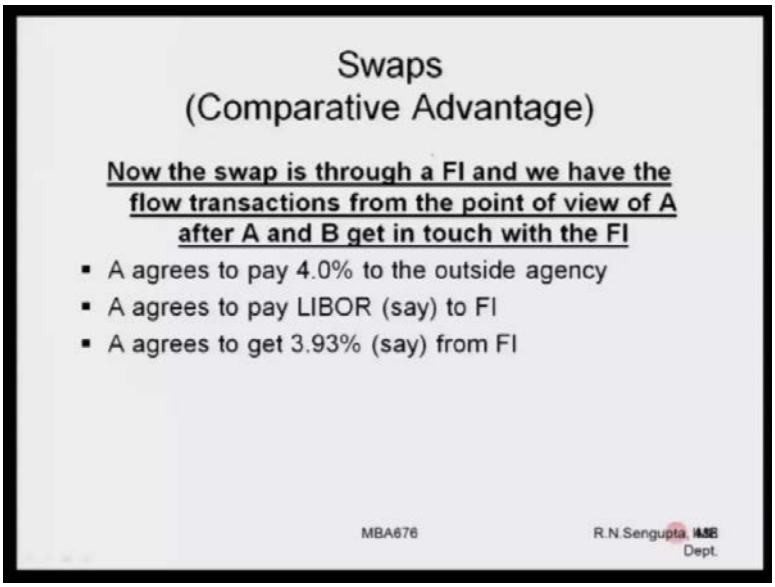

So, considering that A and B get in touch together with FI, A agrees to pay 4 percent to the outside agency. And A agrees to pay LIBOR to the FI and A agrees to get 3.93 from the FI. Now A and B are not transacting been themselves, they are trying to basically transact with the help of a FI. So, what happens? Let us see.

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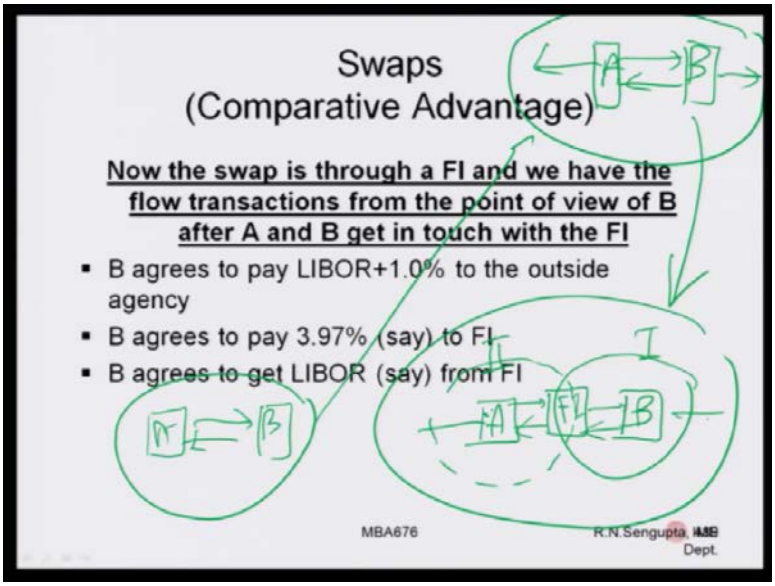

Now, considering from the point of view of the other parties. So, if you have considered A and B, so they are the same thing. Only the plus and minus based on which how you are trying to analyze would be different. So, it is like this; this is A, this is B and this is an FI in between. So, you are trying to consider the transactions one at a time; 1st case, 2nd case. So, this would be case 1, case 2 combine together is when you have the FI, A and B. And later on we will see that technically even though we skipped it, but it is very obvious. Actually A and B initially they were here. Next case they came here, A and B with the third party. So, this was A, this was B. This was A, this was B and once this is done you will come to this. So, the actual flow becomes A and B only. Then, A B along with their respective 3rd parties; then A and B with the respective 3rd party along with an FI. So, is basically stage by stage as we move.

So, here B agrees to pay the LIBOR plus 1 percent, B agrees to pay 3.97 to the FI and B agrees to get LIBOR from the FI. So; obviously, if you consider this transaction you will just place them in the top of the arrow and get to understand, what is the overall flow happening between A to FI, B to FI or A to the 3rd party and B to the 3rd party.

