Quantitative Finance

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Module – 04

Lecture – 20

So, welcome back to the session, we are continuing the concept of order and futures. And till the last class, we discussed how basis risk was important depending on whether there is a mismatch in the actual product which you are trying to buy and sell along with the futures which is not an exact replica of that spot in room one and two as I mentioned. Also they maybe mismatch in the timing, I need the spot four months down the line continue with the same example, but my actual forwarded future are only for duration of three months. Number three can be if you rollover, so as you rollover the overall basis risk we see accumulates. So, if you need the actual product and on twelve months on the line and the forward and futures of for three months; obviously, they would be such four different basis risk which will cumulatively add up to give a much more high level of volatility.

I am using the word volatility, risk, loss, whatever it is interchangeably; obviously, which we see later on that if you try to make a settle difference, there is the difference between them. So, now, we will consider that forward prices how they are calculated and how they are calculated for the cases when they are no coupons are being made no dependence are being made, if dependence are being made or if dependence are being made continuously compounded how do you basically find it out?

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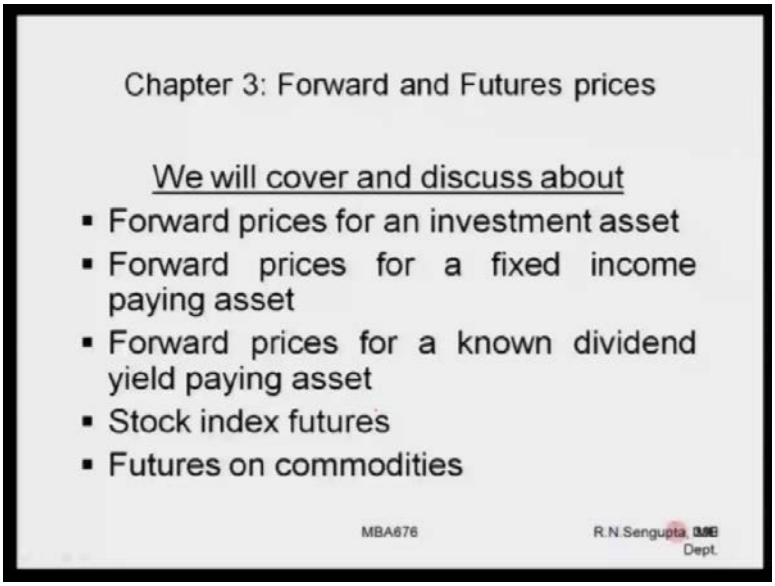

Now, the word forward and futures which I am saying I am using interchangeably. Conceptually they are same even though as you just saw in the last class, there are settle differences of over the counter, not over the counter being traded such that prices can be dictated, but demand and supplying being not traded such that the prices are predetermined you can close it before and you need not close it and the at beforehand we have to be restrict to the time to duration. So, all this things are different and they have a major impact, but when we try to find out the for prices of the futures, we will consider first the prices the forwards, recent being they are hell for a fixed duration of time based on that when you find out the for prices of the forwards, you can easily find the replica of the futures prices based on which you can understand how the futures prices would also fluctuate given the same time of forwards are there in the market.

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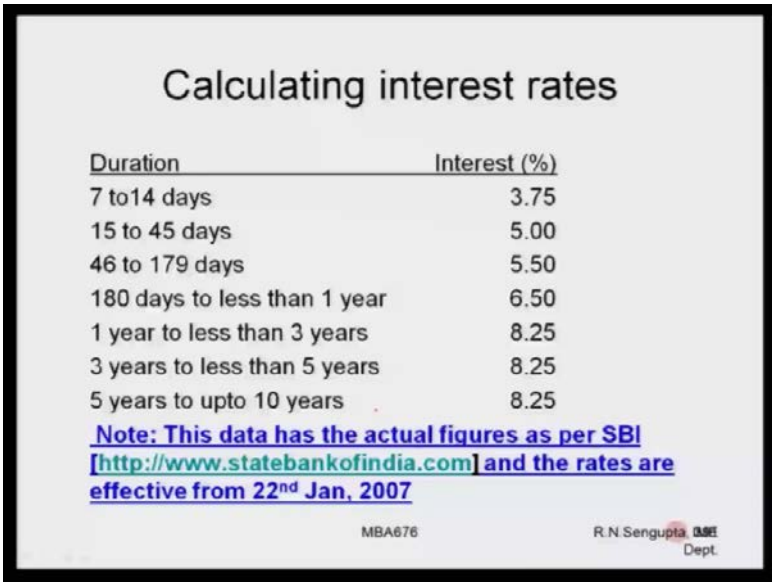

