Quantitative Finance

Prof. Raghu Nandan Sengupta

Department of Industrial and Management Engineering

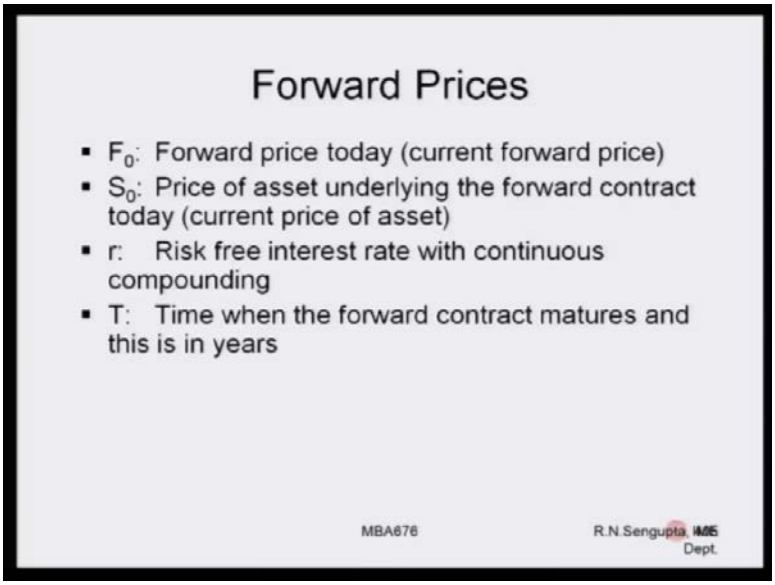
Indian Institute of Technology, Kanpur

Module – 04

Lecture – 21

So, good morning, good evening everybody, so we are back into this Quantitative Finance course, and we were discussing about derivatives. So, initially we discussed about forward features and then very few very rudimentary definition of options call and put what is the concept of long and short and so on and so forth. So, now, we are trying to find out the prices of forward and futures. So, considering the basic difference between the forward and futures that one can be exchange trader another is over the counter another has to have a definite time period before which it cannot be exercised another can be exercise depending on demand and supply when it is depending on the buyer and seller. So, we will consider the simple price concept of how prices have found out for the forwards.

(Refer Slide Time: 01:06)

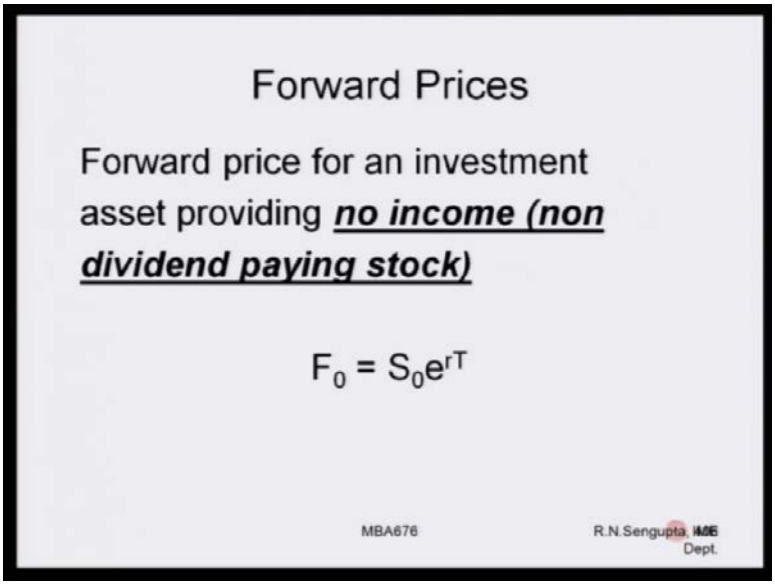


And try to use the same concept when you go to consider the price of the futures, ever though the prices may not be exist, but it will give some good idea that how the prices of

the futures are decided depending on very simple assumptions. So, with repetition I again say the name variable of the symbol F is for the forward price with the suffix basically denoting the time frame, where zero denotes the current time frame. So, if it is one, two, three, four whatever it is the suffix, it will denote the different time frames. S is the spot rate of that particular product which you are trying to buy and sell.

If you remember that I mentioned this room one whether the spots are being traded bought and sold, and the corresponding room two has the derivative of this spots, whether forward, futures options are been traded bought and sold. Then you have R as the risk free interest rate without the suffix f being given. Again we will consider very synthetically restless in lending, barring dates are same. And the fourth so called variable is the time frame which is in years, we will stick to the year, it can be month, but we will convert everything into years.