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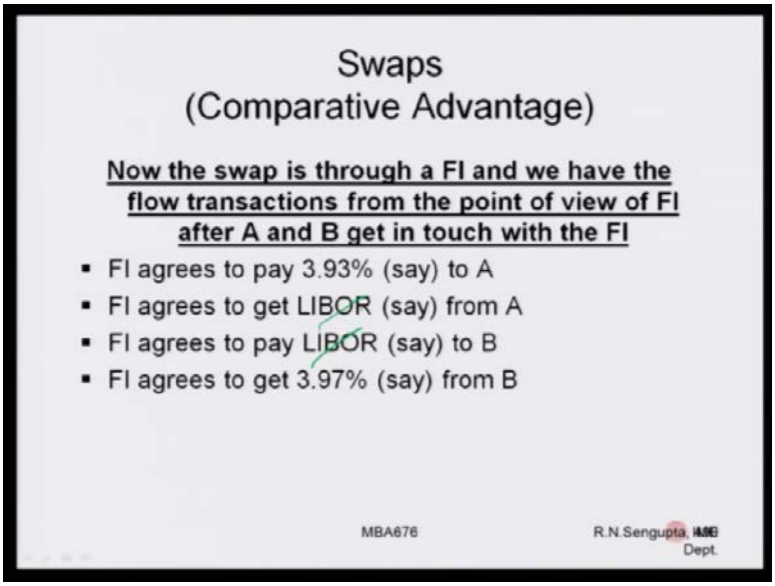

Now, consider the case of only the FI. So, FI is in between; so obviously, there are 4 fold arrows to going outside, to coming inside. So, FI agrees to pay 3.93 to A, FI agrees to get LIBOR, FI agrees to pay LIBOR to parties A and B respectively as written in point number 2 and point number 3. And FI agrees to gets 3.97 as from B. So, if you consider the overall transaction for FI, this LIBOR LIBOR will cancel and this 3.97 minus 3.93 is basically 0.04. So, this is 0.04 which is the profit for FI is being shared equally to the quantum of 0.04, 0.02 by A and 0.02 by B. If you basically draw the diagram at the end of the result we will find out that is the case.

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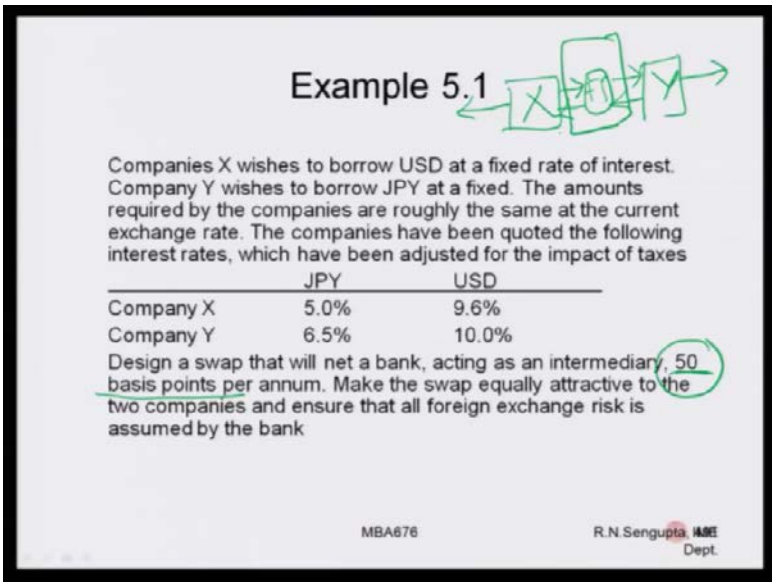

Now we can, we are coming to the interest the currency. Companies X wishes to borrow

US dollars as a fixed interest rate, company why which is to borrow Japanese yen at a fixed interest rate. The amount required by the companies are roughly the same on the current exchange rate. The companies have been quoted the following interest rates. So, basically it is currencies, but we are converting them into the interest rates. Company X if you see the 1st row, it is Japanese is 5, US dollars is 9.6; Company Y is 6.5 in Japanese and 10 percentage in US dollars. So if you compare them, company Y is facing disadvantage of 1.5 in the Japanese yen and it is facing a disadvantage of 0.4 in the US dollars. So; obviously, it is make sense for company Y; if it is case, it is dealing in only Japanese yen it will convert to a US dollars case. So, design a swap, that will net the bank actings an intermediary 50 basis point. So, now, here is the information given. That means, once you have the overall transaction; company X, company Y and the FI. So; obviously, there are arrows going out, coming in, going out, coming in, going out; third party going out, third party, this is X, this is Y.

