Quantitative Finance

Prof. Raghu Nandan Sengupta

Department of Industrial and Management Engineering

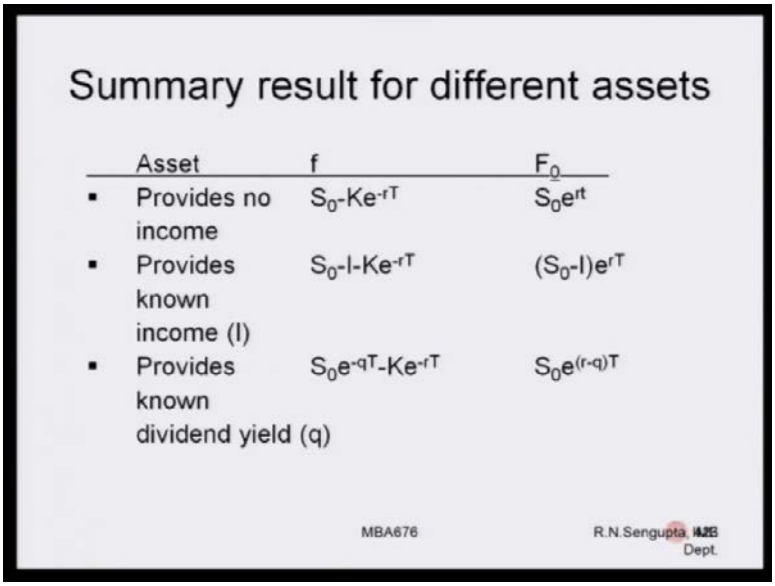
Indian Institute of Technology, Kanpur

Module – 04

Lecture – 22

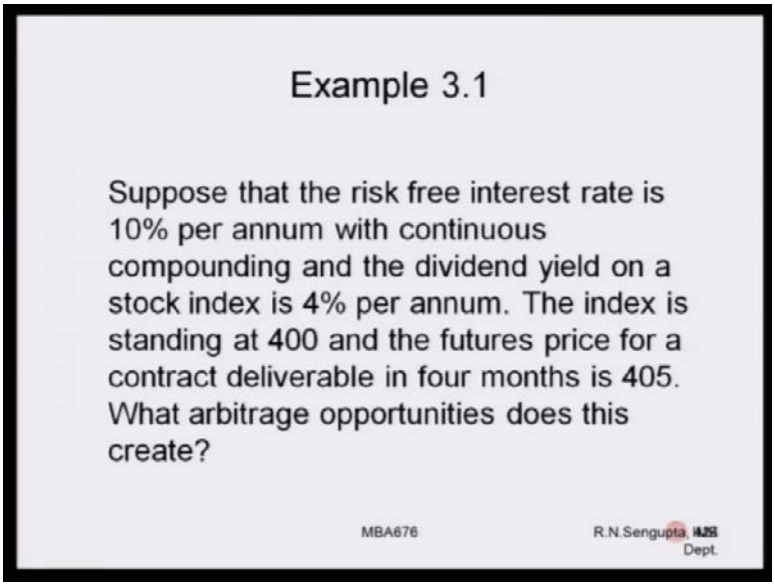
So, welcome back. As you know we are discussing about convenience yields, dividend yields, risk free interest rate and based on that we are trying to find out the values of small f or capital F which is the value of forward and the forward prices.

(Refer Slide Time: 00:27)



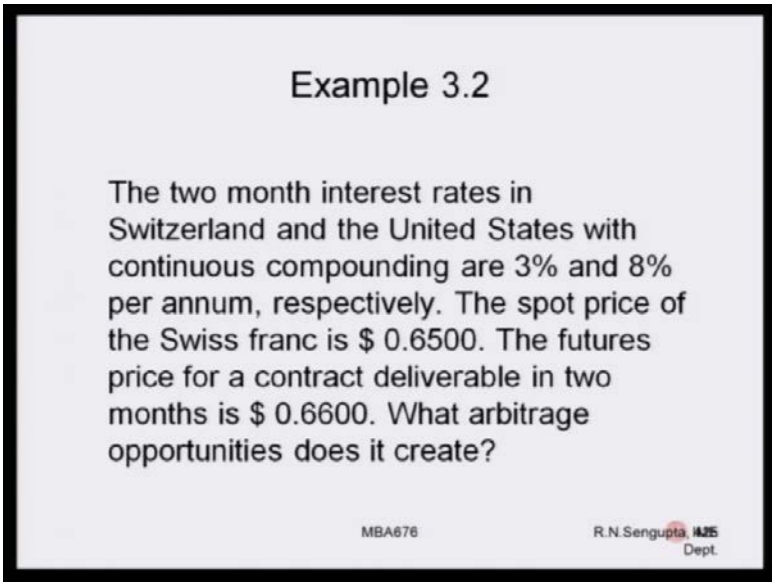
So, this is the basically the PPT slide which you see basically the assets different type of assets, the first one we provide no income provides fixed income which is I 0 as of now and provide some dividend yield and the second and the third column are basically the value of the forward and the forward price.

(Refer Slide Time: 00:47)



So, few very simple example, suppose the risk free interest rate is 10 percent per annum with continuous compounding and the dividend yield calculate being four percent per annum the index is standing at 400 and the future price of the contract deliverable is four months is 405. So, whether there are arbitrary opportunities, you have to say to yes or no. So, remember risk free interest rate is given as 10 percent per annum then the given which is basically q is given by 4 point some annum the index standing as of today as zero is 400 a contract delivery in four months where the time period is now 4 by 12 time period t is given 405 based on the you find out whether this 405 is less than or greater than equal to the value which you are found out which would be S 0 into e to the power in the bracket r minus q T. So, t if you should remember, it is basically four months which is 4 by 12, r is point 1 and q is with the minus sign would be point zero four based on that you can find it out S 0 as I mentioned is 400.