(Refer Slide Time: 02:14)



Now forwards price for a investment as a providing no income; that means, there is no intermediate payment is given by the fact that carefully increase at the same continuous compounding rate as r f. So, it will be given by F 0 which is the price of the forward is given by S 0 into the e to the power r T

(Refer Slide Time: 02:40)

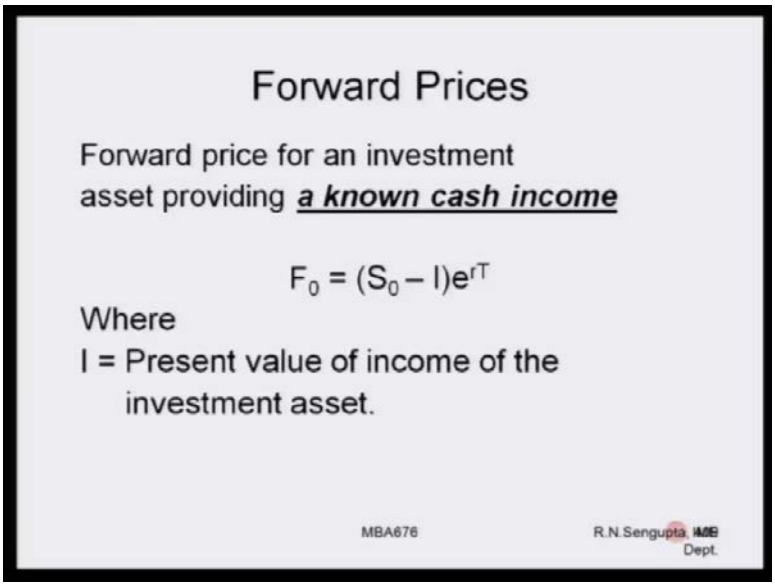
Now if you are the bank of England actual data from 13 July, 2007, and the spot price is given as the 327.528 UK pounds, this risk free interest is given by 5.75 per annum continuous compounded, it is time frame of six month which is six by twelve your forward price is calculated given by this formula which is S 0 into e to the power r T.

(Refer Slide Time: 03:05)

Now, as I am repeating this thing, but it is basically make you to well aware of that. If the forward price is greater than the right hand side which is S 0 e to the power r T, then arbitrageurs will by the asset and short the forward contract in order to make the profit if

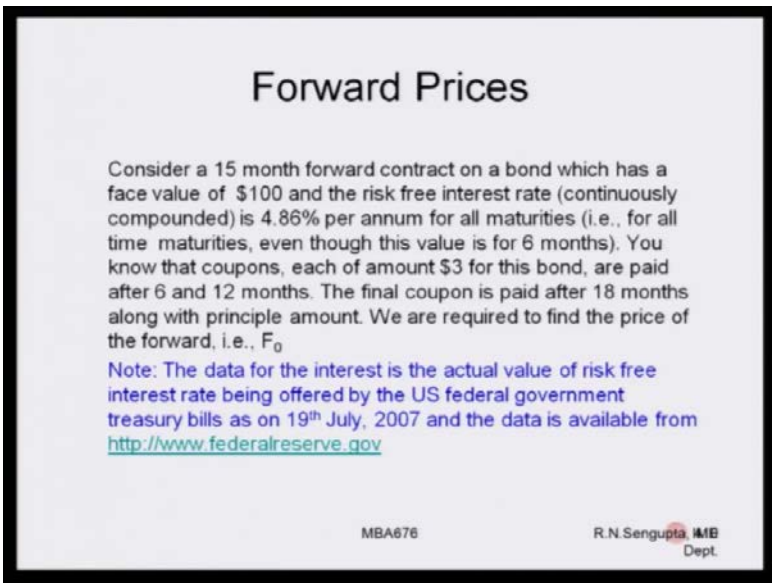
it is just the other way round then he or she would short the asset and by a forward position; that means, the short and the long would be the consecutively different depending on whether the value of f is greater than s into e to the power r T.

(Refer Slide Time: 03:39)



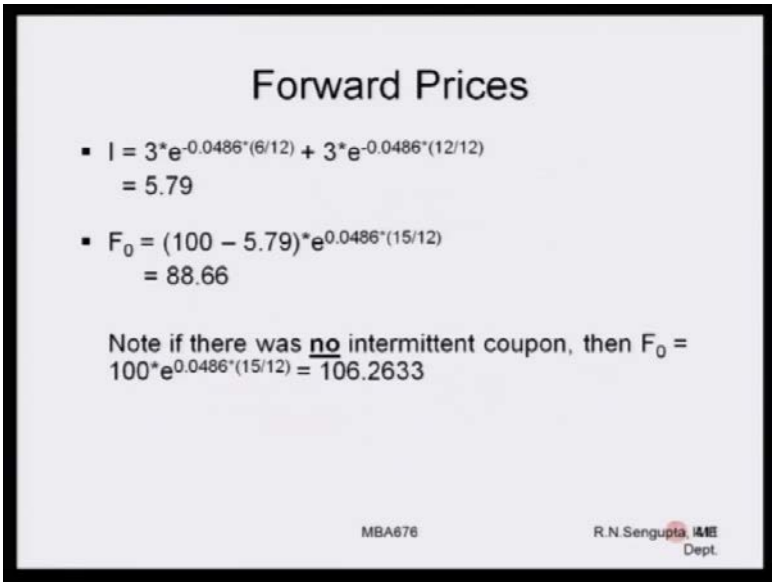
Now, consider you have forward price, but for an investment with price dividend at some intermittent times. So, I which is give is S 0 minus I, I is basically the present value of all the intermittent payments. So, here it is not given a suffix. So, I is a present value of all the incomes for the investment asset.