So, what it says? Once you find out the overall transaction FI, plus minus plus minus everything. The overall percentage of profit which the FI is would be making is 50 basis point that is the end cracks results. So, basically once you draw the diagram, inflow outflow diagram, you should basically concentrate on only the inflow and outflow diagram for the FI is based on. That you will find out that it is 54 basis points and then calculate backwards to find out what is the basic equilibrium equation based on which you can find out the inflow and outflow diagram.

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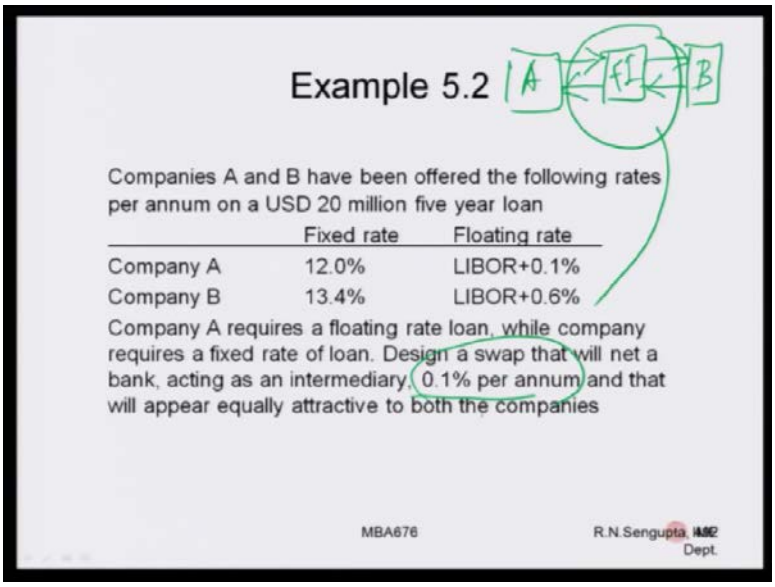

Companies A and B. So, this is a new problem. Companies A and B have been offered

the following rates per annum on a US dollars 20 million 5 year bond. Company A has concentrate on the first row, is 12 percent fixed rate LIBOR plus 0.1 in the floating. company B is 13.4 and the fixed LIBOR plus 0.6. If you compare the difference, company B is facing a extra interest rate of 1.4 in the fixed and basically facing in extra interest rate of 0.5 percent on the floating rates. So; obviously, it means the company B would like to changes its position from a fixed to a floating, hence company A would changes position from a floating to a fixed.

So, it basically means company A requires of floating interest rate bond. While company requires a fixed interest bonds design a swap that will net the bank. Again when you come to the financial institution, it will mean the overall profit the bank would make is 0.1 percentage per annum. Such that it will be equally divided by both the parties, such that A would pay the extra amount of 0.05 percentage to the financial institution and B would pay the same amount of 0.05 percentage to the financial institution. So, in this way you will basically have the swap. Such that if this is A, this is the FI, this is B. So, this is outflow, inflow, outflow, inflow. If you find out this total thing, this value would come out to be 0.1 percentage per annum and this is equally divided between both the parties.

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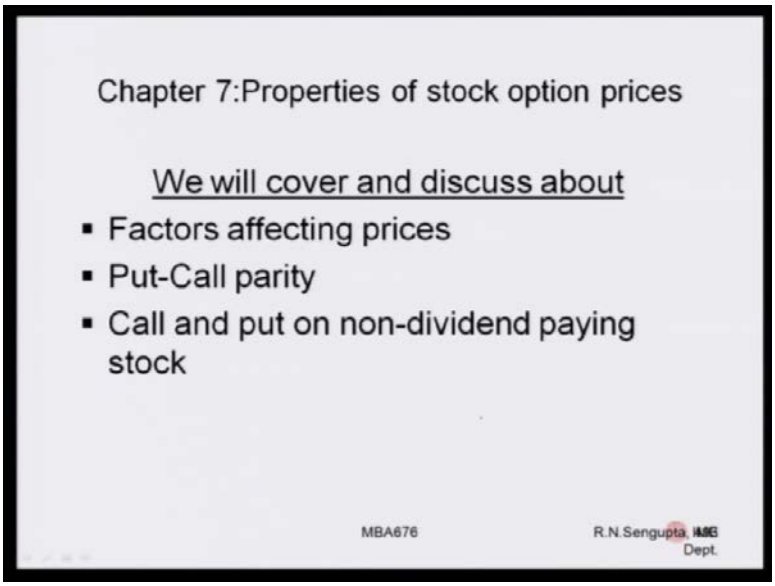