Now, we will consider the interest rates. So, consider this data which you have taken is from the State Bank of India in 22nd January, 2007. So, these are the seven to fourteen days interest is given as 3.75 percentage. And again not mentioning anything related to per annum or continuous compound, we will come to that later on. So, if you remember for the interest rate, I mention time and again the concept per annum and whether is continuous compounded or compounded based on certain time frame or number of times is important, which will see in few moments. Then you have the 15 days to 45 days interest rate in given as five; 46 to 179 days given a 5.5 and so on and so forth.

So, if you see that the interest rate is in a way of proxy of the overall risk which is happening. So, longer the time duration more the risk is obviously people will demand more interest rate, because I want to put that money for a along a duration this money is being lock and being utilize by somebody else if I deposit in the bank. So obviously, the interest rate should reflect such that my overall payback which is happening when I was close my position or try to withdraw my money should be such that the interest rate should reflect the overall risk which was there for that time duration. And if n for the interest rate details, you can go to the Reserve Bank side or the State Bank of India or any other bank and try to find out and considering the duration how the interest rate change.

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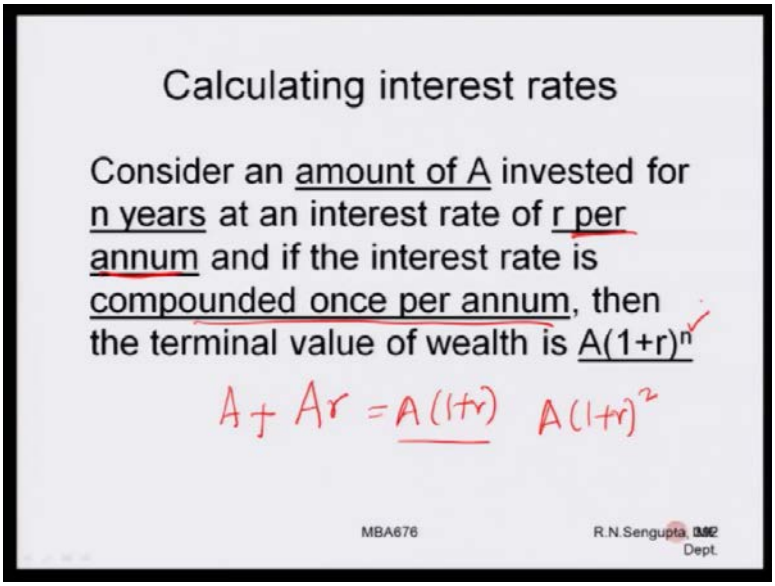

Now, consider now for the first time the case of how you find out the interest rate based on that calculation we can proceed. So consider an amount A principle amount is invested for n number of years at the interest rate of r per annum, and have been interest rate is being compounded only once per annum, so this is the point which have been talking about this is per annum percentage and this compounded per annum. So, then the amount of money which you would have is A you had initially, so the interest rate would now be A into r. So, actual amount which we will get would A is already there plus the interest rate. So, you will basically have A one plus r. Now you consider that if it been compounded two times. So, what you will do is that this is now the actual amount of which is being deposited by using at the beginning of the second year so obviously, they would be interest rate which is A plus 1 plus r whole square.

So, as you continue the over because it is two times, so as you continue doing that you can find out that number of years which is now n remember that symbol should be important for you to understand is the fact do compounding on an annual basis per annum the total amount of money which was A would now increase to A into 1 plus r to the power one if it is for one year; A into 1 plus r to the power two if it is two years and so on and so forth.