(Refer Slide Time: 04:01)



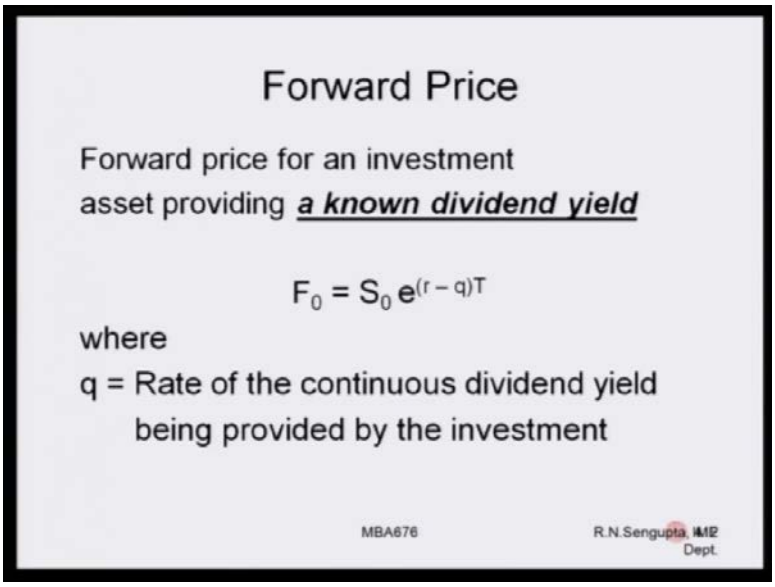
So, consider that of fifteen month forward is there, principle of the values phase values hundred, continuous compounding per annum interested is given by 4.86, this data is as I mention is calculated from the federal reserve. And for the six month, so each coupons are for three, they would be paid as a six months twelve months and so on and so forth.

(Refer Slide Time: 04:24)



So, based on that if you find out I 0 then you can find out the F 0 price also given, the fact it is S 0 minus I zero that is the bracket into the e to power r T.

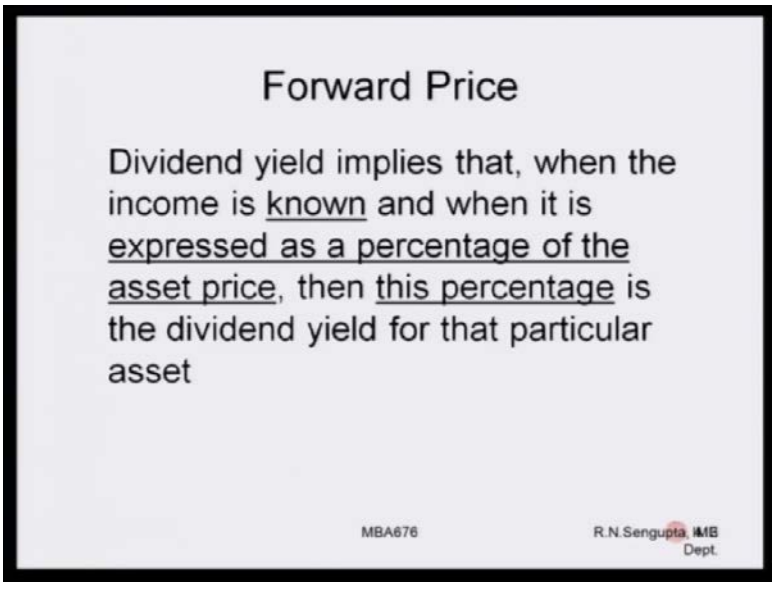
(Refer Slide Time: 04:34)



Now, consider that you have forward price is for investment which is paying some continuous dividend. So, dividend is basically some amount of money which is coming back and you calculating with respect to continuous being continuous respect is paid back to you. So, obviously, when you try to find out the corresponding the investment value as on date which is I 0 obviously has to be factor by the concept of e to the power minus some r 1 T. So, this r 1 and r f which is the risk interested are not the same. So, we are here if consider the known dividend yield is given by q.