Now, we will consider the properties of the stock option prices. So, we will cover these following things. The factors affecting the prices, how we find out the Put-Call parity and how will find out the call and put on a non-dividend paying stock. So, this would be more of a conceptual chapter and if somebody wants to refer definitely all the

information is there in John C. Hull. So, but we will give a flavor of this corresponding to the fact that we have already understood and dealt with the concept of say for example, what is an option, what is a put option, what is a call option, what is a long put long and then short put, long call, short call. What are the difference forwards. And also based on the fact that the interest rate rates which are there are the futures interest rate, the 0 interest rate or the par yield. So, we will only concentrate for the calculation on the 0th interest rate on the forward rate, that will become a evident and it proceed with the calculation and understanding. So, we would not going the detail calculation, we will give you a much more conceptual frame work. How this concept basically can be done? Such that we can understand, how the Put-Call parity and all these concept can be dealt only problems as we consider.

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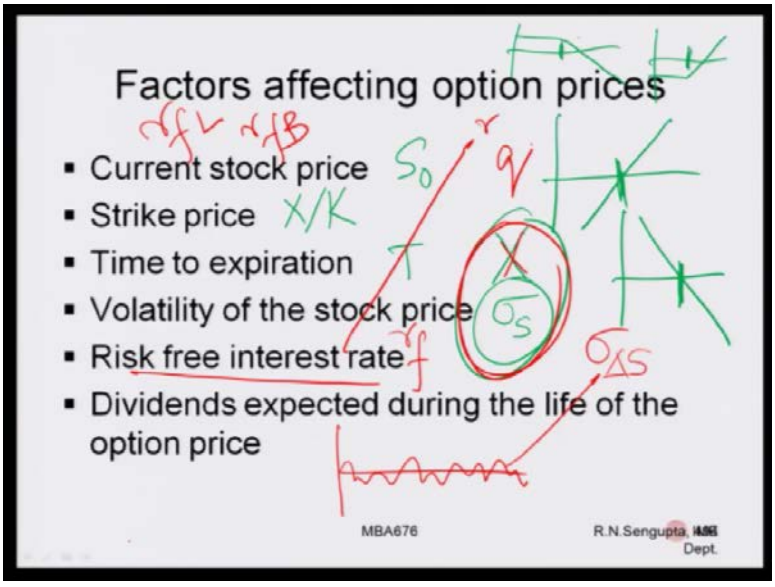

So, the factors affecting the option prices are: the current stock price for which the symbol is S, with a suffix 0 or o; basically means the price of today. Strike price is basically x or t which we are already considered for the forward and futures, if you remember the graph. This are the graph this. The value which we had was basically X or K. Or say for example, we have a option this. So, the price at which they person would basically exchange the option, that is basically this strike price or the delivery price. So, time to expiration will consider that is T, this is X and K as mention. Time to expiration is T, which is basically the European call, volatility of the stock price.

So, let us pause here for 1 minute. By the world volatility will still strict to the standard deviation, but this volatility is not for stock price remember that. This is nothing to do

with this, so you should never consider this. Because actually what will be considering is the standard deviation the rate of change of the spot price. Because if you draw the spot price differences the actually graph would be like this. So, what we are interest rate in the standard deviation of rate of change of the spot price. It also considers the risk free interest rate which in our problem of the portfolio analysis was r f. But with the caveat or important note, that risk free lending rate r f l and risk free borrowing rate both would be the same. Because we will consider that if you go to the bank and borrow money, if you go to the bank and lend money the interest rate would be the same for both the cases even though that is not true.

We will also consider dividends, if we are expected during the life of this options are initially 0; that means, they would be no coupons and hence they would be no dividend being paid, but later on will relax on assumption and consider. That dividends are being paid at some interpreting payments, such that we are able to find out the dividend payment at time T is equal to 0 that is the present value. And later on we can also relax and find out what is the dividend paying yields, such that continues dividend would be consider. As we do the case for considering the continuous risk free interest rate. So, if you remember the problems we consider for finding out the futures and forward prices. We consider 2 values; one was r was the risk free interest rate, this is the one. Another concept we consider was the q which was the dividend yield continues compound. So, will bring that into the picture also.