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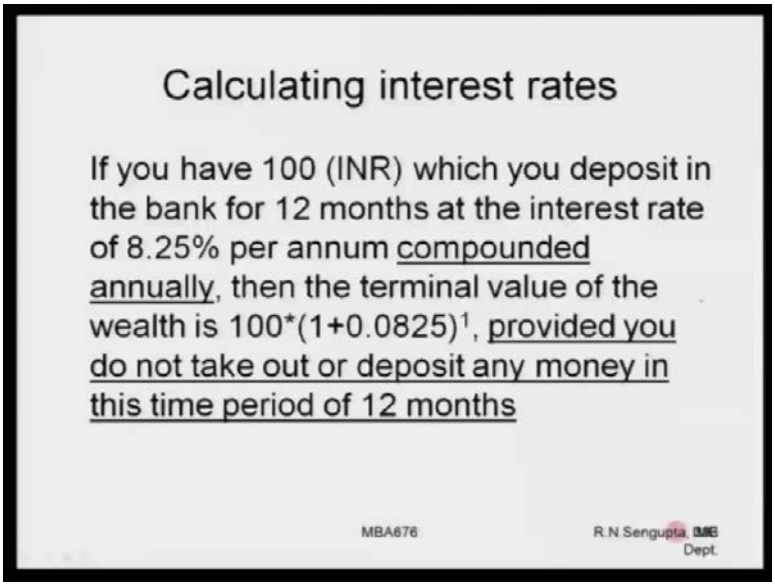

So, if you have 100 rupees which you deposit in the bank and the 12 month period at a interest rate 8.25 per annum compounded annually, then the terminal value would be given by 100 which is A into 1 plus r which is 0.0825 in percentage sums converted into ratio to the power one provided you do not take out the deposit any money or deposit any money during the time of 12 months. So, once you put in, you stay put and then one the time expires you take out that money.

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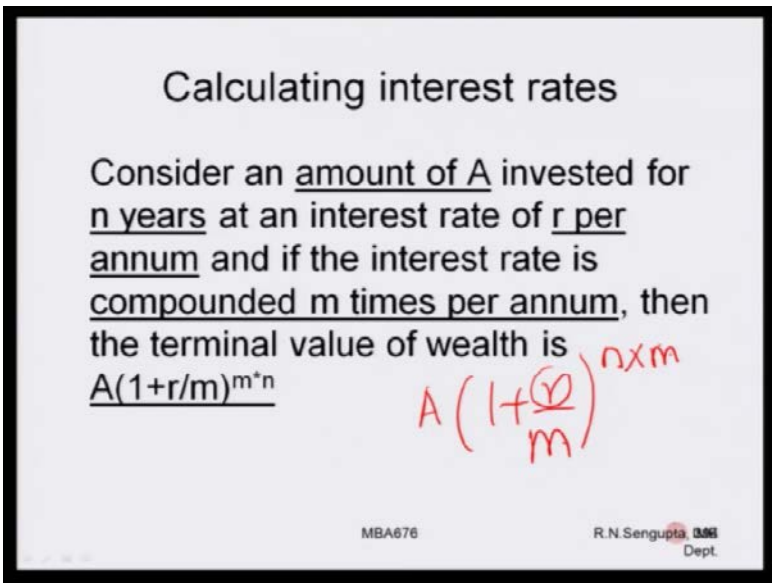

Now consider the next case, same amount of money A invested for n years at an interest rate again for r per annum, but now you are basically compounding it m number of times per years. So what do you mean by m number of times per year, which means that you

will be circulating that money m number of times in that duration of n years or per annum. So, you deposit take it out of after two months, so it would be done for the first two month period, then again you will deposit of the beginning of the third month the total amount of money which you have got back after two months and then again deposit and get it money after say for example four months. So that means, the cycle has continues twice, 0 to 2, 2 to 4. When you take out the money which you get after end of four month, you deposit at the beginning and fifth month I take it out of the end of six month and you continue doing this.

