Quantitative Finance

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Module – 04

Lecture – 24

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So, welcome back, let us continue our discussion about the interest rate, the 0 rate, the forward rate, and so on and so forth. So, the second of this, we will we were discussing that what was the forward rate in the based on that is basically the ((Refer Time: 00:25)), the agreement between 2 parties depending on the rates which are prevailing in the market. So, what rates will consider would be either the 0 rate or the forward rate of the ((Refer Time: 00:33)) that will depend on whatever problem you are trying to solve, and also conceptually have how you are trying to build then interest rate into the market.

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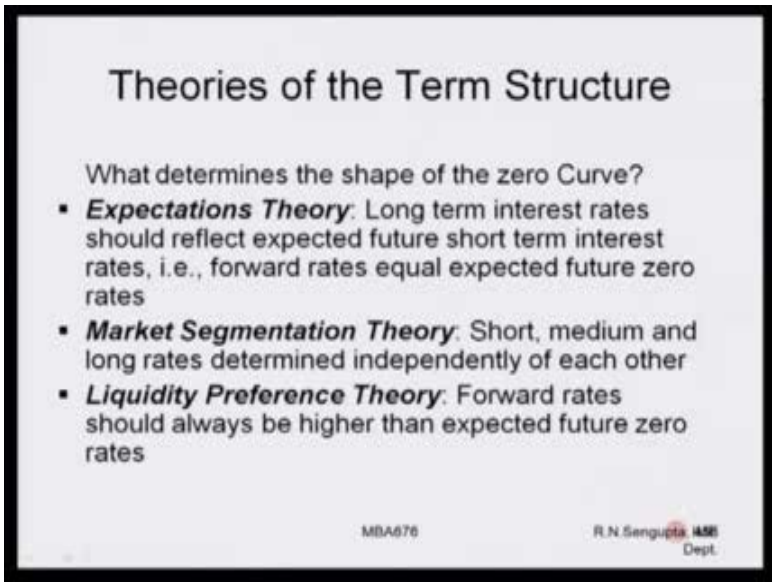

So, we are discussing the theories are term structures in the last class, I just mention the highlight rate points. So, the expectation theory, long term interest rate should reflect the expected future short term interest rate, such that forward rates equal expected future 0 rate, which means that the interest rate based on which you are trying to do your calculation for forward rate should exactly equal to futures is 0 rate, such that any coupons, you will consider for this transactions would be of 0 value. For the market segmentation theory short and medium long term interest rate determined independent of each other of whatever the 0 rate it, whatever the forward rate is on, whatever the long term interest rates are they would be calculate independent of the fact whatever calculation, which you have done so far.

And in the liability preference theory that forward rate should always be higher than the expected futures rate future 0 rates, because that would be true, because in the future will always consider the interest rate I increasing, such a way that it make sense for you to going to forward rate and futures and so on so far, because forward and futures mean are being brought into the market in order to basically hedge your overall risk.

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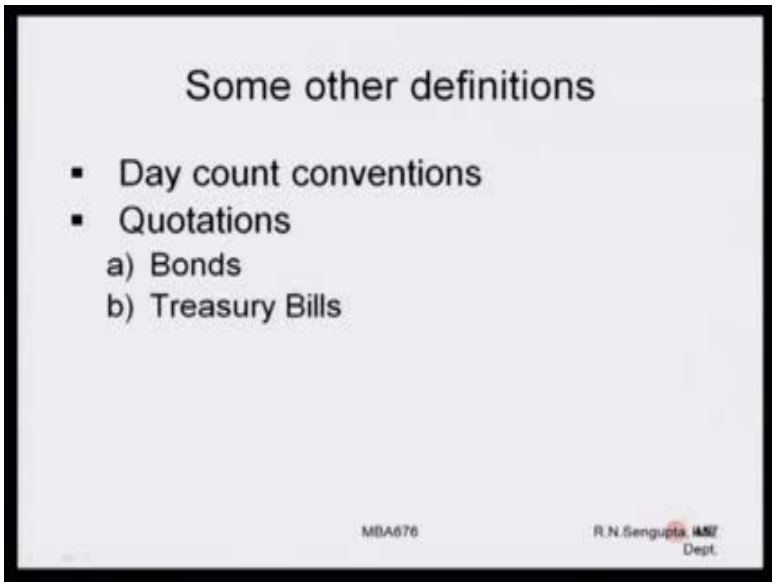

So, some on the definition we will consider what are the day counts conversions, what are the quotations for the bonds, and what is that treasury bills. So, will come to that first letter is going to the details of how the swaps calculation. So, swap calculations is very important at least for trying to find out that how you can basically covert a liability into asset, asset into liability how you can convert of floating interest rate to a fixed interest rate are vice versa or how you can basically convert 1 currency risk to other currency depending on the mutual advantage, and disadvantages of party or a basically I institution phases in the market.

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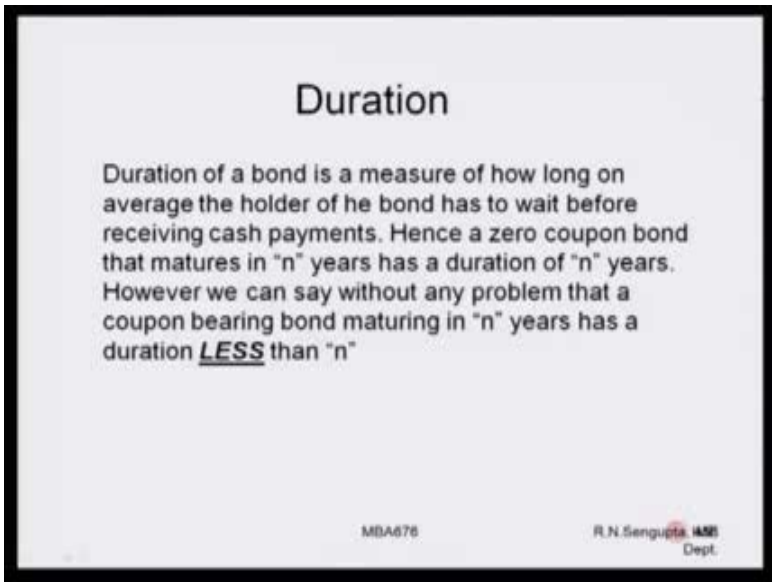

So, if you talk about the duration of the bond; the duration of bond is the major of how long on a average the holder of the bond has to wait before receiving the cash payments. Hence a 0 coupon bond that matures in 8 years has a duration of n years; however, we can say that without any problem that the coupon bearing bond maturing in n years has a duration which is less than n year, because duration means the actual time between which you basically get back the total amount of money, which you have investing. So obviously, they for the bonds they would coupon payments. So, if there are no coupon payment bonds, then the overall duration take will definitely, and because you will get the total amount of money after n years, but if the they are coupon paying bonds; obviously, you are getting back some money intermittently or in between the time frame which is less than n years. So obviously, your duration in the bond would be calculated accordingly.

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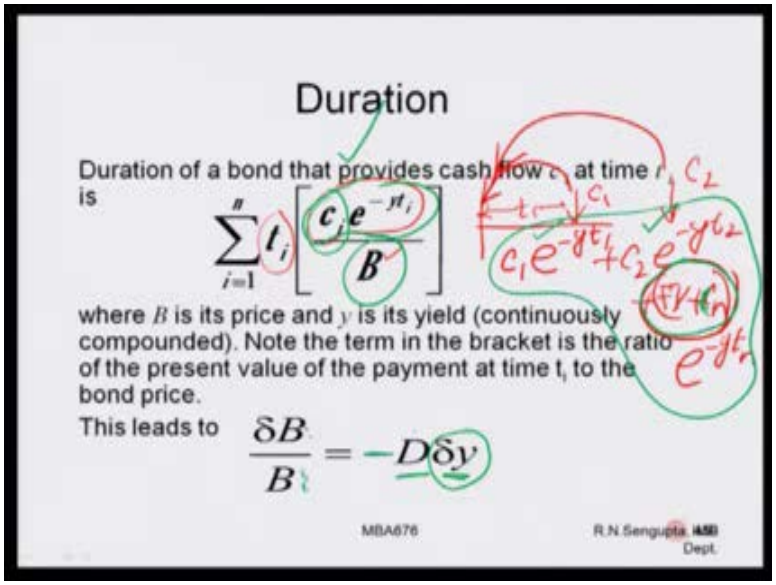

So, now how do we [03:19] find out the duration on the bond. So, duration on the bond is basically given by this formula which is very simple to understand. So, now if you consider that B is the price, and y is the yield which is continuously compounding yield for the bond and per annum basis. So, what you have is 3 terms 1 is the price of the bond; that means, you divided find out that the equality wholes for the duration such that the duration can be calculated accordingly. Now what are these t 1 is the time and the value of c i into e to the power minus y t i is basically the overall value, the present value of the overall investment, which is being made at some point of time t 1 down the line.

So, consider this. So, say for example, you are total amount of payment which you are amount after t 1, it is the t 1 is basically c 1. So, what you will see that you try to find out the present value of the c 1. So, that would be what c 1 into e to the power minus y into t 1 which is the first term, if i is 1. Now if you basically c 2 among which is being among, again you want to find out the present value of the c 2 value that would be plus c 2 into the r 2 power minus y in t 2 which is the value in i 0 2. So, if continue and find out all the values; obviously, the last value be is the case where it is the phases value, if the which is 100 plus the coupon which is being paid which is c n into e to the power minus y into basically t n. This value which you have the value, which you have here, which is the phase value plus c n in the bracket is the c n value, which you have here when i is equal to n. So, that is being given in a general symbol.

So obviously, the first value which you have a c 1 into e to the power minus y, t 1 is the present value of the first coupon c to into e to the power minus y, t 2 is the value of the second coupon which as of now, and if you continue adding this terms which is c to the power minus y t 3 plus c 4 in to 8 to the power minus y t 4 so on and so for. So, the last value which will have according to this equation would be c n e to the power minus y t n, but actually c n value is the phases value, which is 100 obtain whatever it is plus c n value which is coupon which is being paid. So, hence you will basically multiply it e to the power minus y t n and the find out the values.

So, now these are the values which you have many add them up. Now what you need to that value is basically multiplication of 2 terms; one is the time period form. So, one is basically amount terms. So, once you divide by the value of B, which is the price then the actual terms of the image with comes ((Refer Time: 06:19)) the duration based on the duration, you can say what is the time period at which you will get back to exact amount of money, which have been invest for the bank for the bond.

So, now the as I mention if they are no coupon payments; obviously, the duration exactly n years if their coupon payments; obviously, it would be risk by the total duration of when and more the coupons are paid as of now in the initial years lesser would the duration, because you have already getting back to the overall amount of investment, which you doing from the back for the bond. Now the term in the bracket is the ratio of the present value of the payment time t 1 to the bond price. So, this is basically the present value divided by the bond price.

And this leads are to the case that the rate of change of the price with the initial value of the bond is equal to negative during, which respect to the yield rate. So, if say for example, if you ignore the minus sigh for the time consider, if this is negative delta where B is negative the obviously, the science between D and delta y could be search that del y in that also be positive, because this is negative you already have negatives sign. So, the negative in the positive basically the negative direction of moment or delta B into B of this is positive.

So obviously, it can main that this is minus and; obviously, this case you should also be minus say that minus into minus should basically give you plus sign. So, remember that the relationship between the bond is yield rate coupon values, and obviously the duration or interrelated or such that given set of information will can different define the find out what is the characteristic tell the bond which you delivered.

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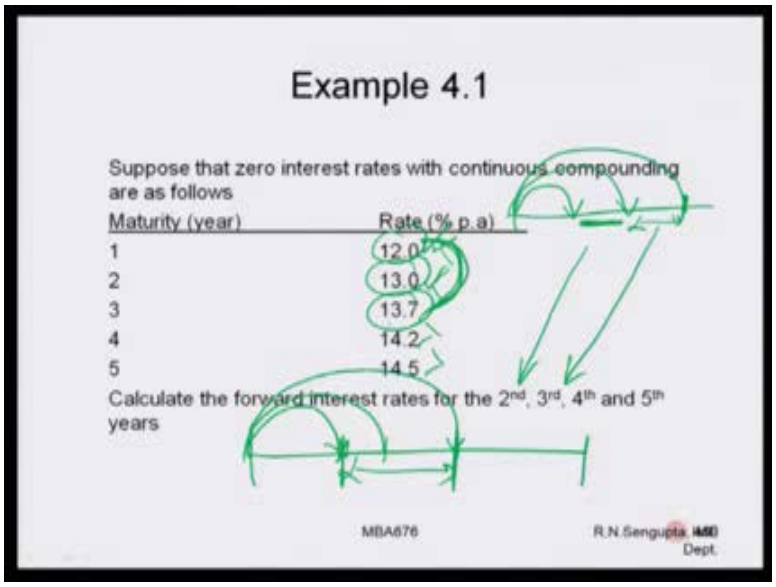

So, now suppose I know other example we are just trying to discuss suppose at the 0 interest rate continuous compounding is given here. So, this is the first column, which you given the maturity in the years the rate interest rate, which is given per annum percentage continuous compounded. So, adequate and find out the forward interest rate for the second year, third year, fourth year, and fifth. So, what you need to find out is basically from 0 to 1, the 0 rate, it is 12. So, basically if you start from 0 to 1, this is twelve this is start from 0 to second year, it is basically 13. So, you require to find out

interest rate between the first and second year, which is this, again if you have the interest rate, zero th rate from 0 to 3, the require to find out the interest rate between the second, and the third year which is basically this. So, if you continue doing this, you can find out 0 right between this time periods, also you can find out into zeroth interest rate; given the zero th interest rate you can find out the forward rate between other type of time period frames.

Also consider this again the same example from 0 to 1 interest rate given as twelve from 0 to 2, it is given as 13 from 0 to 3, given as 13.7. So, these 3 values or given 2 day for example, now what you will find out it is basically what is the forward between the first and third year. So obviously, you do the calculation considering the 12 percent interest rate and 30.7 interest rate which are continuous compounding per annum case, because you can find out the forward rate for the time durations also, but we are ((Refer Time: 09:32)) for and requesting the student to do the calculation. So, on the round in order to understand, how the calculations can be done that given the in 0 rate, you can find out the forward rate or given the forward rate you can find out the forward rate, the 0 rate.

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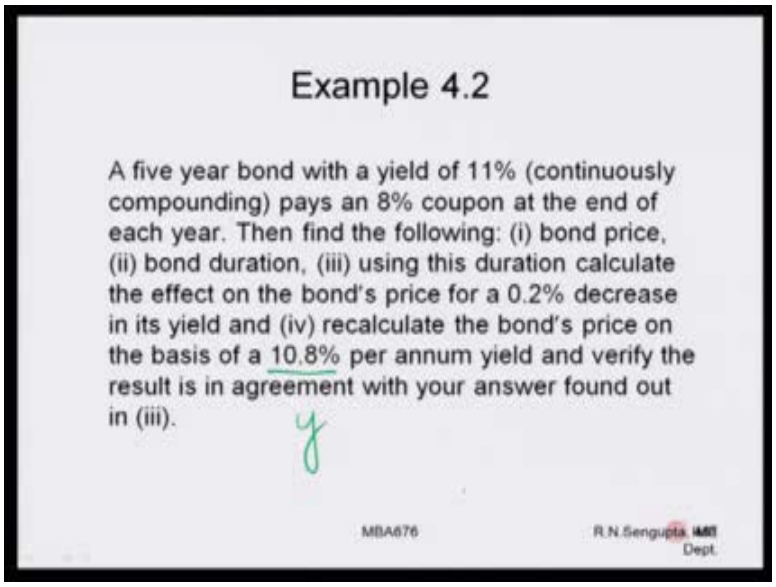

A 5 year bond with a yield 11 percent continuous compounding pays an 8 percent coupon at end of each year, then find the falling bond price, and bond duration using this duration calculate effect to the bond price point to decrease in the yield. So; that means, delta y would be decreased which is minus 0.2 percentage in the minus sign the calculate

the bond price on the basis on 10.8 percent per annum yield, and verify the result in the agreement. So, this value which you have a 10.8 is basically y. So obviously, you will have the rate of change of this bond, you can basically the rate of change of the price of the bond, and given the information or the calculation which you already done depending on the bond price, the duration the rate of change of the duration with respect to the yield. So, you can find out what are the relation between in this value which you already discuss.

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Now, we will start of the discussion of the swaps. So, will cover and this is the little bit about not very different consumption, but it give you whole lot of information depending on how risk can be mitigated, how different concept of quantity finance are utilize in order to find out are swaps are utilize to reduce a risk will cover discuss, and cover interest rate swaps, currency rate swaps and the value currency rate swaps and latter on discuss in a not much detail much in conception sense, how different type of interest rate plus ((Refer Time: 11:09)) swap can be utilize in order to mitigate to different type of risk, what are the risk 1 would be interest rate risk.

That means, interest rate risk would basically portent to the those parties, who are facing some rick on the front of floating rate, and they want to converted to fixed rate or the parties who are facing a risk on the fixed rate and want to basically convert that you are floating rate; that is the one type of risk based on interest rate another set of parties

would be who are facing based on the fact that the risk currency risk; that means, I have Indian rupees and I phase a currency risk on US dollars are yields, I have US dollars and basically I fate a currency risk on Indian rupees. So, I want to basically convert from currency risk 1 currency to the other or form say for example, Indian to dollar rate I want to basically covert this Indian to n rate ((Refer Time: 12:00)). Such that my overall risk basically mitigates that will also going to convert considering different type of swaps, and later on will combine in the interest rate and the and the currency rates swaps in such a way that I can reduce my risk also in two different fronts.

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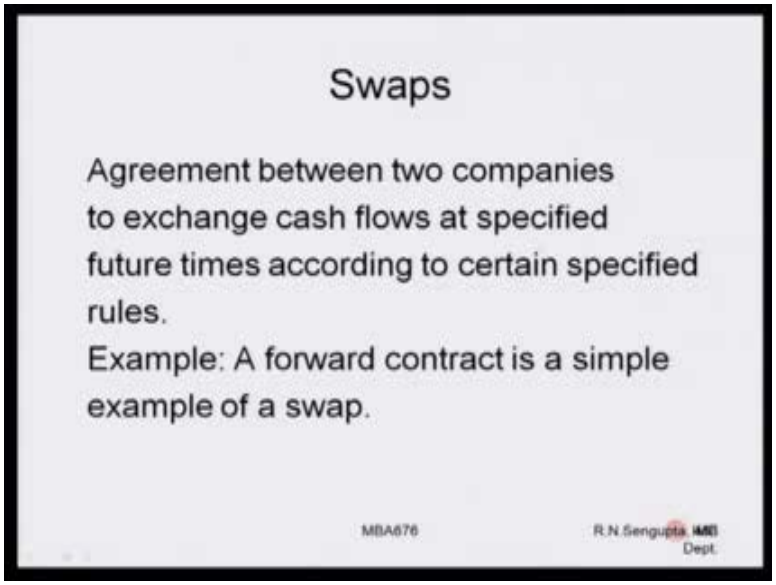

So, swap is an agreement in general between the 2 parts; to companies to exchange cash flows at some specified future times according to certain specified rules, while on the other end of forward contract is simple and example of very simple example of swap, but in the forward rate is only one exchange while for a swap will be basically consider that the exchange are happening at some develop time, and some fixed amount of money is being exchange between these 2 parties why they are going to do that will see that very small.

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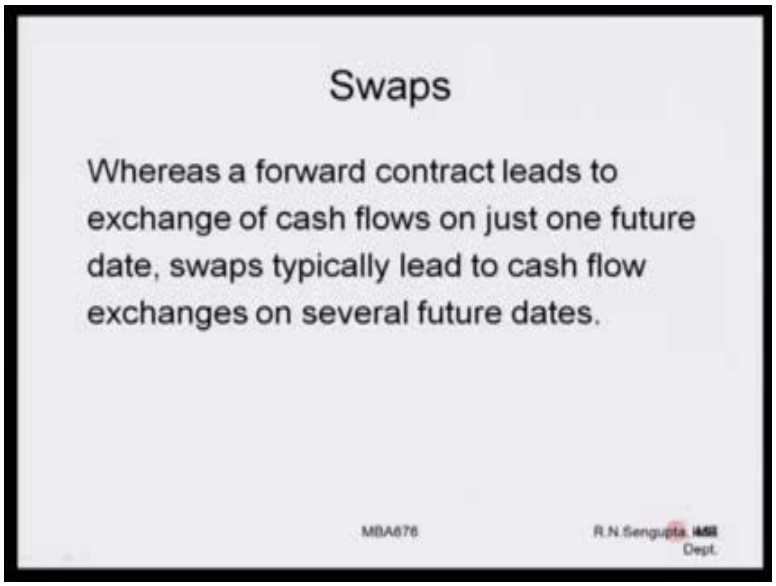

Whereas a forward contract leads to exchange of cash flow on just 1 future date swaps typically lead to cash flow exchange are several futures dates, and it continuous at intermittent times. So, intermittent times are equal or equal duration, but once we decide the swap will basically continue for that duration and equal amount of money would be extend between these parties.

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Example: on first March 1999 company enters in a long forward contract. So, let us highlight this. So, long forward you know what it is basically long forward contract to by

100 bounds of gold at 300 per annum. So obviously, once you apply you can find out what are the price the total amount immediately are getting that gold, it can short that gold for hundred into ST. ST is the spot price which will happen at time t is equal to capital t note it leads to a cash flow on just 1 futures date, but swap typically lead to cash flow and several such future dates, but they would be intermittent and the interest rate and the amount transaction would also be decide by this 2 parties.

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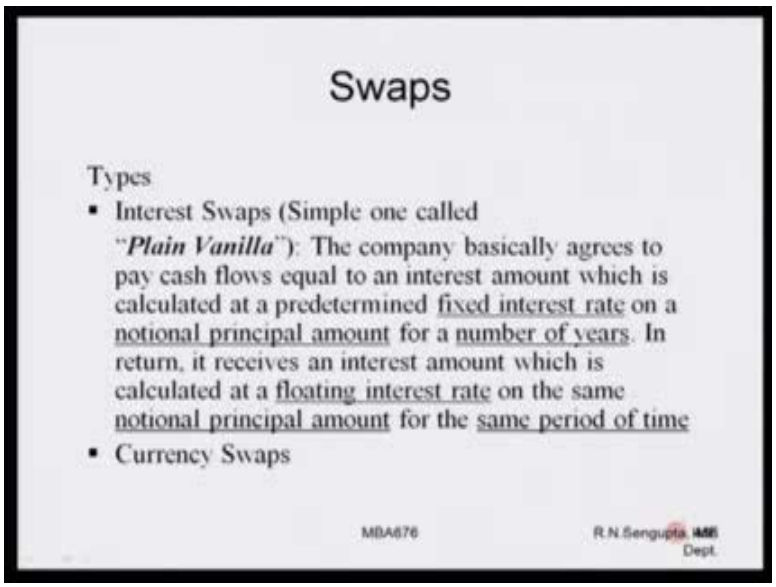

So, swaps can be interest rate swaps simply, they are call the plain vanilla the company basically agrees to cash flow equivalent or equal to a interest amount, it is calculated at a predetermine fixed interest rate 1. A notional principle amount of exchange which is happening between the these 2 parties in return the company receives and interest rate amount, which is calculate at a floating rate on the same notional amount of the same period of time 1 parties, basically the trying to exchange money on a floating rate another party try to basically exchange amount of money a fixed rate, but it mean the… So, happen party which is being whole to do an floating rate, we find it much more advantage to. So, on a fixed rate and party which is being today and a fixed rate it is much more advantages for that party to that a floating rate. So, what they would try to do initially they would be a fixed and floating, which is definitely not advantage to them.

So, now with that condition they would basically go into much more the other transaction with other parties which is basically the swap, such that one is able to cover

this advantages position from a floating. Now to a advantages position which is a fixed and the other party, which was basically facing and disadvantage of the fixed rate will now converted into floating rate. So, how it is done will consider and another case for a currency swap again as a mention some parties phasing risk for US dollars and Indian rupees, it will basically converted into position to Indian rupees to US dollars or you can convert position from the US dollar Indian rupee is to n to rupees also depending on what you thing is based on rest for his or her overall financial transaction, such that the it was to basically minimize this overall risk.

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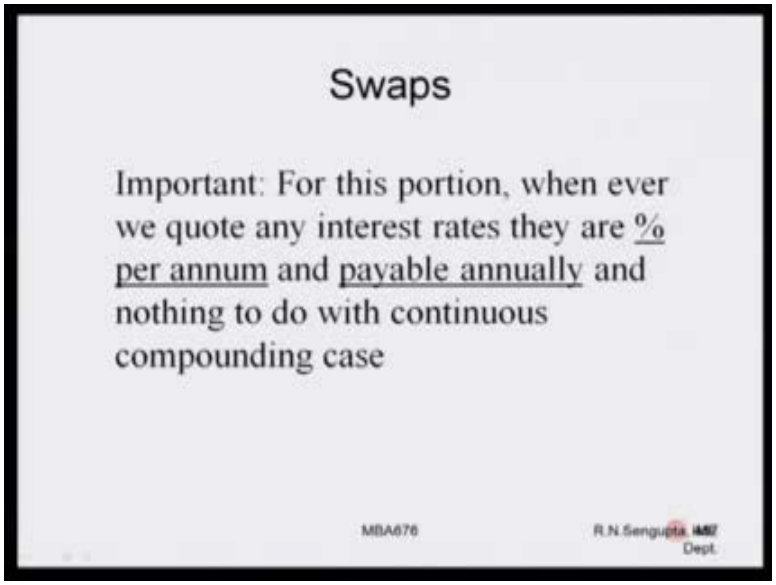

So, important is that for this position whenever we quote and interest rate they are always given for annum continuous compounding basis. So, remember that. So, even if you have interest rate which are given per annum given compounding basis or per annum or a compounding in number of times. First calculate all the interest rate are per annum basis on continuous compounding basis, and then do a calculation the accordingly. So, whether the interest rate is given on fixed rate whether the interest rate in given on floating rate whether the amount of transaction, which is being done if there is interest rate foreign currency the interest rate are giving other terms first without doing any calculation covert that into per annum continuous compounding rate and then do your calculation for the swap.

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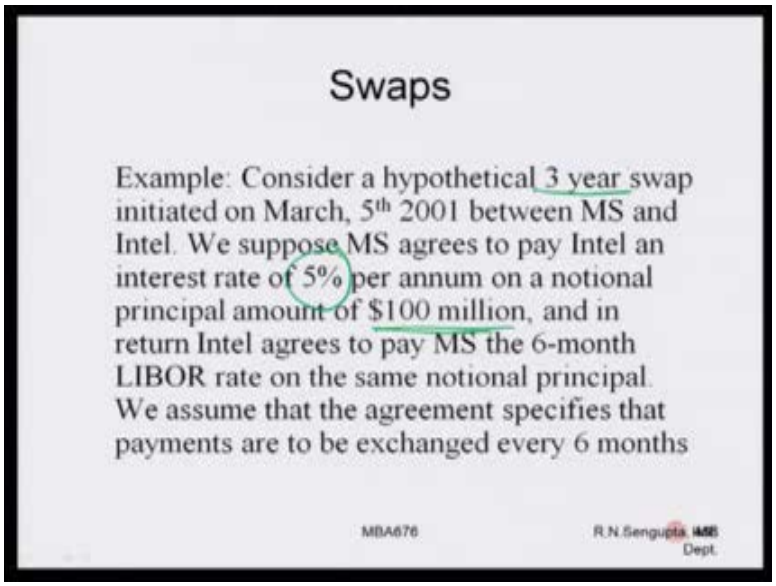

So, let us consider the simple example of a swap consider the hypothetical 3 year swap, which is being initiated on 5th of March 2001 between Microsoft and Intel. Suppose the Microsoft in order to sell his computers in need and the chips are basically made by Intel. So, we suppose Microsoft agrees to pay Intel and interest rate of 5 percent per annum on a notional principal amount. Say for example, of 100 million dollars. So, this percentage which is give is basically fixed and this is the notions level and in return Intel exist to play Microsoft the 6 month Libor rate the labor rate is basically with the floating 1 is, you remember the London interbank of rate in London interbank bit rate or equality in India with the Mumbai interbank for rate, and Mumbai at bit rate or Libor rate in the same notion amount principle amount of 100 millions. We are assume that the agreement specify the payment also exchange every 6 month onwards, basically the 3 years swaps. So, the 6 month period, it will basically paid after 6 month after 1 year after 1 and a half year, after 2 year after 2 and a half year, and the 3 year and behind the overall contract for the swap.

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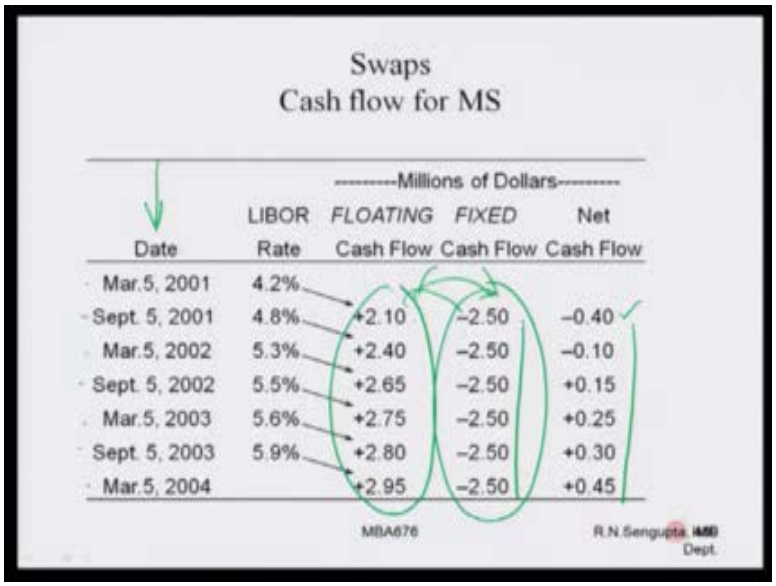