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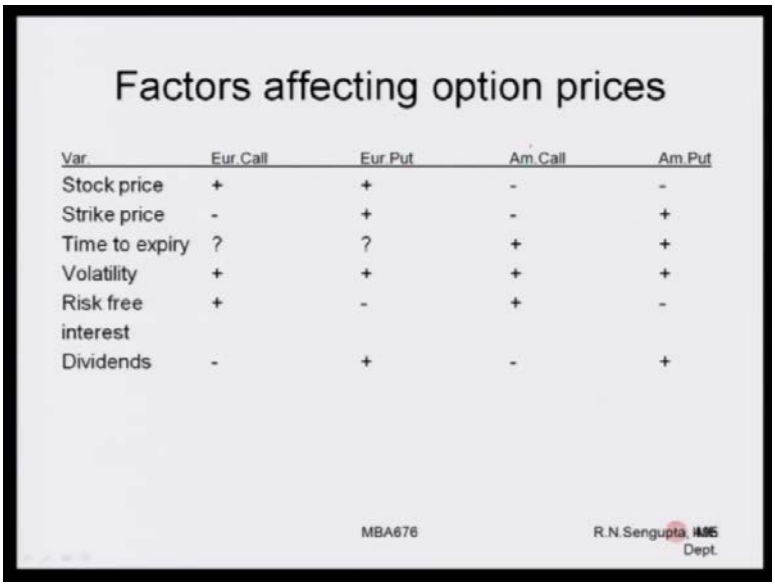

So, generally if you refer John C. Hal, you will understand that if you have this two

different calls; one is call put call option and the put option for the American one, call and the put option for a European one. So, the European we know that the expiration date is capital time T, for a European American you can basically excises it must before.

So, we consider the stock price, it will be considered if in general. The European call and put has positive relationship with the stock price and American call and put has a negative price. Strike price which is X or K, basically would have negative for the European call and in American call would have a positive effect on the European put and the American put. Time to expiration for the European call and put we do not, because that is now immaterial. Because that would basically depend on whenever it expires that is the time frame which is a fixed. And for the American call and American put; obviously, it will depend and when you are try to excise it and the time to expiration would be much before then the capital time T.

Volatility as I mention sigma, is always positive for whichever case you are considering; American put, American call, European put, European call. Risk free interest rate is in this case for the put it would basically be negative for the negative relationship and for the call it in positive; we will see that later on. And for the dividends it will be positive for the put position and negative for the call positions. The reason for the for the dividends, the risk free rate why they are negative positive? As we continue discussing this, i will try to explain it in more qualitative terms; such that you will understand that what is the intensive difference between the positions of a put and call? Leave a side, who is buying and selling, that is not important. What is important? That if you go for a call or if you go for a put the situations would be consider in such a way. That you should basically consider when, that actual transactions will take place. If I am going for a call position it is means what? I am going to call some product or call some financial instrument later on.

So, obviously, I need to go for a, if I am going for a call later on. I will basically go for a put now. So, on the case when I am going for a put later on then; obviously, I will go for a call now. So; that means, long and short go hand in hand in such a way that if I am selling later on, I will buy now. And if I am basically buying now and basically sell later on in order to basically make the position clear such that the overall cycle calculations. Or say for example, if I am going to buy later on I will sell now and if I am basically going to sell later on I am basically buying now. Based on that you will basically have the put and call in such a case; the dividends, risk free interest rate, the volatility, the time

to expire, the strike price on the stock price would basically have a positive or a negative effect. On whether is a American put or European put or an American call and an European call.

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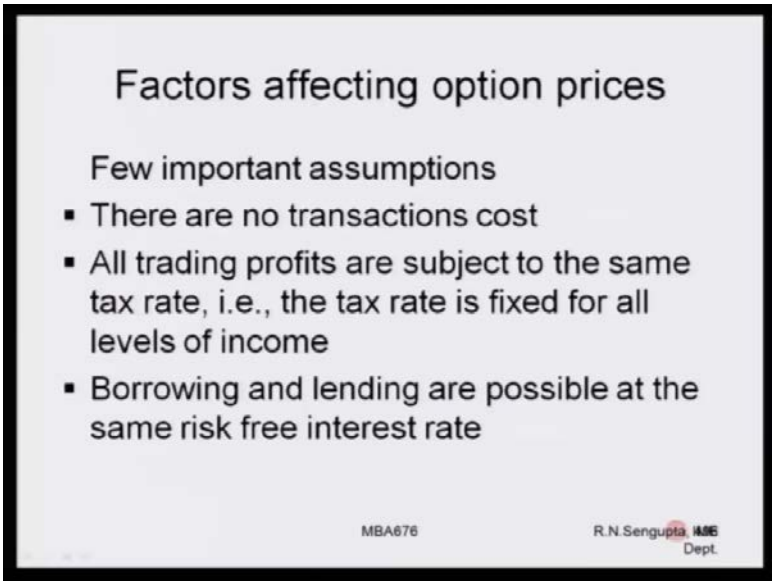

So, few important assumptions for the option pricing as we consider. Is that there is no transaction call? So, whenever two parties are transacting due to themselves, they are no brokerage cost. We would not consider that in our picture. All trading profits are subject to the same tax rate; so obviously, there are no discounts or they are no higher or lower interest rate based on which will to be calculated or there are no higher and lower tax rate based on it overall calculation would be done. We will also consider as I had been mentioning time and again in different discussions, during the course teaching. Is that the borrowing and lending rate which is the risk free interest rate which you have is basically the same. That means, whenever you go and borrow money or whether you go and lend money the interest rate r f would always be the same.

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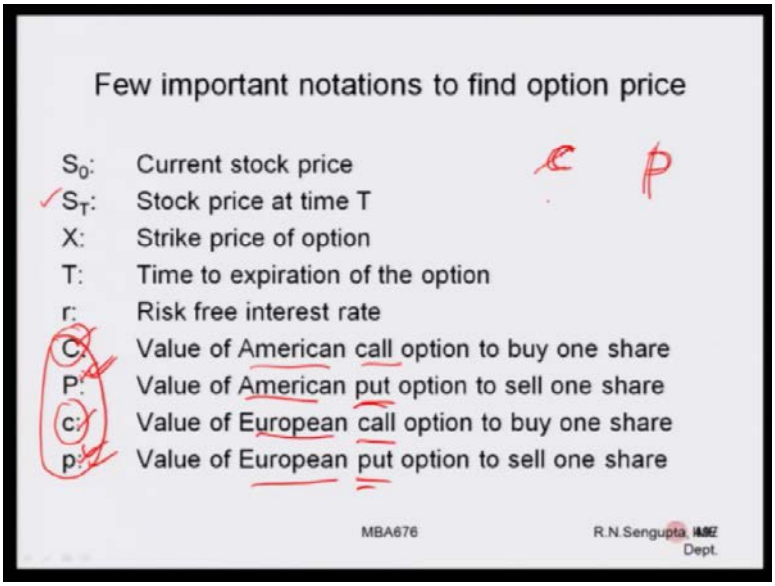

So, for you important notations which are current stock price would be denoted by S with the suffix 0, depending on when the stock price is stock price is basically we calculated. So; obviously, if you go to the second bullet point is S is the stock price and suffix T is basically at that point of time when you are trying to find out the actual spot of the stock. X or K whichever symbol we use is the strike price for an option. T is the time to expiration for the option for the European call and obviously, for the American call would be much before. r with the suffix f or r only r is the risk free interest rate which you utilizing for our calculation. And these 4 symbols capital C, capital P, small c, small p are the actual call put call put prices. Only thing is that, we are using the symbols of capital C and capital P for the American one and we are using the small c and small p for the European one; as point number 1. Point number 2 also remember C is being use for the case for the call option and P is be utilize for the case of put option.

So, obviously, the capitals are being used again I am saying for the American one, small symbols are being used for the European one. C is being used for the call one and p is being utilize for the, so the put one. But; obviously, you can use the same symbol later on also like c would always be the call, p would always the put small one. And you can basically have this call price and the put price depending on the buyer and the seller. You can basically use different type of subscript depending on how you want to basically notify does those symbols in the problem. But we will still stick to the suite to the variables of capital C, capital P, small c and small p as just discussed in the slide.

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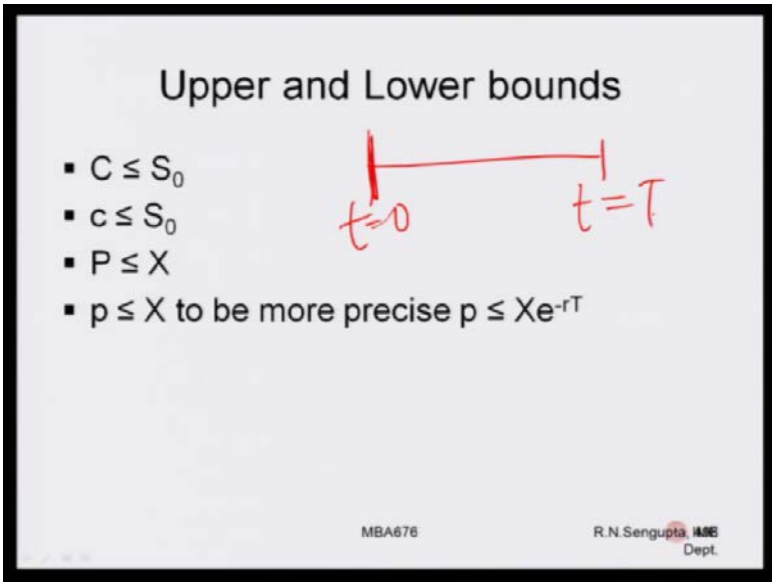