So, if you continue doing such thing, so your compounding i a one year six number of times, because first is for two month, second is for two to three, three and four, next is for five and six, next is for seven and eight and so on and so forth. So, if you do the compounding m number of times of years then actual formula now becomes A into 1 plus r divided by m. So, this rate is interest rate per annum remember multiplied by you already had n, now it would be multiplied by m. So, this is the case where you try to find out the total amount of money which you will get or deposit depending on the per annum case of r, and how many number of continuous compounding at this. So, if you remember the first problem we did, if continuous compound is being done for once in a one year n become and so on, m becomes and so on we get back to the original formula.

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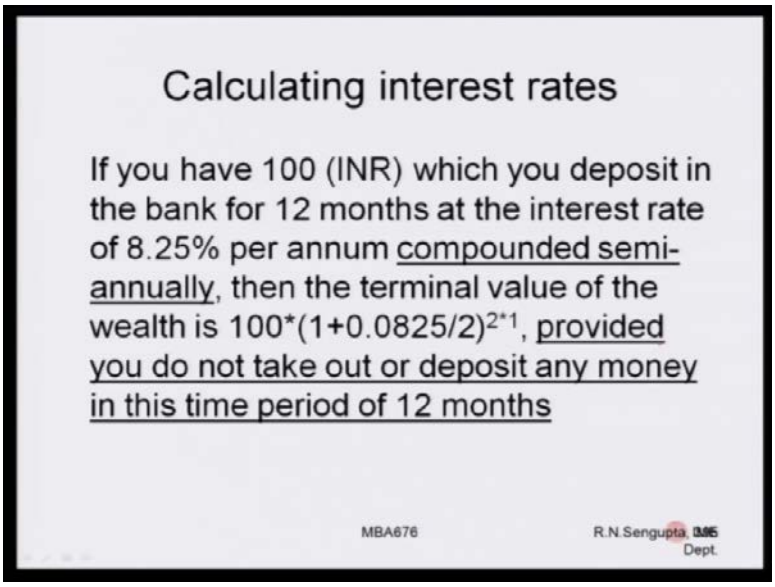

So, now if you have 100 rupees deposit for twelve months in interest rate 8.25 per annum compounded at semi-annually that is twice. So obviously, this formula would become 0.0825 divided by 2 and to the power one is the time duration and two is the number of

time of compounding you are doing. Provided you do not take out any deposit or take and deposit money and take off money any money during the time period of 12 months.

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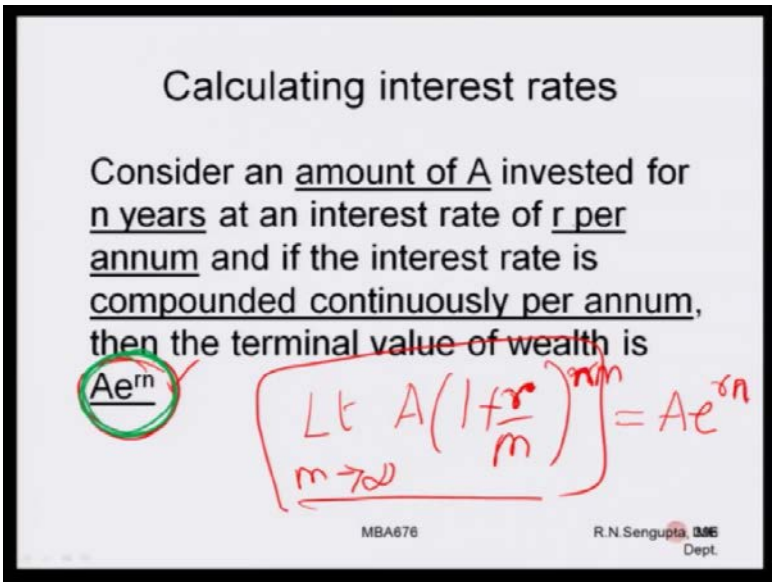

Now consider that how do you want to find out the continuous compounding case that is being compounded exactly the same thing be. So, actual formula was this, now consider continuous compounding mean you are doing continuously. If you consider that the actual value which we will have there is n here this is n; the actual formula which we have is as m terms to infinity. So, this is the interest rate which you have per annum and this n is the number of years, this you will see formula which you get this is the simple concept of the limit; check any mathematics books you will find out this formula to be true.