Now, the cash flow of Microsoft. So; obviously, the Intel would be it will just the reverse. So, what you have the reverse first 6 month, 1 year, 1 and half years, 2 years, 2 and a half years, 3 years. So, all the date are given in the last column the Libor rate, if they are as this. So, it is 4.2, 4.8, 5.3, 5.5, 5.6, 5.9. So, this is the Libor rate which is happening the total amount of transacting, which is going to take place would be the amount of money cash flow and this is positive, because cash flow of point of you Microsoft. So, this will be on 6 month basis. So, plus 2.1 2.4 plus 2.65 plus 2.76 plus 2.8 plus 2.95 and the other hand, because payment by Microsoft. So, would be done in a fixed rate if you remember I stored about the fixed rate, which was there. So, there are values are fixed do it is minus 2.5 throughout.

So, if I want to find out what is the overall net cash flow; obviously, it will be difference on there. So, this minus… So, this would basically be net outflow of cash are minus 0.4 for Microsoft the first year, first year, 6 months then this first duration to this 6 months, then from the 6 month to 1 year, it is again a minus than from the 1 year a 1 from the half year this plus from a 1, and a half year it is 2 year, it is plus and so on. So, values are given. So, this minus and plus basically means what is the inflow and outflow can which is happening between Microsoft and Intel, and if I look at the cash flow for the point of view for Intel, it will be just the other way round.

So; obviously, the values which you have here for cash flow from Microsoft they get swapped swap means they can exchange. So, this would be the inflow for Microsoft for Intel in the other diagram and this would be the outflow happening for Intel. So obviously, Microsoft and Intel when they add up the total tradition and should come out to 0.

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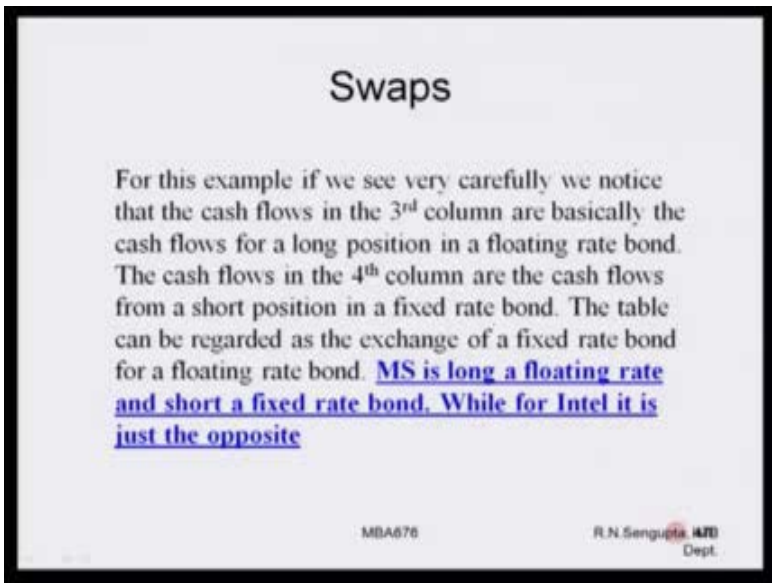

For this example, if you see very careful we notices that the casual on the third column basically the cash flow this for a long position floating rate the cash flow in the fourth column with the cash flow in the short position on a fixed rate point. So obviously, the cash flows or for a long and a short the interest rate based on which you are doing the calculations are fixed or floating. The table can be regarded at the exchange of a fixed range of on for a floating rate bond and a Microsoft is ling, and a floating rate and short and a fixed rate while Intel is just the vise versa position. So, remember that.

So, first we will try to analyze what is the position of the both the parties whether they are long and short on a floating rate or a fixed rate or say for example, we are considering the solar to rupee rate or say for example, dollar to n rate you will try to find out that parties are long and short on respect to what exchange based on the exchange it try to basically constant, which is advantage of party 1, which is a advantage of party 2 basically exchanged the position with respect to this and try to recalculate the calculate such that risk is minimized.