(Refer Slide Time: 01:53)

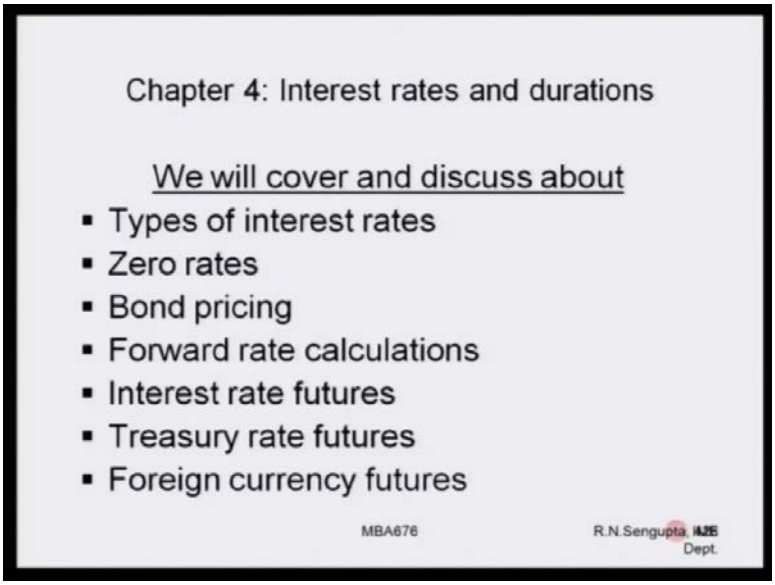


Coming to the next example, the two month interest rate in Switzerland and the United States, I given 3 percent and 8 percent; remember if I am standing in Switzerland then my domestic currency is Swiss franc and the interest rate is 8 percent, and the foreign currency is given at a dollars, hence the interest rate it is 8 percent. And if I change my position, if I go to US then US would be the domestic one, Switzerland would be the foreign one. The spot of Swiss franc is 0.06500. So, depending on where you are you will basically try to find out the foreign rate based o the domestic one. So, if you are in US then obviously will quote the price accordingly; or if you are in Switzerland, you will quote the price accordingly. The futures prices is given for the contract in two months 0.6600. So, whether there are arbitrage opportunities for that you have to basically find it out.

So, remember the time frame is given as two month, which is 2 by 12, and in this case remember very categorically, so if you are using the Swiss franc in dollars spot S 0 as point six five then obviously, e to the power r minus r star would be the corresponding interest rate where r would be for the domestic one, r star would be the foreign one and then you will basically calculate and find this value and compare with the value of 0 point 6600 and then say whether there is arbitrage opportunity is there yes or no. In case, if you change in your position go to the other country then the corresponding values would be very simply calculated spot now would be 1 by 0 point 65, and the futures would be the value would be 1 by 0 point 66 time duration remains the same which is exactly would be two months which is 2 by 12, but now the r and r star would just be the

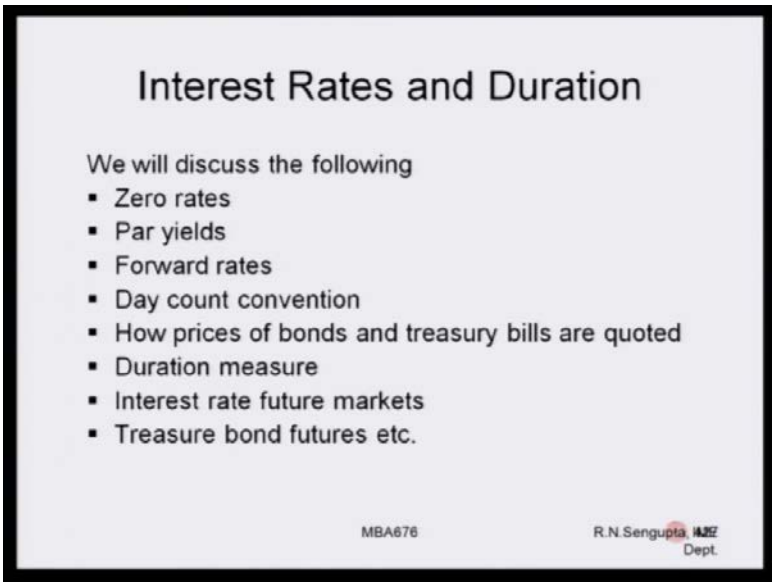
reverse. If in the first case, if it was three percent and eight percent, now it will be eight percent and three percent depending on how you are trying to look at the diagram or the whole situation.

(Refer Slide Time: 04:02)



Now, we will cover the following concept which is types of interest rate, what do you mean by zero rates, bond pricing, forward rate calculations, interest rate, par yield and so on and so forth. So, go very slowly and with the example such that you can understand it clearly.

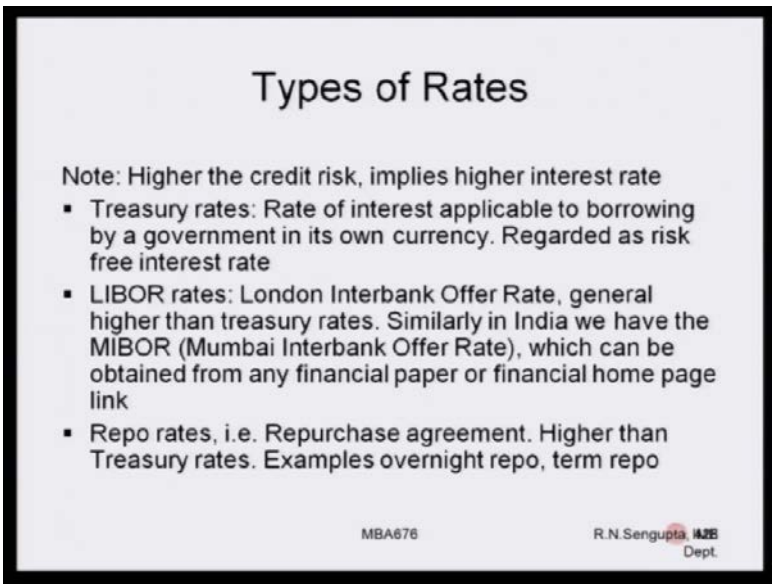
(Refer Slide Time: 04:15)



We will discuss as I mentioned the zero rates, par yields, forward rates, day count

convention, interest rate future markets and treasury bonds futures.

(Refer Slide Time: 04:24)

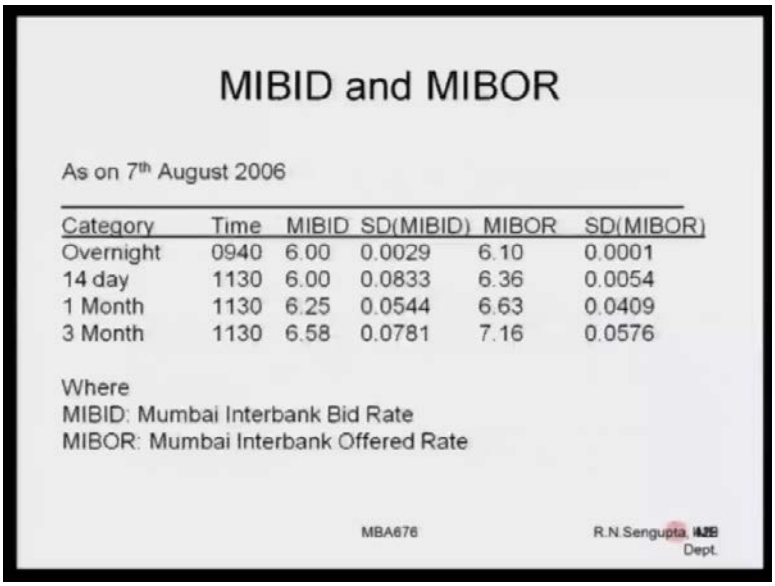


Now before that we will consider the two simple concept which is the LIBOR and the LIBIT which in India we have the MIBOR and the MIBIT. So, LIBOR means London interbank offer rate and LIBIT means the London interbank bit rate. In India, similarly have the Mumbai interbank offer rate which is the MIBOR and the MIBID which is the Mumbai interbank rate. So, this basically this means the amount of money which is being exchange by banks what is the interest rate which is being charge, so obviously, we will think that why they are two different interest rate. If you remember, if you go to the bank, and if you deposit money or if you withdraw money, the interest rates are different or if you go to the bank and sell dollars or buy dollars the rate of which we will sell and buy are different. So, in the similar way, they would be a MIBOR and MIBID depending on what is the interest rate of buying and selling in India, similarly different foreign banks with the trade they basically trade that based on the LIBOR and LIBIT.

So, LIBOR are also would be find out later on in the problem is the basically repurchase agreement and depending on the repurchase agreement which is happen two financial institution of the banks. And examples of the reports rates can be over night they can be term repo rate, they can be for weekly basis and so on and so forth that means I give some money, I want to repurchase that amount of money. So, what is the interest rate which is being charged. So, remember one thing, if the demand and supply is very fluid the symmetric in the market then the difference between the MIBOR, MIBID, LIBOR, LIBIT all the buying in the selling interest rate is very small, the more the difference in

the demand and supply is greater would be the price difference, we will see later on, and also time frame also plays a big factor.

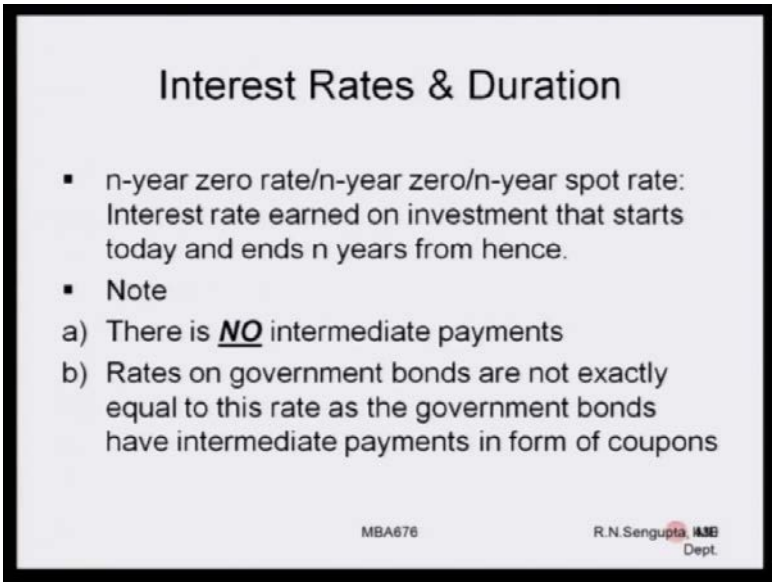
(Refer Slide Time: 06:04)



So, as on 7th August, 2006, these are actual data whenever I basically giving some information or the data they are actual data which I am taking from the net. So, you have the MIBOR and the MIBID rates are given the overnight is given at time 940, then 14 month at 1130, 1 month at 1130, 3 month are 1130, and the time of the trading and the third, fourth, fifth, six columns are MIBID and its standard deviation, MIBOR and its standard deviation.

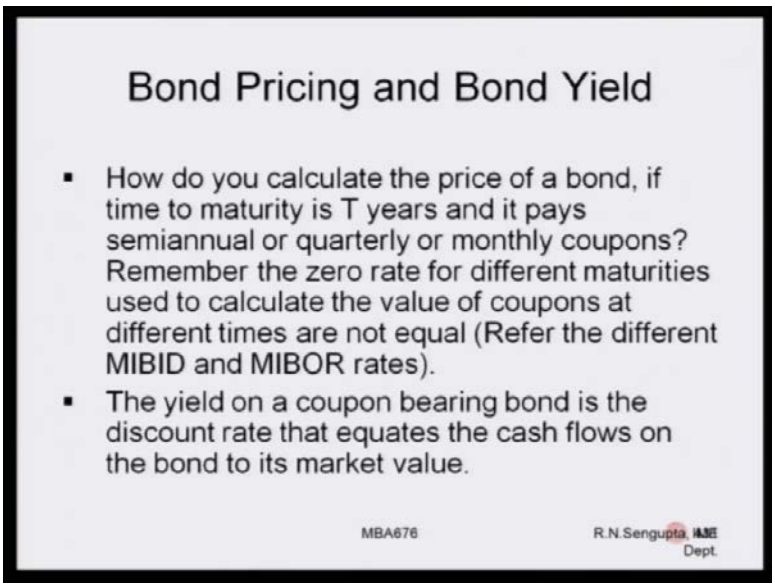
So, if you look carefully, as the time increases, hence the MIBOR and the MIBID interest rate also increases which means in a way that time had some amount of uncertainty, hence we would like basically have a higher interest rate. But on the other hand also this has a high interest rate is huge amount of fluctuation, so that disposition basically sub soon and the picture comes out to very clearly if you see the standard deviation. So, standard deviation for MIBID or standard deviation for MIBOR, for overnight one or for the fourteen day one when compare with the three month one is very slow; that means, for three months as you go down the line the MIBID and the MIBOR in the standard deviation basically in cases, similar would be the case for the LIBOR and the LIBID.

(Refer Slide Time: 07:18)



Now we will find out the n year zero rate n-year zero rate or n-year spot rates. Note for calculation, we will try to find out will consider the no intermittent payment is being paid that means, there is no coupon payment. Rates on government bonds are not exactly equal to the rate as the government bonds have intermediate payment of the coupon rate which is being create by the government bonds.

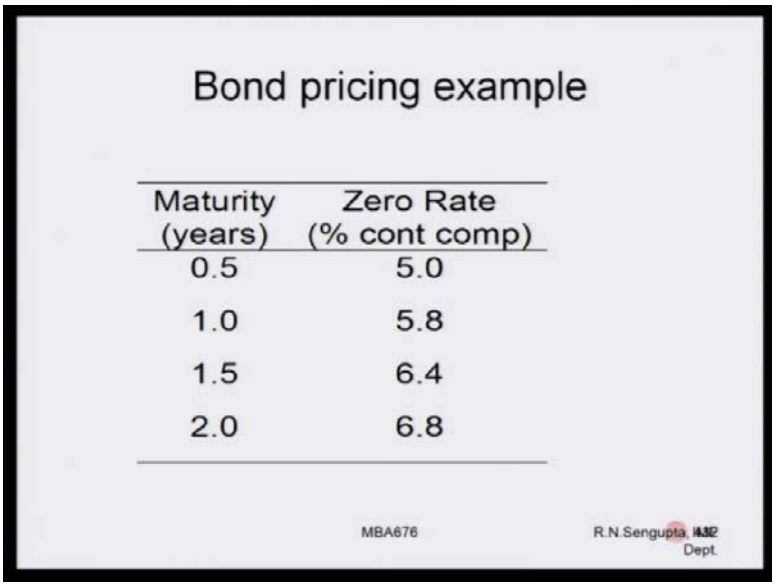
(Refer Slide Time: 07:42)



So, now the question is that how do calculate the price of the bond. If time to maturity is T years and it pays semiannual or quarterly or monthly or daily interest rate whatever that is there or coupons, remember the zero rate for different maturities used to calculate the value of the coupons and different times and not equal and they may change because

of reasons of time frame and volatility and fluctuation which is happening in the market. The yield on a coupon bearing bond is the discounted rate that equates the cash flow based on which you are getting exactly equal to the market value. So, there are two things one is basically the bond pricing how we will do that another is basically yield on the coupon based on which you will try to find out what is the value of the bonds such that demand and supply exactly matches considering the some coupon payments are being paid, but for the zero coupon one will consider zero rate is based on the fact that this no intermediate payment which is there. So, given one you can find out the other or vice versa.

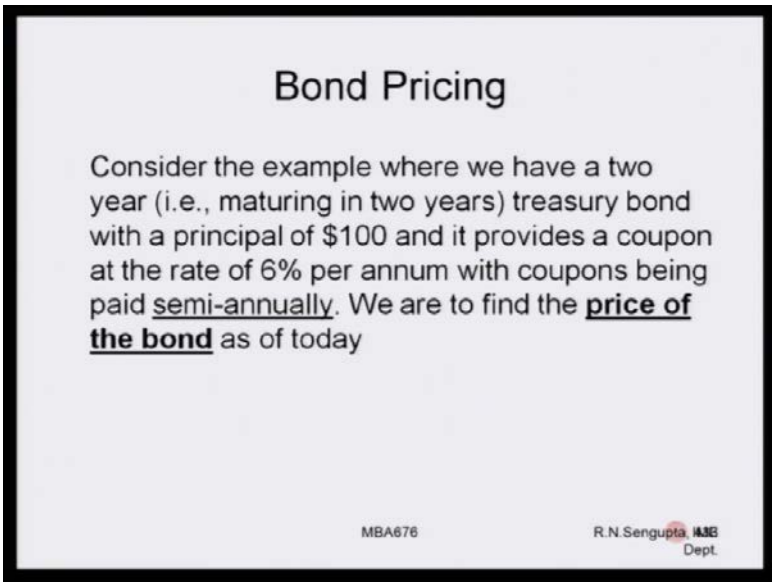
(Refer Slide Time: 08:40)



So, now, consider the bond pricing example is given and see here the second column very specifically mention this is a zero rate. So, these are the zero rate means there is no coupon being payment is being made, and the first column you have the time, they are in years as always. So, we have half year, 1 year, 1.5 years, and 2 years, and the zero interest rate are continuous compounded per annum are given as 5.0, 5.8, 6.4, 6.8. Again you will see as time increases, the zero interest rate also increase. Now what does this 5, 5.8, 6.4 and 6.8 means, it means that we have basically investment some any money for a time frame of half a year, the interest rate based on that considering this no independent payment for the principle amount would be 5 percent. If you put that money for one year, considering there is no independent payment the principle amount basically now gets interest rate of 5.8 percentage per annum continuously compounded. So, these are continuously compounded. Similarly 1.5 year, you put that money no intermediate