So, consider an amount of A invested for n years at a interest rate of r per annum and with interest rate is continuous compounded continuously per annum then the terminal value would be given by this. So, for all your calculation, remember we have to use the continuous compounding rate. So, let me highlight it such that people will be aware of that. So, the value of the continuous compound interest rate which will utilizes will be, so this formula, you will be utilize true are the calculation and if in case any rates are given or compound of m number of time or m 1 number of times would be calculated accordingly.

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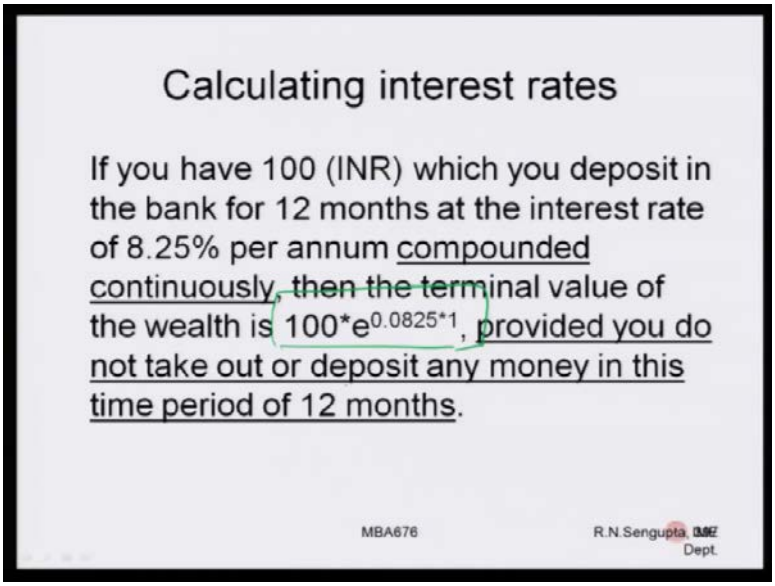

If you have 100 rupees deposited for again for 12 months interest rate in 0.0825 continuous compounded then the amount of money which you have is this. Considering the fact the provided you do not take out, you not deposit any money within this time period of one year.

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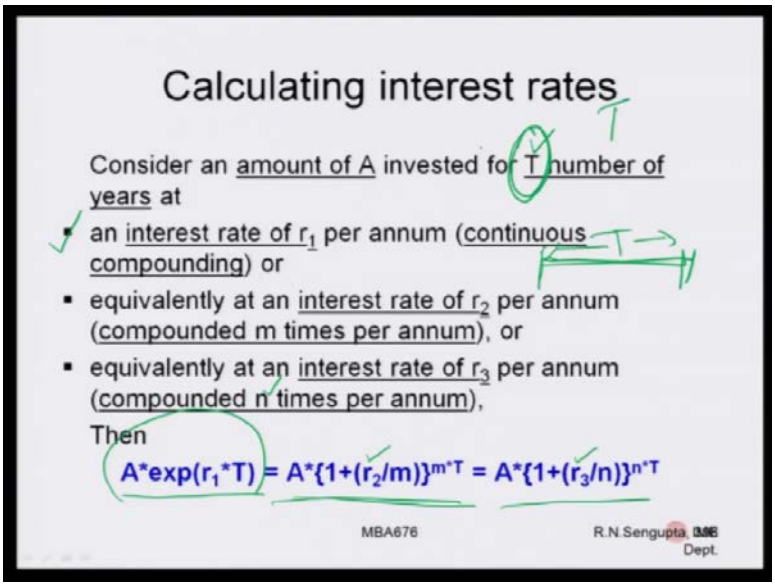

Now, you want to convert any compounding rate to the continuous case or the continuous case to the compounding rate, the concept is very simple. