Quantitative Finance

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Module- 04

Lecture - 23

So, welcome back to this quantitative finance NPTEL mooc online course. So, hope everybody is fine and enjoying the course. So, we have covered a lot starting from portfolio arrangement to different concepts of how portfolios are made, then we went in to derivatives, we did discuss about different type of averaging techniques. So this is a whole game out of things we have covered there are whole let, whole lot of things to be covered. So, I am sure that this this course is interesting to all 1200 plus students we have registered.

I want to say few things because as the course was continuing do understand the coverage's are happening a little bit fast, but on the other hand I would definitely mention that once you get the interest you can pick up all the information from the net which is really very helpful, to give you a good understanding about the concepts which we are discussing. And also you are your are most welcome to send your comments, you are most welcome to send your quarries and I am sure that in with little delay, may be a delay of 1 1 1 and 1 and half days maximum and definitely going to answer all the quarries which are there.

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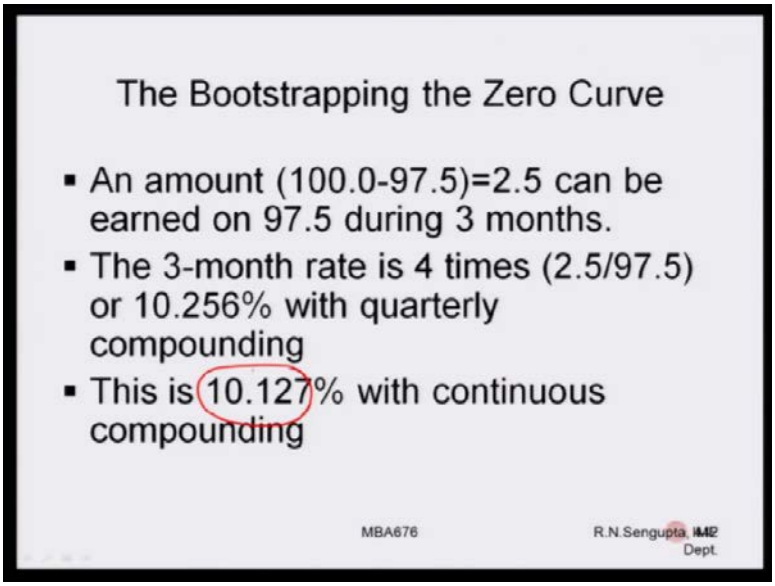

So, continuing that we are in the area of par yield, then you want to find out what is the actual coupon rate of the bond, how the zero rates are there, how you are find out the forward rates and so and so for. So, with the discussion we are having in the last class was, there are 5 different bonds in the first 3 the coupon payments are 0, for for the fourth one the coupon payment is 8 units, dollars rupees whatever it is being paid semi annually. For the second one is 12 dollars paid semi annually and the interest rates was given, and the price of the bond was also given base on their actual demand and supply. And the face values are also given on the first column.

So, for the first bond for a face value 100 you are getting 97.5 which is 2.5 which can be earned out of an investment on 97.5 in 3 months, because you remember it was for 3 month duration. So, what will say exactly is that the 3 month rate, for this one and then we convert in to a yearly 1 it will be 4 times of that, so it will be 4 times 2.5 divided by 97.5. So, 4 why if it is being compounded 4 times? Why 2.5? This is the overall amount which we are receiving, what is 97.5? Is the overall investment which are making. So, this comes out to be 10.256 which is per annum quarterly compounded. So, again if you use the same formula, which we have been discussing continuous compounded. The continuous compounding rate would come out to be 10.127. So, this will use later on in our calculation. So, if you if you remember. So, this is important which will be utilize by a in our future calculations.

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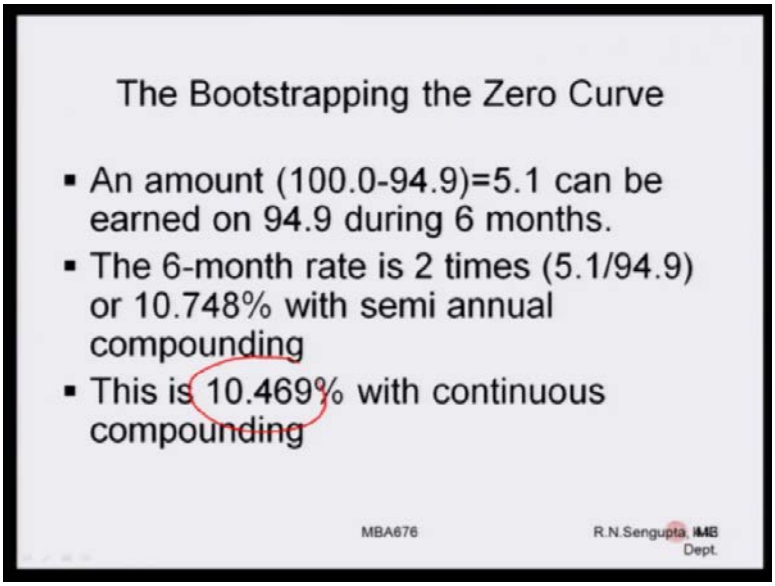

Now consider the second row. So, here again the face value was 100, the actual value of the bond depending on demand and supply is 94.9. So, this comes out to be 5.1. So, now you will ask the question this 5.1 which you are getting after how many months? It's basically six months, which is half a year. So, you will find out the six month which is semi annually compounding percentage per annum would come out to be 2 multiplied by 5.1, 5.1 is the total of value which you are getting the difference and the investment is 94.9. So, that comes out to be 10.748 with semiannual compounding. Now, again if you convert into continuous compounding rate, we use the same fundamental principle and that comes out to be 10.469 per annum continuous compounding. So, this is the value which is important for us.

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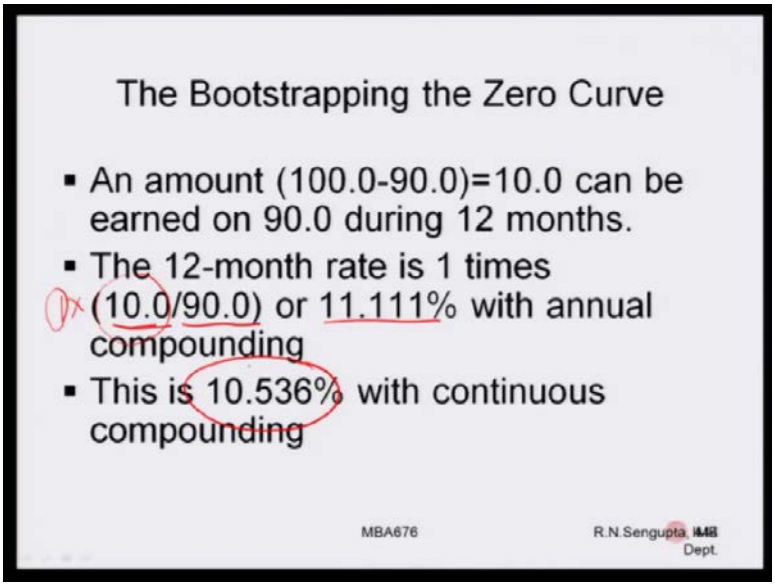

Now consider the thirds row. So an amount face value 100, total amount which you getting after demand and supply can taking a consideration is 90 which is 10. So 10 is the total amount in units of rupees or dollars whatever it is, which is being obtained after 12 months. So you are compounding is being done once in 1 year. So, hence the percentage is calculated comes out to be 11.11 with annual compounding and it is basically per annum. So, if you find out; so what you are doing is exactly this, to find out 11.11. Which you are, the same fundamental principle you are doing for the other calculation also. This is the total amount which you are getting investment is this. How many number of times compounding is 1, so it multiplied by 1. So, in other cases it will basically multiply by the number of compounding you are doing, multiplied by the total amount you are receiving and in divided by the overall demand and supply value of bond which use. So, again if you convert it into a continuous compounding case it comes out to be 10.536 per annum continues compound. So, we have found out all the values of continues compounding case for what for the case when it was be done on quarterly basis, for the case when it is being done on a semiannual basis and for the last case which is this which you are seeing on the slide is 10.536 is for the annual basis.

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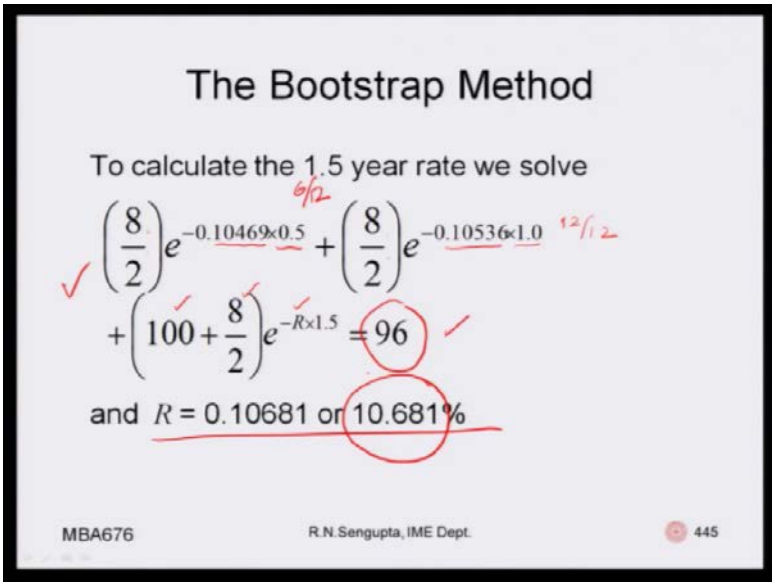

So, now, what we need to do is that… Now you want to go to the next step. So, if you remember we are given for quarterly case which you have already found out for the monthly for the semi annually case we have found out, and the last one is being for the yearly case we have found out. So, with this is 3 sets of information you want to proceed 1 step forward and find out what is for the 1 and a half years, then once we find out the 1 and the half years we want to proceed one step further and try to find out what is a 2 years interest rate depending on the calculation which we have already done.

So, if you remember the fourth row was there, it was being paid a coupon of 8 dollar and it was being paid semi annually. So, the total value of 1 which we have getting after 6 months is 8 by 2, then the second coupon we will be getting is again 8 by 2 which is being paid after 1 year. The third one would be basically paid along on the principle amount which is 100, so you will get a value of 100 plus 8 by 2 and that will be paid after one and half years. So once you equal the left hand side all the 3 values which I have mentioned, that is 8 by 2 after 6 months, 8 by 2 after 1 year and 10 plus 8 by 2 after one and half year. So, if you equate including the interest rate, which we have already found; the overall demand and supply value at which the bond is being traded is 96. So, now let us go for the calculation slowly.

So, this is the coupon which you are getting after 6 months. So, now, the interest rate which you based on which will find out the present value is the 1 which you found out

for the case of half a year. So, that is basically 10.469, so obviously would be minus because you are trying to find out the present value. And multiplied by time, time is basically 6 by 12 which is half a year. For the second case which is being paid after 1 year, again the coupon value is 8 by 2 and the time frame is now 12 by 12 which is 1. But if you check the interest rate, that is 10.536 which is continues compounded per annum interest rate for 1 year. So, this is minus, minus being because you are trying to find out the present value, minus of 0.1036.

And for the last calculation which will need to do on the left hand side, so this is the left hand side which we are doing. It is 100 is the face value, 8 by 2 is the coupon payment. Now you will ask yourself, what is the interest rate? You do not have that, interested for 1 and half year; that means, if you invest at time t is equal to 0, you want to get back total amount of money whether is coupon whether is principle of amount there has been immaterial, you want to get the money after 1 and a half year. So in between there are no payment which have been done to you, neither you depositing any money in the whole transaction.

So, considering the interest rate which is continuously compounding for annum interest rate and the total time duration is 1 and half years. If you need to find out you equated to 96, which is the demand and supply value. So, if you find out that r value comes out to be 10.681 which is the continues compounding interest rate for the time duration.

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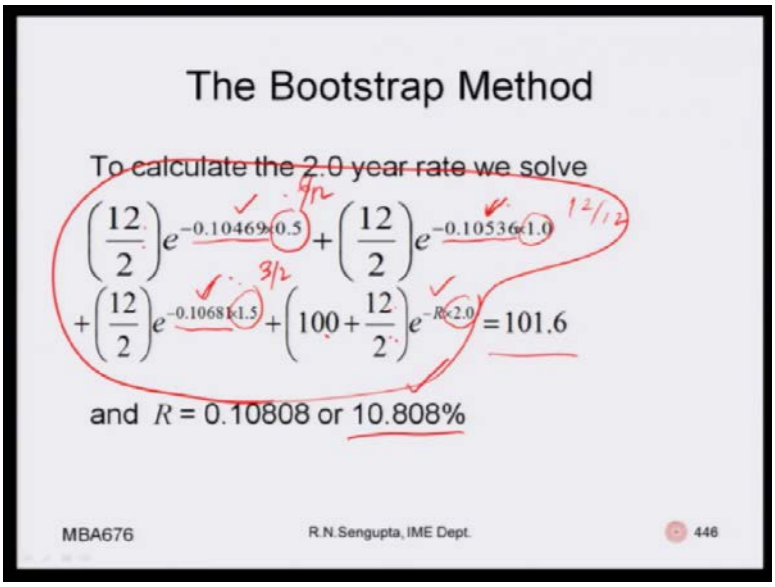

Now, having said that, what you have done is that? You have going step by step. So, you have already found out again I am repeating for the first quarter, then for the half year, then for the 1 year, then for the 1 and half years; and the last row is basically for the 2 years. So, what you will do? You will first find out what is the coupon? Coupon is basically 12. Now will ask the question, how many times is being paid in a year? It is being semi in being paid once after 6 months, then the next one is after 1 year, the next one would be again after 1 and half years and it will continue in the sixth sense. So, if you see the total duration is 2 years, so how many such payments would be there within the 2 years time frame? So, first would be after 6 months, which is 12 by 2 and the time frame which you are trying to find out is t; is basically 6 by 12. I will come to the interest rate later on, the next one is again being paid 12 by 2 the total amount and it is being paid after 1 year which is 12 by 2.

The next one, again the coupon of 12 by 2 would be paid after 1 and a half year which is 3 by 2, and the last one which will be paid after 2 years interest rate. So, but in the last we are remember there are 2 payments being paid, one is the coupon 1 12 by 2 and one is the face value. So the next question would be what are the interest rate? So, we already know 10.469 is continues compounding interest rate for half a year. So, from the actual calculation, so you utilize it here.

Next question is that, what is for the one year? We already know, it is 10.536 continuously compounded per annum; so you will you will use it here. The next question is that what is for the 1 and a half year? So, we have already just calculated, that comes out to be 10.681 we use it here; that means we are proceeding one step at a time. So, the last information, which you just got the 10.681 you utilize that. And utilizing this whole value, r is unknown for the two year period; that means, you investing in 0 time getting back all the amount of money after 2 years without any per payments being made in between, or without any payments being made back to you in the 2 year period. So, that is interest rate which is continuously compounding being done per annum. So, that you equate, that is are every value which you have done is basically on the left hand side, so you equate that on to the right hand side. What is the value? That is basically the demand and supply value, which we have is already noted on the last column in the table which is 101.6 utilizing that you find out the interest rate at 10.808.

So, what you have done is that? Using this concept of going 1 step at a time, is basically like the bootstrapping. When you have a very high gumboots and when you tie up the laces, you basically start from the bottom most level and then basically tight in a as you go up to your ... So is exact with the same thing, in the bootstrapping you go 1 step at a times. So, first you found out for the half year, which is 10.469; based on the calculation is you proceed to the next step, then you find out the one year which is 10.536. Then in the last slide in we found out, we found out basically the 1 and a half year it is 10.681 and in this slide we basically find out the interest rate for two years, is continuous compounded which is 10.808. So if you had basically 2 and a half years, 3 years equal to be have basically calculated the interest rate based on the same principle.

Now remember one thing, before I proceed further on to basically have a sense of what you mean by the forward rate. 0 rates are very important to understands means the 0 rates means 0 is not the interest rate or something to be interest rate. Is basically the coupon payment or in between payment which is being made to you or the deposit which you are being made you are you are making within the time prime frame it is 0, that means it is no transaction. Hence, they are considered as the 0 interest rate based on which you do the calculation.

So, the value of 10.469, the value of 10.536, the value of 10.681 and the value of 10.808 are the 0 rate continuous compounding for the case half year, 6 months, next for 1 year, next for 1 and half years and last one for the two years period.

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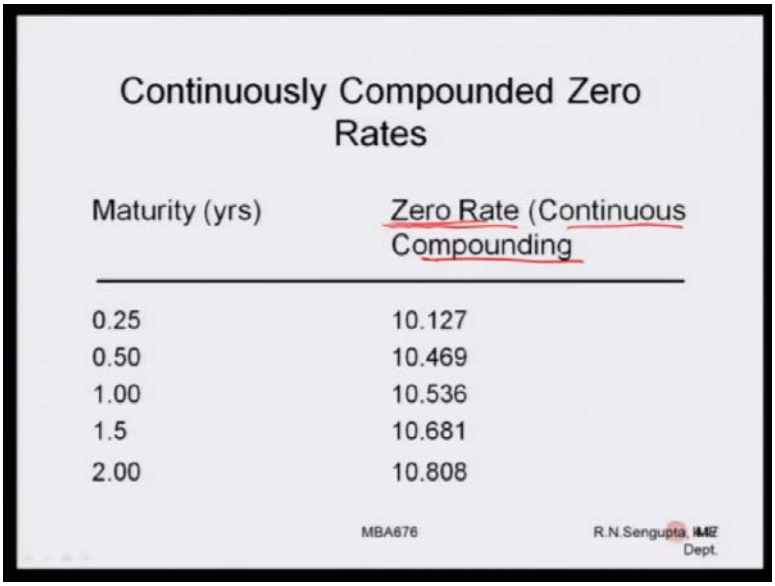

So now, once you note down you in the 1st column you have the maturity in years, which is given; which is one 4th of a year, half of a year, 1 year, 1 point 5 years, 2 years and the interest rate which you have already found out are noted down in the next column which is in the 2nd column. But note down again, I am again repeating, these are the 0 rates and there are being continuous compounded and the values are begin found out on a annual basis or a yearly basis, that is percentage per annum.

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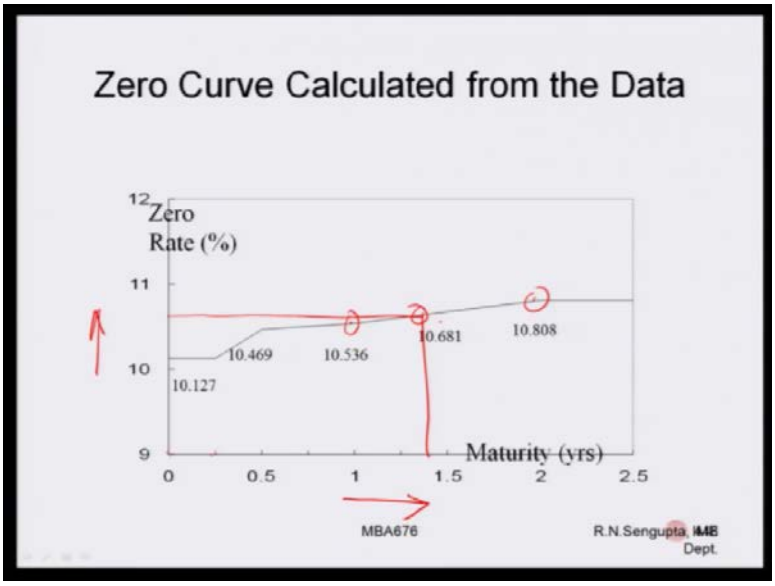

So, if you draw the curve, the curve for the maturity which is basically important on the X-axis and the 0 rate which is on the Y-axis; this is value gives you. That in between if you want to find out any 0 rate between was 0 month to say for example, 1 by 4 of a year; the now is you will take the interest rate as it is which is given as 10.127. If you want to find out the any interest rate between say for example 1 point 25 to 1 point 5; so obviously, you will find out what is the slop based on the slope you will find out. So, it will basically be given by the values which are there.

So, you to basically broad them and find it out. So that will be just simple interpolation. So, you have already these values, so in the you will interpolate and find out what is the 0 rate. Not that they will give you the exact calculation, but they will give you a good approximation of whatever the existing value of the 0 rate is.

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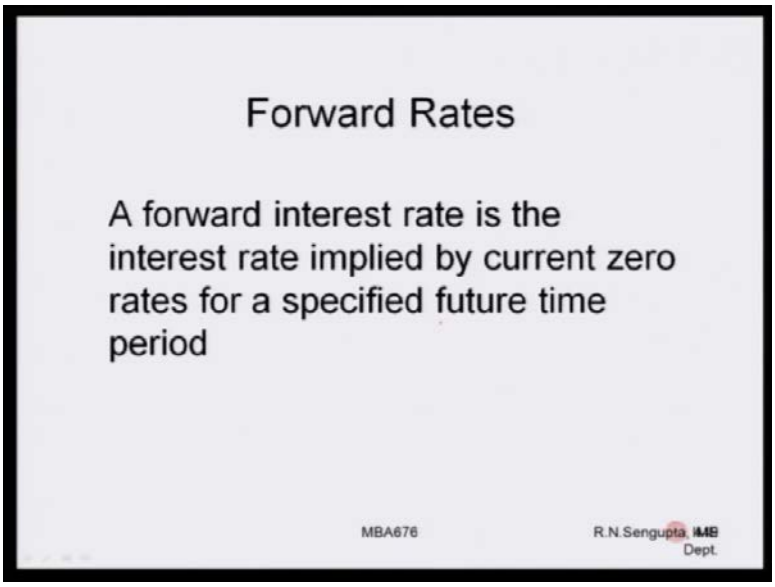

Now give you, having discuss about that 0 rate the coupon rate and all those things. Now next question is that how do find out forward rate? But before you answer the question that how we calculated forward rate the or main concern or question would be what is the forward rate? So, forward rate is the interest rate implied by the current 0 rate for a specified future period of time. So, let me explain with the slides which you already discuss. So, we had 0 rate for 1 by 4 of a year, for 1 by 2 year, for 1 year, for 1 point 5 years and 2 years.

Now what if I ask you the question that what is the interest rate, which is being paid in between the time from 1 to 1 and 1 by 2 years; that means, we already known 0 rate from 0 to 1 year. You already know the interest rate from 0 to 1 point 5 years. So, what we are interest rate know is that, what is the interest rate between 1 to 1 and 1 point 5 years. Or in the similar way, we know the interest rate for 5 months, we know the interest rate for 1 year. This interest rate means from 0 to 6 months and 0 to 1 year. So, what we are now interest rate find out is that, what is the interest rate which is applicable for the time frame from 6 months to 1 year. So, these interest rate in between depending on the 0 rate which we have already found out and another forward rate. That means, once you read this 0 rate, if you want to go 1 step forward, what is the forward rate based on which you can do the calculation. So obviously, the concept would be the same as we have utilize for the continuous compounding concept and the compounding concept n number of times. So, concept would be same that will try to basically equate 2 equations; 1 on the left hand side, 1 on the right hand side and based on that you will try to find out what is the interest rate. Such that given the 0 rates you can find out the forward rates or given the forward rates we can find out the 0 rates. So, let us see how we can do that.

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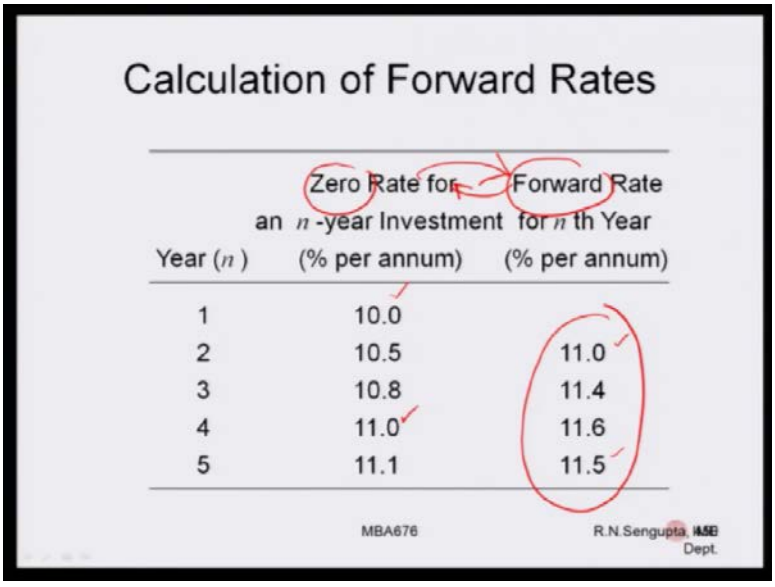

So, consider another example. So, all of these example are taken from the Joncial's, so you did not be too much bother once you understand and then you read Joncial book it will definitely give you a lot of information how the calculations are been done. So, in the 1st column you consider the year, so they are given in units of years; it could have

been 1.5 years also, 2 years also, 2.25 years also that immaterial. So, once you decide the time frame it is years, stick to that and do your calculations accordingly.