So, if I try to find out the known dividend in present value, it will be S 0 into the e to the power minus q T. So, obviously, you will be getting some money back. So, the overall amount which will find out in order to find out F which is the forward rate would be S 0 into e to the power there are two terms r is the risk free interest rate which is the money coming back to the you and minus q is the amount of so called money which you would be finding out at the present value to the find out all the amount which will be taking in place till the time to maturity. And obviously, t is the time to maturity in years. So, here if you note the last line it says that q is the rate of continuous dividend yield being provided by the investment.

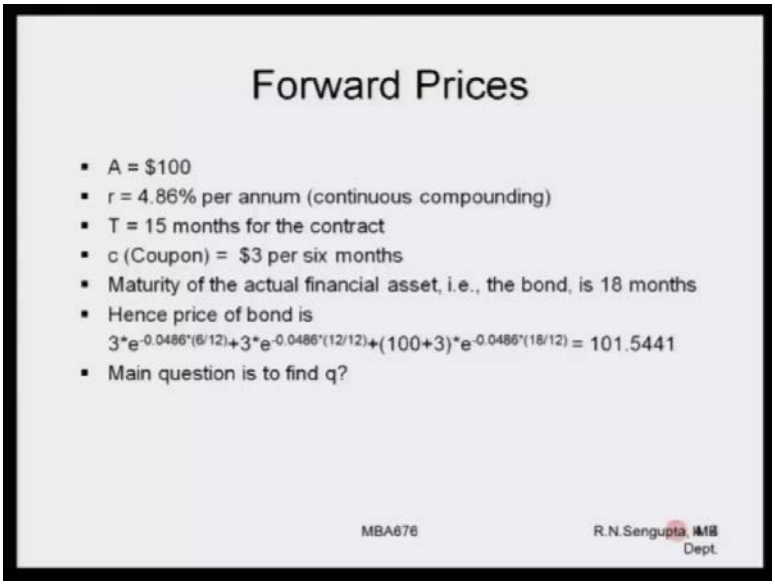
(Refer Slide Time: 05:58)



Dividend yield implies that, when the income is know and when it is expressed as a percentage of the asset price then this percentage to the as the dividend yield the particular value. So, the value of q and r are on percentage term per annum continuous

and the dividend yield is being carried on the expressed as a asset price based on which you are trying to find out the dividend yield.

(Refer Slide Time: 06:19)

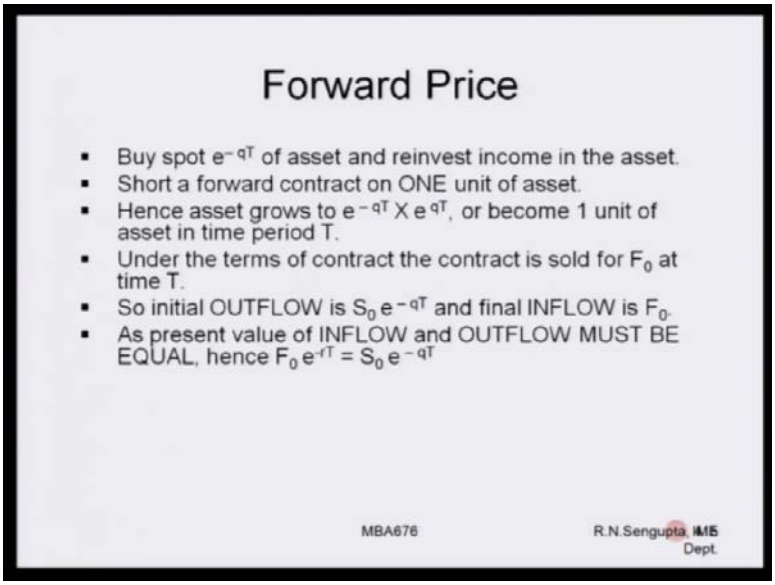


Now again consider A is given as hundred r risk free interest rate is given, T is time 15 month which is 15 by 12, the coupon are given as 3 per six month, so three for the first six month, again for three next six month and so on and so forth. Maturity of the actual financial asset that there is the bond is happening in eighteen months. So, what you need to find out is, what is the total payment? So, if you see the first value which is basically three in to e to the minus 0.0486 into 6 by 12, if the present value of the coupon which is being paid after six month and what is the value now. The next term is again three, because three would be paid after twelve month into it is the bar r t, which is also the same minus r T, because we try to find it now. But here the time frame will change, it will now become twelve by twelve because that amount of payment which will is happening after one year. So, you are trying to basically bring that whole amount of money as of now and try to find out the present rate.

And when this total amount of duration of 18 months is over, so obviously, they would be two per months; one is after 18 months you have the coupon payment of three and also the principle amount which is 100. So, it will be 100 plus 3 which is 103 which you will get after 18 months and you will try to bring that money value as of now, so obviously, it will be minus r into T, where T is now 18 month period which when convert

into years would be is 18 by 12. So, the actual value can find out would be given by 101.5441, and you can find out the value of q also depending on this value of 101.5441.

(Refer Slide Time: 08:02)



Now basic what is the fundamental principle of forward price. So, let us given a very simple example in order to understand how the balancing is done depending on you have a spot in the room one and some forward in room two. So, I am not mentioning what is the quantum was spot, it can be anything it can be one unit, it can be other unit also similarly this forward can also be of one unit or less or more than that, but we will try to basically the balance of portfolio as the overall risk comes out to be zero. What I am mean by risk is basically the overall fluctuation the price is done in such a way that the portfolio which is formulate using some quantum of the spot and some quantum of the forward balances.

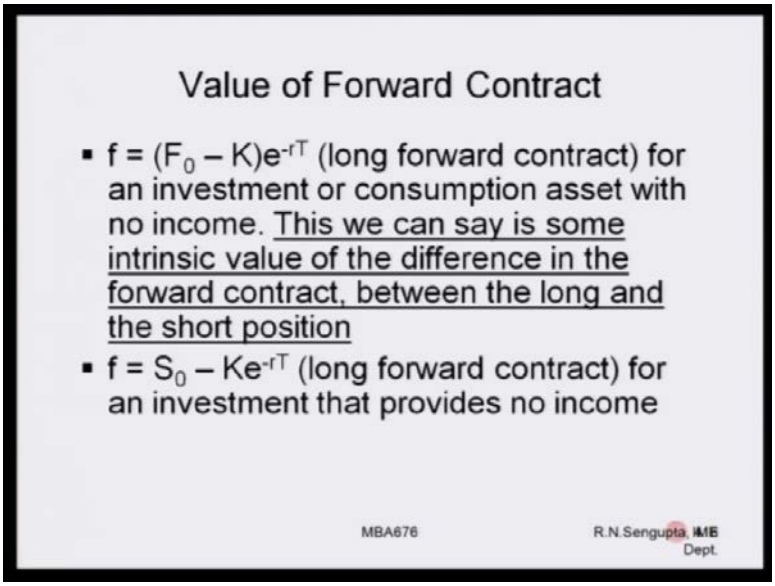
So, buy a spot of what value e to the power minus q T. I will come to that why we are taking e to the minus q T, and reinvest that total amount of asset income in that particular forward. So, now, what you do is that by a spot of some amount due to the minus q T. Now short of forward contract of one unit of that particular asset. So, you are trying to basically plane both of the rooms; the first statement being means you buy a spot in room one, second statement means you short sell a forward contract of one unit. Now what happens, in room one you are basically bought a spot of this quantum e to the power minus q T, so that will be increasing. So, at what rate is increase, it increases at that rate

of e to the power plus q T. So, the total quantum is e to the power minus q t is increasing by the rate of e to the power q T, so if the cancel out so at the end of the time period it becomes one.

Now another term of contract, you sell a forward at F 0, when the time expires. So, what it happen is that in spot, you basically invested that amount become one total quantum become one including the increase; and when you sell that in room two which is exactly could in the forward. So, if you consider the outflow and the inflow initially what you had, you had basically bought a spot that was an outflow a money then when the time expires you buy back that spot which is inflow money and simultaneously you basically sell of forward contract also. So, initially they was outflow at time T, there is a inflow and outflow; this inflow, outflow are happening in two different rooms spot in room one, forward in room two, so the initial outflow which is given for the particular sport because now that quantum was given by e to the power minus q T. So, what are the sport price, spot price for S 0. So, the overall outflow was S 0 in to eight to the power minus q T, and the final inflow which is happening is f zero.

So, at a present value what you have that S 0 which you have had increase by this value which is S 0 into e to the power minus q T, which the present value that has increased by the quantum of risk free interest rate which is S 0 into e to the power minus q T, in to e to the power plus r T. So, if that is exactly equal in the forward then what you will have is the forward which is F is equal to S 0 into e to the power in the bracket r minus q; the r term is coming for the increase and the minus q is coming to the fact that you are basically getting some dividend back at time T equal to zero you are trying to basically find out of that value. So, at the end of the day, if you see the equation, if you should take e to the power minus on to the right is exactly the same equation. So, F 0 is equal to S 0 into e to the power in the bracket r minus q into T, where T is the time frame based on which you trying to find out the forward price.

(Refer Slide Time: 12:14)



Now till now we have been considering about capital always that was the forward price, but this is the different concept also which is known as the value of the forward. So, the value of the forward and the forward price are different things. Price of f basically means depends on the demand and supply in room two plus the relationship between this part and the forward, and the value of the forward is what is the intrinsic difference between these two prices at which it will be bought and sold. So obviously, if there is a intrinsic difference in this spot on the forward, the value of the forward will be different, it can be either positive or negative. If it is positive or negative, somebody will make a loss somebody will make a gain; that means a long or a short position in one of this cases would basically make a profit. So, if the long makes a profit the short makes the lose or vice versa; that means, who is basically selling and buying.

Now, the small f which is basically the value of forward considering is a long forward contract remember whatever I am saying is basically for the long forward contract for the short forward contract will just be the reverse. So, long forward contract is, if you see this strike price is K. So, the long forward contract, if I try to find out the present value of the net ward when the time expires what I do, the actual difference would be F 0 minus K at that point of time that is t. And when I try to find out the present value of what I do that basically, I multiply it with e to the power minus r t two basically bring it as of today. So, the term which we have inside the bracket is F 0 minus K is the price of the actual difference all the forward considering F is the price and K is the delivery price

when the contract expire, and the second term which is e to the power minus T is basically the multiplying factor which brings back that amount of money as of now.

So, this we can say is in some intrinsic value the difference in the forward contract during the long and the short position such that if it is zero it means absolutely perfect balance if this is a positive and negative it will mean the short or a long position would be making a profit or loss. Now if I am considering the long forward contract as I mention if this is buyer, this is the seller, hence for the long forward contract it was F 0 minus K whatever be the multiplying factor outside, but when I basically consider the other way round the sign just reverses. So, what I am trying to calculate is, how the value of the forward contract changes depending on the short and long position hence is the value of the contract is zero then obviously, your long or a short position does not make any profit or a loss which basically equally balanced.

(Refer Slide Time: 15:14)

When I am considering the value of the contract for a dividend paying one, then also the same concept comes. The first term which you have is S 0 minus I minus K e to the power r T. So, S 0 is spot as of today, I is also the value of the investment which is basically will be paid in between, but I am trying to find out what net present value of that as of today and that minus of the third terms is basically the delivery price is basically K. So, whatever the delivery price would happen at time t is equal to capital t is been brought back here at present time. So, it will be basically multiplied by e to the

power of minus r T. So, if the actual value which I am getting for that contract is K e to the power minus r T plus or minus is depending whether I am in long position or the short position.

And the overall net value which I have in room one which is the spot minus the intermittent payment which I am getting. If both of them balance; obviously, it means either the long position or the sort position would be making any profit. If I go for a long forward contract considering the long forward contract have been discussed where it is basically S 0 minus I minus K to the power r T, if you see that for the short position, it will just be the reverse. So, the short position would be the person who is selling the contract to the long position hence the profit and loss which is they make at different point of time would just be the same value quantum wise, but with the sign change.

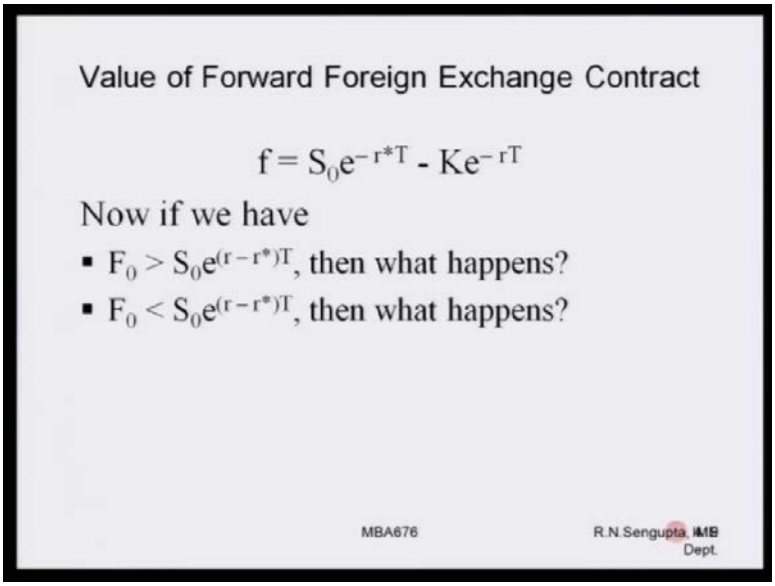
Now finally, if you come to the dividend paying one again we follow the same concept, they are in the dividend you had basically minus q two which is the dividend yield. In the same light, which is you see the value of the forward contract for a long forward contract is S 0 is the e to the power minus q T as of now for the overall spot and the next term is the minus K e to the power minus r T is basically the delivery price of the contracting room two which is being brought back to the present value as of now which is time T equal to zero. So, now, the short contract would be again the same terms, but with the sign change.