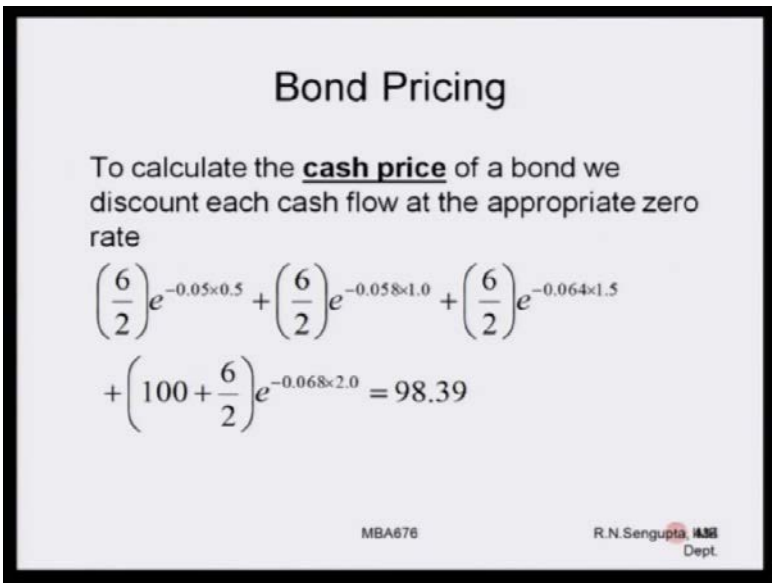
payment, the principle of amount increase by the interest rate are 6.4, what is per annum continuous compounded and similarly for the value of 6.8.

(Refer Slide Time: 09:55)



Consider an example, where we have two years maturing treasury bond with a principle amount the phase value which is given is 100, and it provide a coupon at the rate of 6 percent per annum with coupon being paid semi-annually. So, remember this word, they are being made 6 percent per annum which is a coupon is being paid twice in one year. So, one after six months, another after another six months we have to find out the twice the bond as of today based on the interest rate chart which has just been shown to you.

(Refer Slide Time: 10:28)

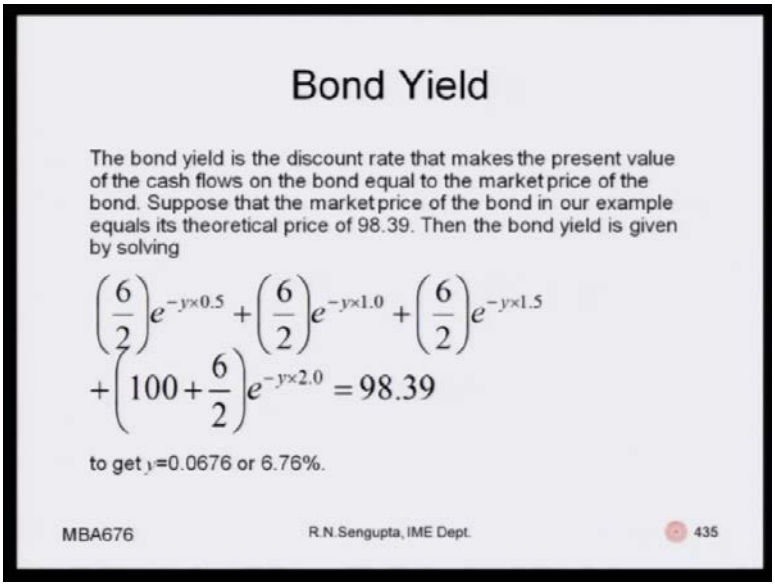


So now let us come to the calculation. So, calculate the cash flow or the total amount of been given back to you, remember once the clock starts after six months you will basically get that some amount. So, what is the amount, first you note the amount is six by two, because you have been paid semi-annually, and the coupon data was six per annum. Now that three rupees three dollars whatever it is being paid after six months has to be brought back as of today and try to find out the overall value. So, what you do is that for the six month period which is half year, the interest rate was five percent, so basically you will multiplied by six by two which is the three dollars by e to the power minus r which is minus 0.05 into t. So, t is the time frame which is basically six months which is basically six months by twelve which is 0.5. Now second, the total duration was basically given as 2 years, now you will find out the second coupon is being paid after one year.

So, what is the total amount of coupon payment 6 by 2 which is 3, but when it is being paid after one year. So, if you find out the value as of now, it would be 6 by 2 which is the amount into e to the power of minus r t; r is now not five, but it is 5.8 depending on the interest rate, which is for one year and t is not five months, it is six months is basically one year hence it will be 12 by 12. Now if you go to the one and half years again will find same coupon.

Now the actual interest rate would not be five not be 5.8, but it will be 6.5. So, it is 6 by 2, which is the coupon value in to e to the power of zero point zero six four in to the time frame which is t which is basically one and half years and finally, when the total time duration comes you will get the coupon that true which is by two plus that you will get the phase value. So, the total amount of money which you are getting back after two years which is 100 plus 6 by 2, but when you basically bring it back as of today. So, now, you are basically trying to utilize two different things time frame is now two years and the interest rate is not 6.4 is basically 6.8. So, your actual calculation would have 100 plus 3 in the bracket into the bar minus zero point zero six eight into two and that value are to be 98.39.

(Refer Slide Time: 12:50)

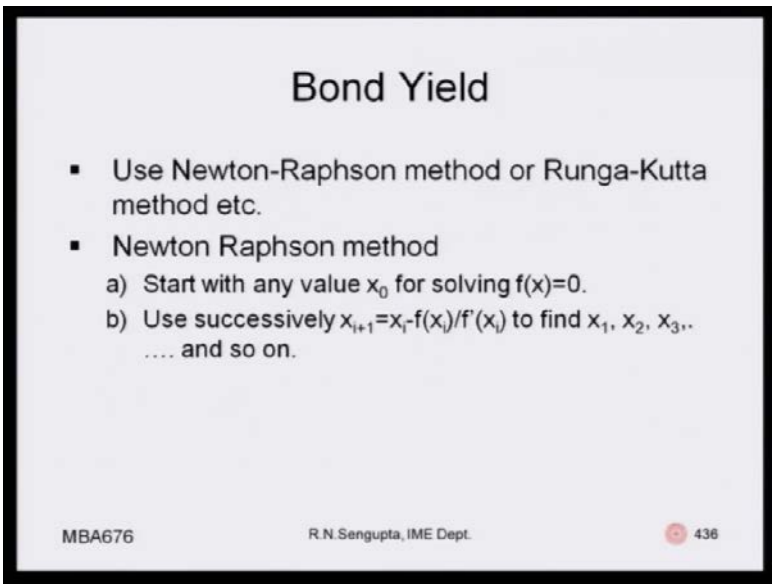


Now given this 98.39, you want to find out what is the bond yield; that means, what is the interesting value of the yield of the bond is coming back to you. The bond yield is discounted that makes the present values. The cash flow of the equal to the market price of the bond exactly. Suppose, the market price of the bond in our example is as given as ninety eight point three nine then you want to find out what is the basically the bond yield. The bond yield the basically the interesting value of the coupon which is happening, if you remember we have said that the coupons are being paid six months sense, so considering the bonds coupons which you are getting back is six. So, it is basically six by two, but now if you consider the bond yield initially it was 5 for 6 months, now you will consider constant which is basically per annum on continuous compounded interest rate. So, it will be six by two e to the power of minus y into half.

The second term is again value of money which we are getting after one year, it will be 6 by 2 into e to the power minus y into 12 by 12 which is one. The third you are getting after one and half years, it will be 6 by 2 into e to the power the minus y into 3 by 2, which is the time duration 3 by 2. And the last one, again you will have the coupon payment which is six by two plus hundred, so this 100 plus three in the bracket into e to the power minus y into 2. So, hence the value comes out to be ninety eight point three nine which we already assume to the true based on that you can find out the bond rate yield as 6.75. So, remember one thing the more the bond yield is laser with the value on the right hand side less the bond yield is more than the value the bond yield the value on the right hand side. So, the yield comes out to be 6.76 percentage per annum

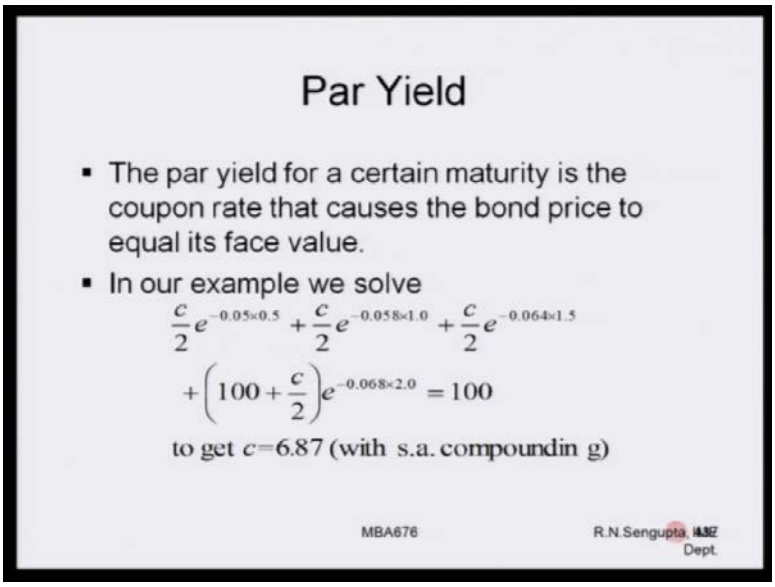
continuously compounded.

(Refer Slide Time: 14:32)



So, there are different matters on Newton Raphson methods or Runga-Kutta method different type of iterative methods using this you can find out the bond yield as done in our example.

(Refer Slide Time: 14:42)



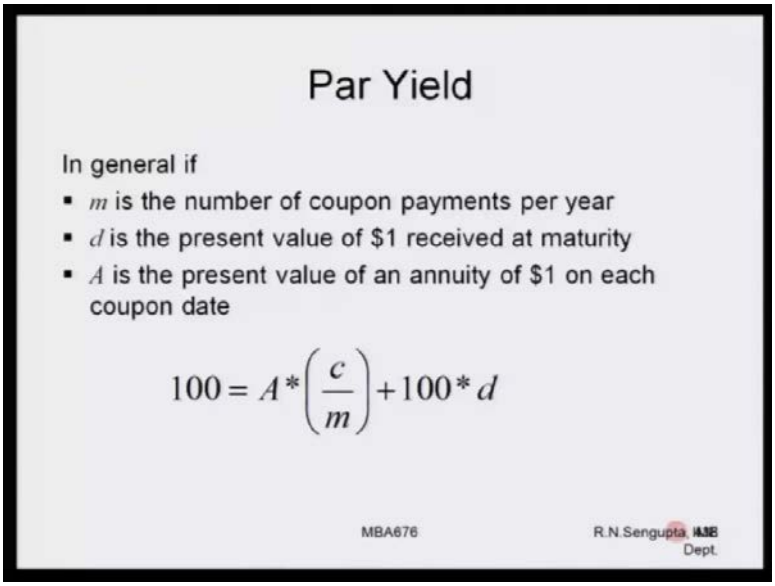
Now the third question will try to answer what is the par yield. So, par yield of a certain maturities the coupon rate that process the bond price to be exactly equal to is phase value. So, if you remember the phase value is not the value of ninety eight point which is the demand as a price based on which is the market price is dictating for the bond phase

value is the value of the of the of the bond which is return with technically we consider as hundred and ten. So, this case it is hundred now will consider the par yield based on the facts such that in exactly it was hundred. So, we have to find out basically the interest rate which is already given the table. So, for six months it was five percent for a time duration one year it was five point eight percent one and half years it was six point four percent and for a for a time duration two years it was basically six bar eight percent. So, utilize that value now using the par yield of a fix value of c.

So, now, if you remember mention vary categorically that is being paid semi-annually. So, the c amount would be made after six months which is c by two and the interest rate based on which you will find out and the time duration would be now five percent and the duration of a year second again will get back after one year c by two and the interest rate is again five point eight percent and one year third one again we will get back c by two and the interest rate the time duration is 6.4, 1.5. And the last case we will get back the par yield plus the principle value, which is hundred so that would be multiplied by interest rate into a minus then interest rate 6.8 percentage per annum continuously compounded the time duration is it basically 2 years. So, if you equated to the phase value the value of the par yield concept to be 6.8 percent. So, if you consider par the value of six and six point eight it will give you whether the bond is doing well and good or it is not definitely not going.

So, in the first case coupon paid on six would actual value phase of the bond was coming out to be ninety eight from three nine and in this case which is exactly to that phase value to the bond which is hundred. So, now, if you compare the value of the bond and as well as based on the par yield and as well as based on the coupon payment then you can make a decision whether the demand and the supply the bond is such that you definitely invest in the are you not definitely invested in that depending on what the interest rate values, but remember also the interest rate are fluctuating remember also the yield also fluctuating. So, you have to basically make a judicial decision based on the coupon payment based on the par yield based on interest rate and based on the yield the bond such that you can basically make profit or a loss depending on your actual a scenario is whether you want to basically increase your risk, decrease your risk, increase your return or decrease your return.

(Refer Slide Time: 17:24)

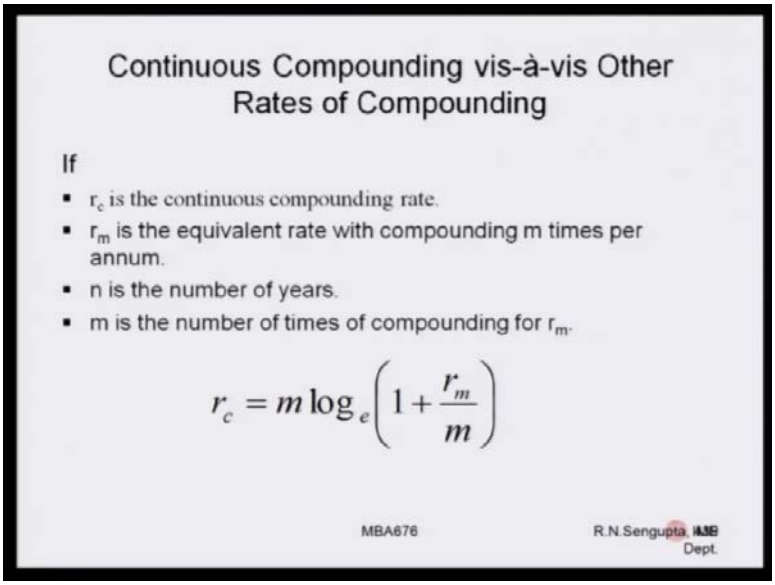


Now in general the par yield calculated by this value formula where hundred is the phase value that is equal to A into c by m plus 100 into d. Now what are this values m is the number of coupons which being paid. So, if you remember c is the par yield which is being calculated and here there in a problem, we consider semi-annually, so m was 2. So, m is the number of coupon payments which is happening per year; d is the present value one dollar received at maturity. So, if the hundred is the value, so it will be 100 into d which is the present value that will among which you are getting, and a is the present value of the annuity of one dollar on each coupon date payment, so obviously, you will multiplied that amount that is c by m coupons which paid multiplied by a such that you can basically balance the equations. So, exactly this is the same thing you will basically have a coupons which have been paid for some interval then of the last stage we basically have the coupon plus the principle amount.

So, if you coupons were being paid once in one month, so obviously, coupon would be c by twelve first month again broad back as of today then it will be c by twelve for the second one brought back today continue doing this in the twelve last term. We will now basically two terms this c by twelve plus hundred which is the phase value so; obviously, the interest rate would be for when we basically find out the present value of the first coupon which is being paid after the one month would be that present value would have basically three terms one is the coupon payment, which is being paid which is c by twelve multiplied by e to the power of minus the interest rate for the one month multiplied by time duration would now to the one by twelve that is one way. Similarly

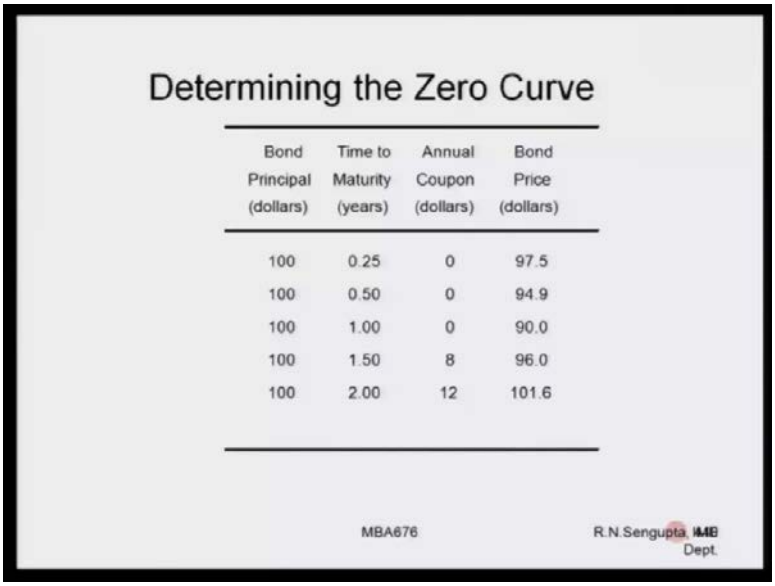
for next case, it will be coupon payment multiply by two terms e to the power minus r where r is the interest rate for two month period multiplied by the time which is basically in our two month period which is two by twelve you continue doing it. And the last sense, the value which we will had have would be 100 plus c by 12 in the bracket e to the power minus interest rate which is for one year multiplied by 12 by 12, which is the time duration.

(Refer Slide Time: 19:36)



In the continuous compounding sense, again you can convert into continuous compounding the sense given by that is values where r c is the continuous compounding rate, r m is the equivalent rate with compounding m number of times per year and calculated percentage per annum; n is the number of years and m is the number of times of compounding. So, if you go back to the actual formula, which we have already discussed either we have the continuous compounding interest rate or we have the compounding interest rate the done m number of times whichever it is you can convert one formula to other and do your calculations accordingly, but remember very clearly that it is based on do all your calculation the continuous compounding rate, but also remain the interest rate which is given should basically we specific on two accounts, one is it is percentage per annum and it should be continuous compounded; if it is percentage per annum, but compounding is being done m number of times, you will immediately converted into continuous compounding case and do your problems accordingly.

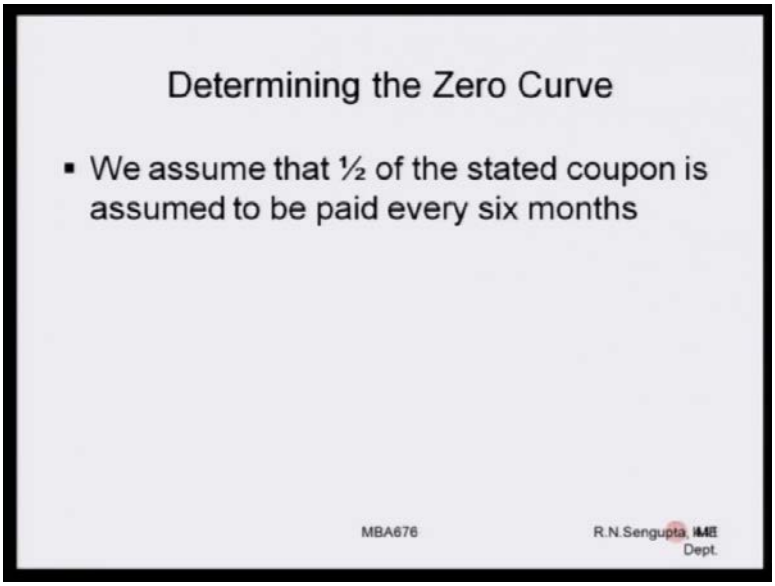
(Refer Slide Time: 20:30)



Now you want to determine the zero curve; so in the first column, consider we have the bond principle amount, we have the time to maturity- second column, third one is annual coupon and the bond price dollars. So, whatever the values now before I proceed to the problem, let us pause and see what values are given bond principle amount is basically the principle amount of the phase value which you have return in the on the bond. So, this is always hundred are ten whatever the value is now time to maturities given first is one-fourth the year second is basically half of year third is one year fourth one is one and half years and fifth is two years. So, all the time duration have been converted into time duration are in years it could have been months it can be weeks and so on and so forth.

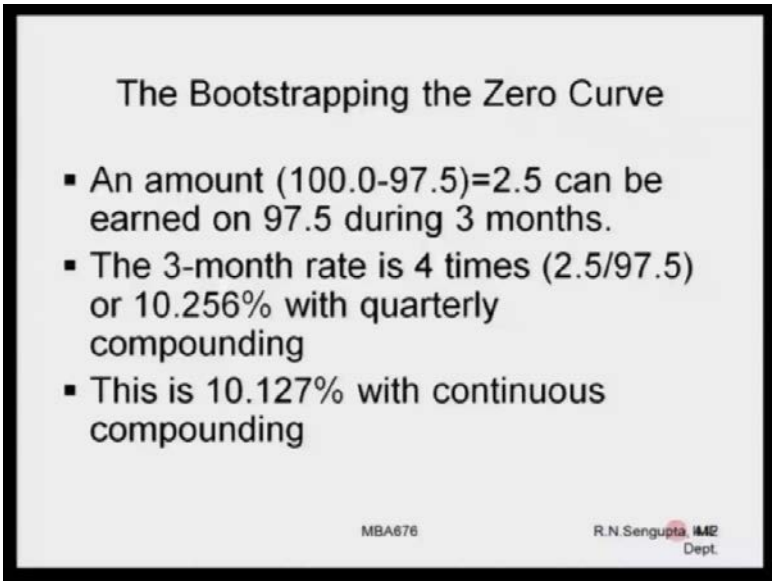
So, if it was given in weeks, so if it is three weeks, it would have been three by fifty two it was in months so; obviously, as we know it will be number of months you had better value if it is days will be number of days divided by three sixty five. So, whatever it is trying to convert the time duration time scale of year third column is the annual coupon rate in the first cases three cases see there is no coupon hundred because their all zero now in the fourth and fifth you see the annual coupons which are been paid is twelve; obviously, when i mention would be their annual payment at the week paid annually at the being paid semi-annually at the being paid daily at the being paid quarterly. So, whatever it is that it will be space it is clearly specified in the problem in the bond price considering very simply they are the actual bond and supply price based on which you are trying to dictate try to find out what is the interest rate given the zero curve has been find out and given the forward have to be found out. So, you will find out both of them

(Refer Slide Time: 22:15)



Now we assume for this problem that half of the stated coupon is assume to be paid every six months. So, for the fourth one, the eight dollars would be paid in two dollars four and four after six months. And for the last one, it is twelve dollars, it will be six dollars per six month then six dollar six month then six dollars in the in the one and half year and when then time duration comes for two years, the principle amount which is hundred plus six dollars would be paid back to you so obviously find out the present value of the total money.

(Refer Slide Time: 22:42)



So, in the first case, let us do the problem. In the first instant, the amount which will be getting is hundred is the phase value and ninety seven point five is basically the demand

accepted price. So, this is difference is two point five. While ask yourself this two point five can be earned on 97 point 5 if you buy today, but what duration, duration is three months. So, what you are trying to find out and try to understand is that this two point five would be earn by on a actual the investment the ninety seven point five on a three month period, hence the three month interest rate is four times this value because this will be paid four times in one year, if you basically expand this time scale three month to one year.

The interest rate concept to be 10 point 25, this is percentage per annum, but remember very, very clearly that these payments are being made after three months, hence you will say this is 10 point 256 per annum interest rate being compounded quarterly. So, the number of time times it is paid in year is four m is 4, so obviously, will try to convert into a continuous compounded interest rate. So, we use the formula which we have already done based on that find out the value comes out to be 10 point 127.

So, with this, we will end the class today, but I will just reiterate what we did. So, given the phase value, given the coupon payment is being done given what is the demand and supply price we will first step and found out what is the continuous compounding interest rate for the zero coupon for one-fourth of a year, and based on that I will proceed to find out the continuous compounding interest rate and the forward rate for the subsequent calculations such that we are able to understand what does the zero rate and what does the forward rate give us the information among the box.

Thank you very much.