In the next column we have basically the 0 rate for n number of years of investments; which means that if you see the first value which is 10, it means for a time frame from 0 to 1 the interest rate for no amount of transaction taking place between u and the bank, where you deposit the money. That is the hypothetical case we are considering, that no amount of of money is being transacted between you and bank that neither you depositing or you taking out. Then the interest rate of 0 rate for the time duration is 10 present per annum continuously compounded.

If I see the value of say for example 11, it means that if I invest in time t is equal to 0 and stick to that investment without any exchange of money taking place from the 0 time to the 4th year, then the interest rate based on which we can find out the calculations and do all the calculations is 0 rate and which is known as the value is 11.0 which is per annum continuously compounded.

Now having said that we want to find out, what are the interest rates in between. So, if you look at the 3rd column with the last column which is basically forwarded for the n-th year, and again it is basically it will be given per annum continuously compounded. So, what does this 11 means? This is 11 means basically what is the interest rate, which is optical for a time duration from 1 to 2. Or say for example, if you find out 11.5, we want to find out that what is that value or what is that value of 11.5 and where it is applicable? Is it basically applicable from the 4th year to the 5th year; or is it applicable from the 3rd year to fifth year. We will see that how the calculation can be done. So, once you do the calculation, you can immediately understand 0 rate which I given the last column, would now make sense such that based on the on the forward rate. Sorry, for the forward rate which we have found out in the last column, would give you a information that given the 0 rates; how you can find out the forward rates? Or given the forward rates, how you can basically go back and calculate the 0 rate.

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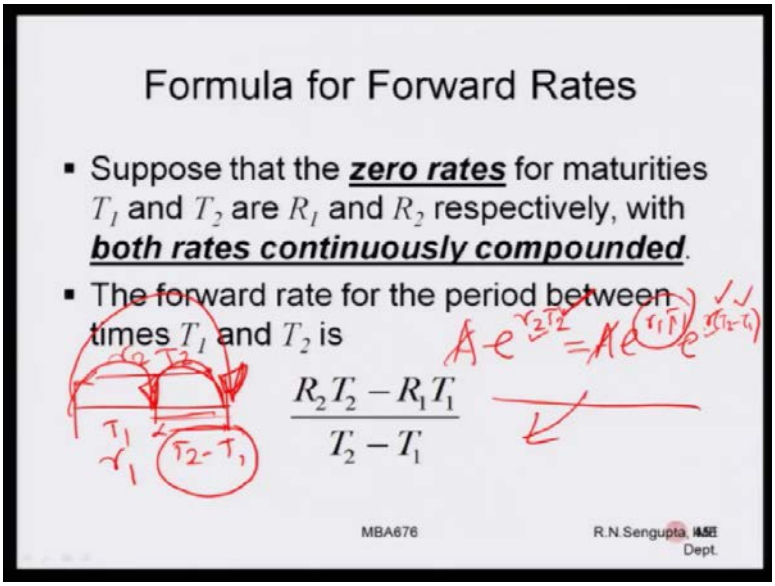

Now, suppose that the 0 rate for the maturity for T1 and T2 time period and R1 and R2 are interest rates which are given. With both rate being continuously compounded and they are being given and per annum basis, then the forward rate for the time period in between T1 and T2 would be given by this formula. So, the concept is very simple. What you do is that, consider a is the amount of the principle amount and consider there are two time frames. So, one time frames are given by T1 is this time frame and consider T2 is time frame. So obviously, this time difference would be T2 minus T1.

So, what we are given is the 0th interest rate for time period T1 and 0 interest rate for time period basically T2. So, what you have is that, e e to the power r2 T2, which is the total amount of money which is being done investment for this time period should be equal to this amount which is now; A e to the power r 1 T1. Then again this is being invested for this time duration, so this note that time duration is T2 minus T1. So obviously you will have, e and this r is the forward rate which is happening between the time frame of ending of T1 and basically ending of T2. So, this is the time duration which you want to find out. So, this interest rate if it is r, so it will be given by T2 minus T1. So, A A will cancel, so it basically all r r to the power of e. So, you can base equate them, then trying to find out what is the relation which is given.

Similarly, if you have T1, T2 and you have you have the r 1 value. So, you have r 2 values and r value known the time T2 and the T2 minus T1, similarly given that you can

find out the relationship and find out what is r value. So, remember here r 1 and r 2 again I am repeating of the 0 rate, for time duration of T1 and T2 and r is the forward rate for the time duration from between T1 and T2; that means, the time difference in between which is happening. So, all your datum for the 0 rate is basically 0 and for the datum for the forward rate is basically for this example is given as T1.

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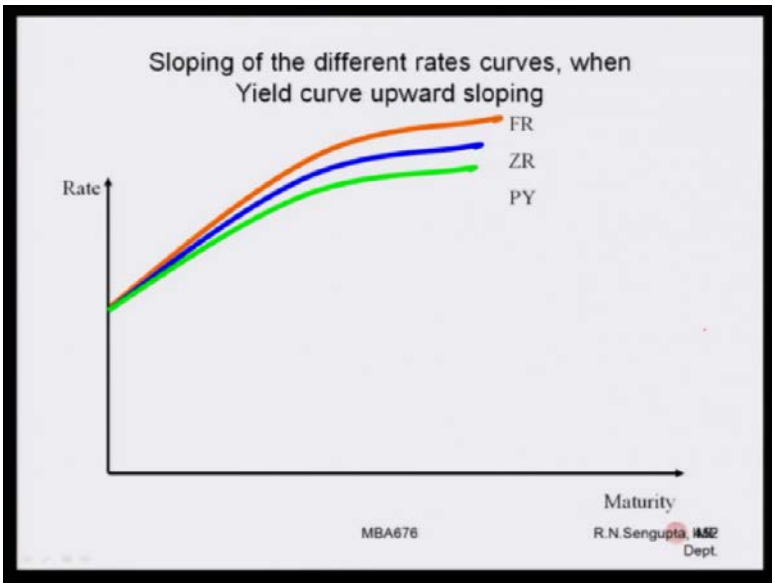

Now if you draw the slope for the different rate curves, so the rate curves are forward rate if f is FR, ZR is the 0 rate, P is the par yield. So, if you basically have or yield curve which is slopping upwards then the values of the forward rate, zero rate and the par yield would basically look like this. Where the forward rate would be the red one which is on the top most, then you have the 0 rate and par yield. And if you have a yield curve which is downward sloping; so obviously, in that these values will just be the reverse mirror image, but when it happen the mirror image is basically make sense to say then the forward rate would be the downward sloping then you have the 0 rate and then you have the par yield.

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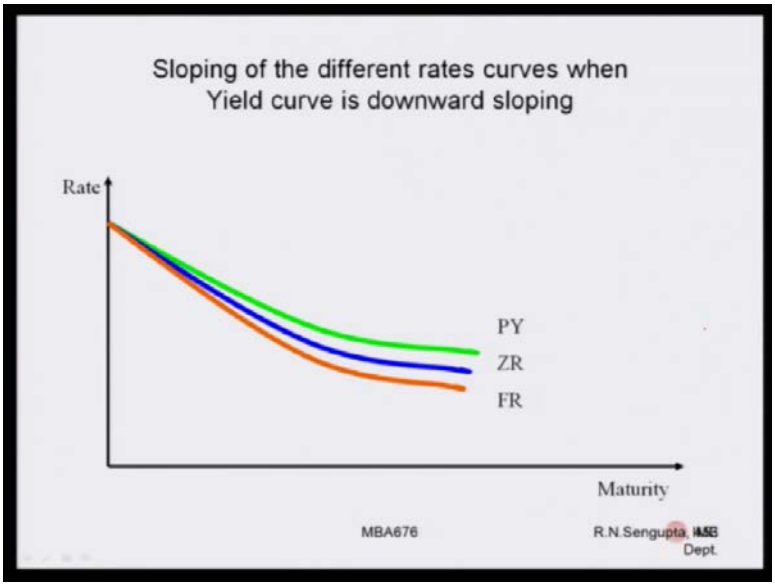

So depending on how the values are changing? Whether the yield rate is upward moving or downward moving you will basically have this values. Because remember, they are being calculated using the same concept which we are just discussed in last class and today's class. So, once you understand the calculations, you can definitely say depending on whether the yield curve is upward moving or downward moving, the positions of the par yield 0 rate and the forward rate would be exactly as is it given in these two slides. And again remember we are trying to basically measure the maturity on the x-axis and rates on the y-axis.

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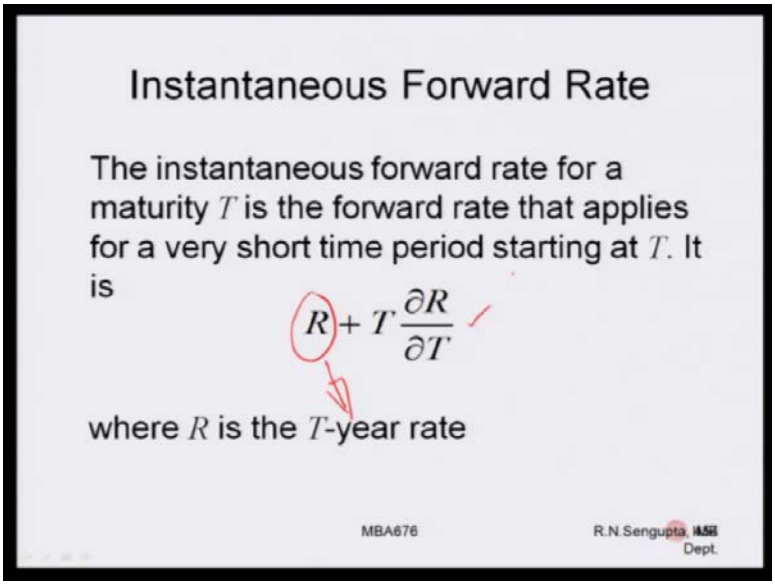

Now, the instantaneous forward rates if you consider that the 0 rate is from 0 to T and another 0 rate is from 0 to T plus delta so; obviously, the delta time period interested which we have basically known as the instantaneous forwarded rate. So, how do find out? The instantaneous forward rate for a maturity is the forward rate that they apply for a very short period of time for a time period starting T and ending a T plus delta T. So, if you find out the 0 rate for T and you find out the 0 rate say for example, for T plus delta you can easily find out what is the actual forward rate for that delta T time unit of time. So, remember this R is basically T R interest rate, zero rate and the and the and the value which you find out, is basically will give you the value of the 0 forward rate.

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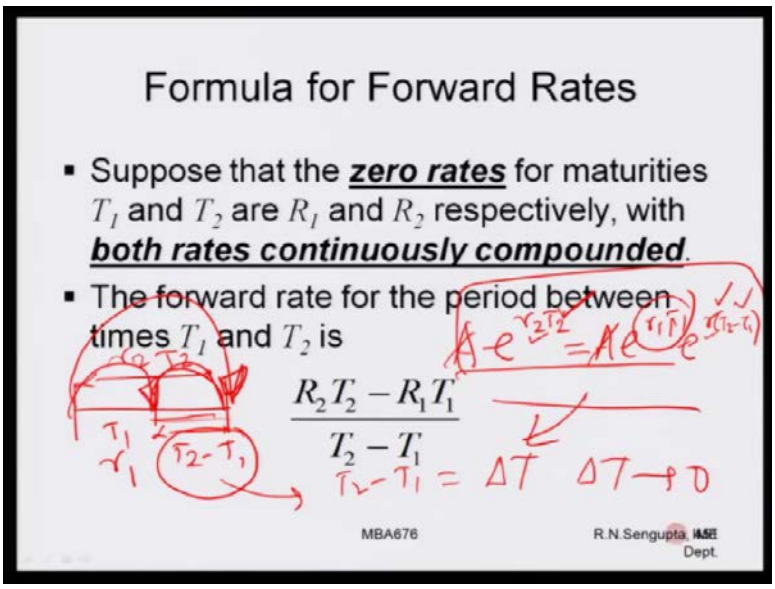

So, if you remember if I go back to the previous two slides before; so this formula when you consider the time difference T2 minus T1 is basically slowly going to delta, where delta basically goes to 0, you can find out what is rate of change of that function; such that you can find out the 0 the instantaneous forward rate.

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Now, what is the forward rate agreement? A forward rate agreement is a forward rate contract where the parties agree that a certain interested will apply for a certain principal during a specified future time period. So, all the calculations which you have done for the par yield, for coupon bond, for the 0 rate, for the forwarded rate all these things can be utilize in order to calculate and find out what is the forward rate agreement based on which two parties can go and sign a contract based on a forward. Now having said that when you all the calculations which you have done for the word forward remember that the difference of the conceptual difference between the forward and futures for our discussion is not much. So, once you able to find out the forward prices and the forward rates, similarly you can find out the interest rate based on which you can do the calculations for the future markets also.

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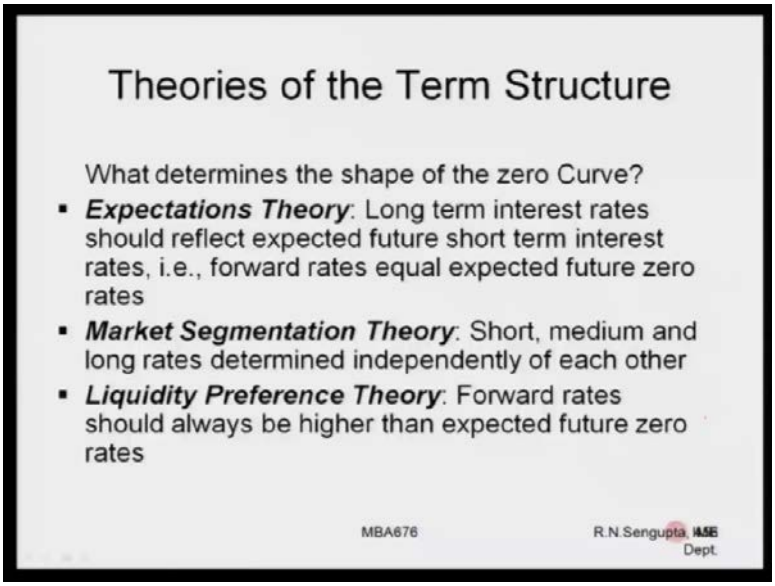

Now, theories of term structure, so before you proceeds I would like to discuss we were a few very commons and very simple details. So, we will just mention the highlighted points and in the next class we will discuss, what are the actual implications of this? The things we will discuss in a next class are; what is the expectation theory, what is a market segmentation theory and what is the liquidity preference theory. So having said that I will urge students again to read Jonical and understand what are the different types of interest rates. Based on which we will do all the calculations and we will try to see, how will the interest rates derived based on this.

Thank you very much. We will start of the discussion on the term structure in the next class.

Thank you. .

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