(Refer Slide Time: 17:28)

Now, consider the foreign exchange contract. So till now we had considered assets, where they were paying no dividends, where they were paying dividends at equal intervals and continuous rate. Now consider you are dealing in foreign currency consider the countries are India and Japan. If you see in the formula, there are two interest rates one is r and one is r star. So, remember both r and r star are risk free interest rate, but for two different countries so that make a distinction which should be very clear. So, F 0 is the forward price measured in domestic currency. So, if I am standing an India by domestic countries is India, and the foreign currency or a foreign country would be Japan. If I am doing my calculation and trying to understand the problem standing in Japan, my domestic currency would be the end and the foreign currency or the foreign country would be India and Indian rupees. So, make a distinction between one, I am in when I mention domestic currency and foreign currency.

So, is the current spot price measured in the domestic currency again the same concept of domestic and foreign currency of one unit of foreign currency and r star is the foreign risk free interest rate. So, r is basically r is the foreign is the domestic risk free interest rate and r star is the foreign risk free interest rate. So, whenever I trying to calculate the value first find out where your datum is, if you are in the domestic country and you consider that domestic country to be India then r star and the spot and the forward would be basically be calculate accordingly. The moment you switch your position you go to that other country again the same example of Japan then obviously, the foreign currency becomes India and then you can find out the r and r star accordingly.

(Refer Slide Time: 19:20)



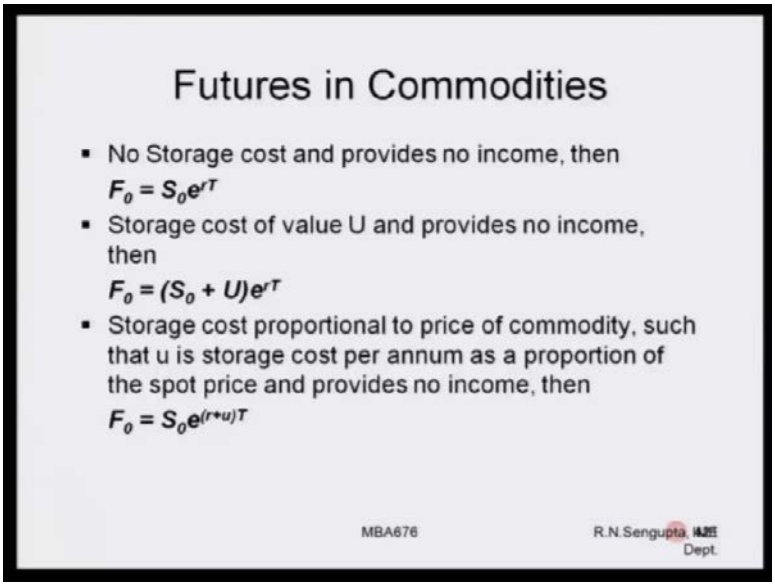
Now, value of the forward contract which is small f depending on the foreign currency which you have domestic and currency, so now, if you see the formula it is S 0 is multiplied by some multiplicative term where r star is there and case we multiplied by some multiplicative term where r is there. So, these are two risk free interest rate into different currency. Now what you should understand is that again room one and room two basically the same thing; now room one is basically in country one, and room two is in country two. So, if you denote country one or room one as the domestic one so immediately you should be aware that country two is the foreign currency.

So, you are interested which you are trying to find out based on r and r star or with respect to that. So, if room one is domestic one, there the interest rate is r; if the room two is the foreign country then the interest rate is r star. If you move back to the second room then in that case r star and r basically just reverse and you do the calculation accordingly. So, in that case rather than expressing Indian rupees to Yen, you will try to express Yen to Indian rupees. So, if you have domestic and foreign currency is UK and India, so in that case you will express r and r star accordingly, and you will try to find out the rate of UK pound to rupees. And when you reverse you position then again r and r star would be reversed, reversed in the sense you definition r and r star will be reversed. So rather than finding out UK pound to Indian rupees, you will try to find out the value of Indian rupees to UK pound to do your calculation.

(Refer Slide Time: 20:59)

So, stock index futures are there. So, here in this case of the stock index futures we will consider the q which is the dividend deal would be the rate of return of that particular stock not the risk free interest rate. So, risk free interest rate as I mention is the treasury bill of ninety days for India and obviously, the q would be with a minus sign because this money coming back to me would be interest rate based on which how that stock prices are increasing or decreasing whatever it is. So obviously, it is decreasing minus of minus q would be significant plus. Now examples are ((Refer Time: 21:33)) average standard course 500, standard course Nikkie 225 stock and so on and so forth. So, you can have all this look at this website, and try to find out how do the stock index features work in the value terms then you can use those to do your calculation accordingly.

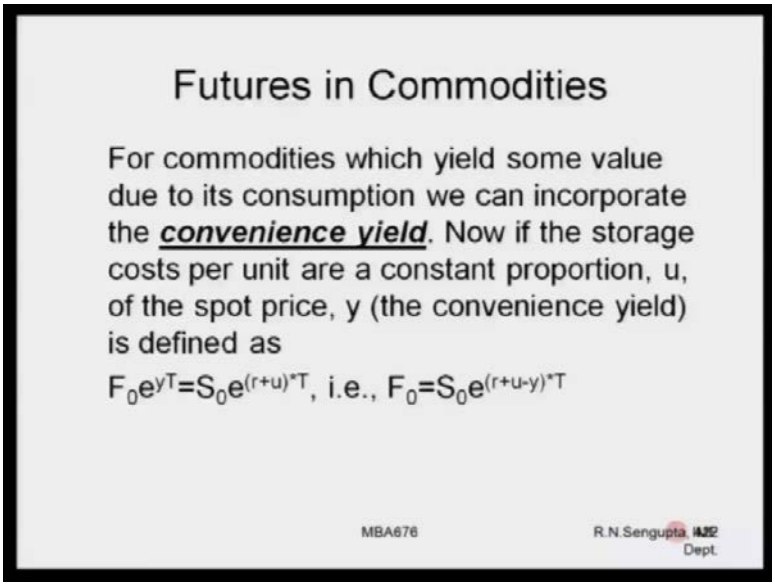
(Refer Slide Time: 22:01)



Now, let us switch our discussion from actually financial assets to commodities, commodities are actually physical objects. So, now if there is no storage cost that and they provide no income now what you will have is that the overall value of the forward would calculate it accordingly. So, F is equal to S into e to the power r T, again r is the risk free interest rate. Now consider this is a storage cost fixed storage cost or is this storage cost per unit time and you are trying to find out the storage cost as of now. So, what is what we mean by u and i 0 or almost the same, but in i 0, remember it was the amount of money which was coming back to me while you would basically the total outflow of money considering this inventory.

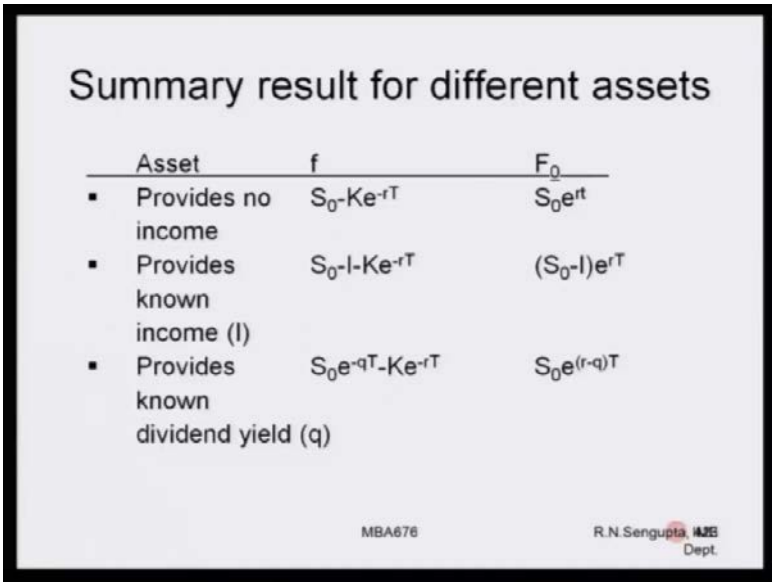
So, in this case you should basically add up is zero plus you not minus u where u would basically with the overall investment and inventory or such cost of caring the commodity as of now which is today then again if you have basically the concept of storage cost per annum continuously compounded then in the similar way as you are the dividend for the stock which was basically pay to me this is the outflow money which is I basically calculate using the concept of u small u which is the storage proposition to the price of the commodity and it will be found out continuously commodity. So, rather than having S 0 into e to the power in the bracket r minus q it will now will basically be S 0 into the eighth bar r plus small u because the overall amount of money it has to be paid by the person who is basically by the particular spot based on the commodity.

(Refer Slide Time: 23:28)



For commodities which yield some value due to consumptions you can find out the convenient yield and if they are been consume that means, commodities are being stored as well as being consumed. So, consumption means that you getting some benefit. So, you will basically have now three things risk interested based on rate of increased of the particular commodity small u is the storage cost per unit continuously commanded based on the spot price, and they would be convenient will also depending on the consumption. So, now as you consuming obviously, in the minus sign come soon. So, the actually you are again with changes to S 0 into eight to the power three terms r is the risk interested small u is basically storage cost based on continuous compound concept is y with the minus would basically means the convenience in the amount of consumption which you doing per unit rate converted to continuous compounding case.

(Refer Slide Time: 24:22)



So, now I basically made a table where if you see the first column, it is the asset, second column is the value of the forward, and third column is basically at the prices based on the which you trying to find out the. So, with this, we will close this session, and in the next class continuous with the summary and the consecutively different type of discussion how it is the futures.

Thank you.