Now, consider the upper and the lower bounds. So, consider that capital C or small c is less than equal to S0. So, what is it means? That if I am going for a call position and I am buying now, then the price should be such that in the case if the price starts increasing; that means, S is greater. Then obviously, we have to take a action in such a way that any positive and negative movement in the price, would definitely mean that I am not willing to go for a short or long position depending on the put or the call. Is simply like this, like consider the situation is this. This is time t is equal to 0. This is can time t is equal to capital T.

So, if I am going for a call position, it means that call means I am going to call a product later on. At time t is equal to capital T. If I call the product which means the; obviously, I would have shorted that actual one now. So, if I short now and call later on, I am selling now and again buy it later on. So, hence if the price fluctuation is such that there is a difference between call and the short position for the same product; obviously, that will basically the disadvantage for the payers who is basically going for a call and short position. Similarly if I am basically going for a put position, if I am going for a buy position later on so; obviously, have to sell. So, the sell and the buys or not not matching each other in the in the quantum terms so now obviously, they would be have huge amount of loss in the two transactions.

So, similarly if you consider the delivery price X or K, then the small price of p which is the put option whether for the American or the European one should be such that they should not be any mismatch. And how these things are done? I will explain that in the

sub sequence slides later on.

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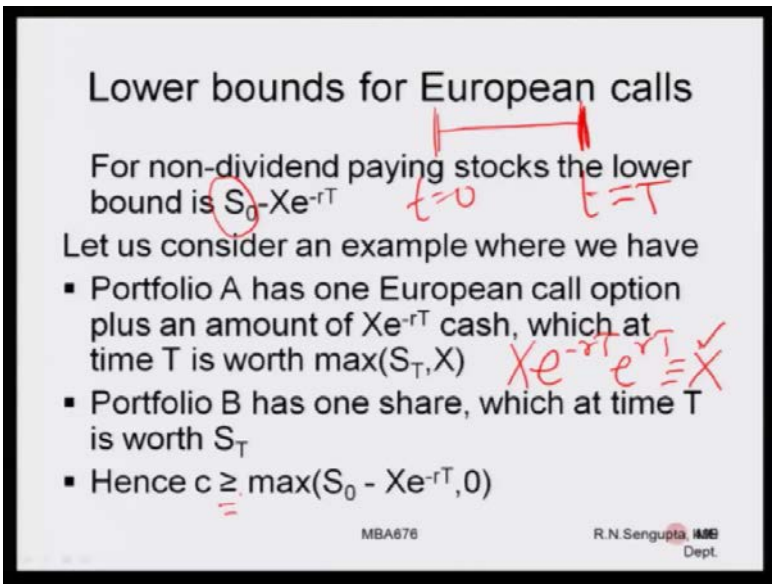

So, now go, let us go one by one and basically understand the lower bound for the European call. Remember in the European call; obviously, the prices as such that, that you can only excise when the time to expire it comes. Now, this is remember, second important point is they are for non-dividend paying stocks. Now, the lower bound is always is S0 minus X e to the power minus r T. So, let us consider these term separately. The first value is S0, so what is S0? S0 is the price of the spot as of now. So, now, S0 is spite of the all the stock of the spot as of now so; obviously, I have decided on a price which is X, for the transaction to happen T time frame down the frame. So, this is T is equal to 0, this x is equal to at this point of time.

So, what I am trying to do is that, that expires I am trying to convert to a present price. How do I do that? I basically multiply X into e to the power r T where r is the risk free interest rate, T is the time. So, if the difference between them is positive and negative; obviously, some would you make a loss, or some would you make a profit. But why it is a lower bound? Because in any case if the low if this calls for the European one, it this is the not the lower bound it would mean that they maybe prices which would be higher than this. So; that means, lower bound is we cannot basically go below this. So, if the prices are below this; obviously, it mean that the calls would now basically have a different position such that the overall profit now basically changes. That means, from a profit it would become a loss and from a loss it would become profit. Such that it would make sense to change your position in order to basically make extraordinary profit.