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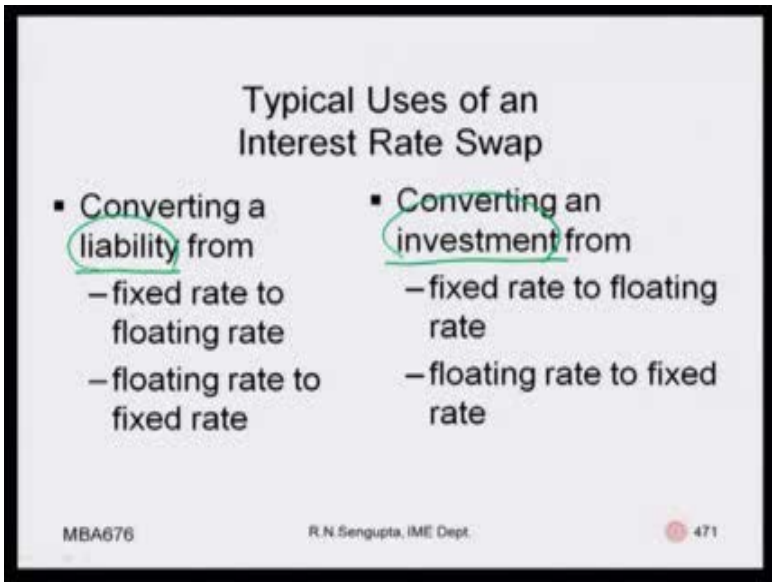

Typically uses of interest rate swaps are done to convert liability from a fixed to a floating, and floating to a fixed and for other instant to convert from the investment, you are basically from the trying to convert from the from a fixed to a float, and also the floating to a fixed. So, you are main concern into basically convert into the liabilities convert the assets. So, whether they are basically the assets are the liabilities you want to basically reduce you are overall base from a floating to a fixed or fixed to a floating, this means saying assets are liabilities are same the overall risk based on which you are going to term, and find out you total liabilities are either the floating fixed or a fixed or a floating, such that you are able to minimize the overall risk.

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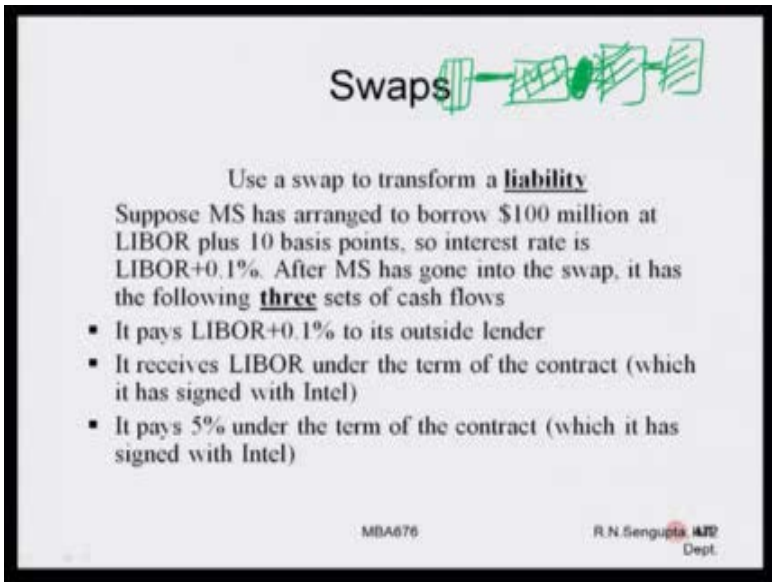

So, now will basically use a swap to transform liabilities. Suppose Microsoft arise to borrow 100 millions ay Libor plus 10 basis points, 10 basis points, 10 divided by 100 percentage or point over all number the Libor. So, interest rate is Libor plus 0.1 for percentage after MS is gone into a swap it is followed 3 sets of cash flows which is pays Libor plus 10 basis points should the outside lender. So, now, a third lender is also coming for Microsoft and a third and a fourth lender will call also coming for a Microsoft for the Intel. So, this is Libor under the terms of contract which is sign with Intel in the first instant and it base 5 percent on other terms of another contract, which is basically sign with the Intel. So, Intel and Microsoft word exchanging from forward and future.

So, now in the second instant lntel will go with another transaction with the third party outside third party to itself and Microsoft would also third party to itself. So, now in the total number of players were there in the market would be Microsoft Intel and 2 third parties. So, initially if you see the graph thus transaction, it was basically Microsoft and Intel.

Now if you see they would be a third party for Microsoft and a third party for Intel. So, I am ((Refer Time: 23:05)) just trying the line, I am denoting the arrows in which detection they would. So, we will slowly consider that considering that initially it was Microsoft and Intel, who are going into use as transactions in the first step in the bring to

third party, and later on will also see that in order to mitigate; that is will basically being a financial institution between the Microsoft and Intel. Such that the overall transaction is not anymore between 2 parties, but now they would be 5 parties 1 would be the third party from Microsoft, second would be the Microsoft itself, third case would be the 5 which is basically in between Microsoft and Intel, forth one is Intel and fifth party would be them, third party for Intel. So, will see that how you can basically deals the overall ways as you basically go step by step in order to formulate, and swap will continue the discussion of trying to formulate the swap in more details for this example, and then basically going to the currency rate swap. And then also see how currency interest can be combine together in order to formulate very sophisticated swap. So, such that the overall risk is minimize for both the parties who are there in the transactions.

Thank you very much.