So, let us consider an example for this. So, let us go through the bullet point which has stated. Portfolio following a portfolio A, which has one European call option plus and amount of C e to the power minus r T is in cash. Which at time T is would be worth what? So let us see. Now I have only one European call; that means, I am going to call means I am going to buy it back. At what time? A time T is equal to capital T. The cash which also I have is x into the e to the power r T. So, if I am going to buy back, what I will do in the cash? The cash I have already in hand, I immediately go to the bank deposit that cash. What is the amount of money which I deposit in the cash? It is X into e to the power minus r T. So, if I deposit X into e to the power minus r T into the bank; the total amount will increase.

What is the rate of change of that amount will increase? It would be now X into e to the power minus r T into e to the power r T. So, the overall value now becomes X. So, once it is X then what I will do? I will immediately with that amount of money I will go, I will go to the bank withdraw that money of X and immediately go to the option and buy it. That means, because I am want in a call position which means, the amount of cash which I has is exactly compensated by its rate of change of the interest rate with increase. Which is X and with this X amount I basically buy back that particular call option which is there already sign by me.

Now consider the portfolio because one share which I time T is what s t. So if now we have another share which is price s t, s t means actually there is a price. So, now, if I consider the price of that particular stock and if s t is less then X and I am trying to call it. What I will do is that? I would not exercise that. So if I do not exercise that it will mean basically I will go to into the market buy it and a buy it a s t, because if I had gone for and and gone for that buying that at particular asset at a price of X; if X was greater than basically s t, then obviously I will make a loss. That means find the point of view of the buyer. If it is other way not; obviously, I would exercise that basically able to buy it at a price of X and immediately sell into the market at the price of s t.

So, hence the overall price of c would be the difference between as of now, S0 is the spot price X e to the power minus r T is basically the Delhi prices of now. The difference of that and 0 whichever is basically greater. Hence my value of c should be greater than that because, if it is not; and obviously, mean the seller who is going to the market is going to make a loss. So; obviously, in order to compensate that the exact price should be equal. So, in the case if it is equal no party basically makes a loss, no party makes a gain.

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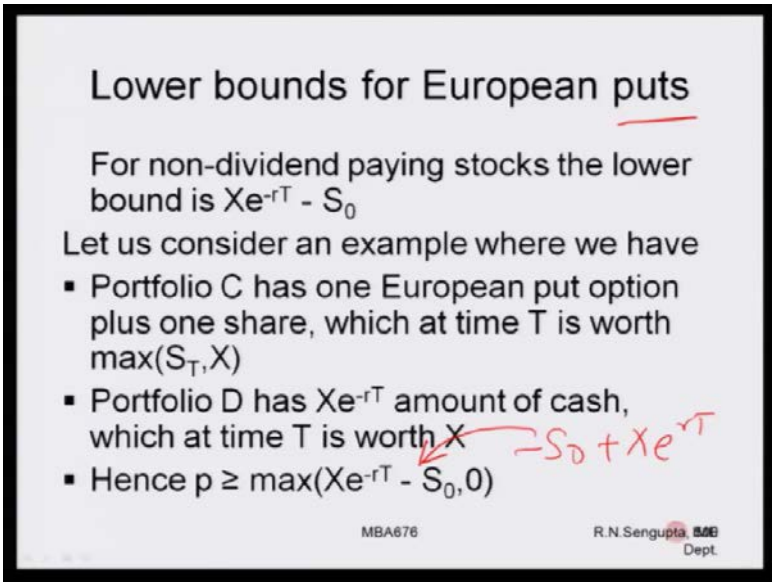

Now, consider the lower bound for the European put option. So obviously, change the scenario. So, if you have put; that means, you are putting and order to sell. So, again consider the same scenario, but in the other way round. Here you have a portfolio c with an European put option plus one share. So now, the share value would increase, what you will do is that? One is expires you sell in the market. So, if you consider the overall scenario, initially if it is a call then it was S0 minus X e to the power r T. Now is a put, basically put a minus sign in front of them you get this; based on the maximum value you will exercise that. So, the value of p in the case of the put option should be greater than equal to max of X e to the power minus r T minus S0 or 0, whichever is higher. You basically have that value of p or else you want have that value of p to zero. And in the case when you have basically the call option, it will basically max between the differences; now it is just the reverse. So, the movement the prices is more than in 0 value; obviously, that is the value of the call option.

So, with these two slides I would urge the students to basically go through the slides and I also try to go through the John C. Hull; and based on that this we will continue the next class of discussion.

Thank you very much.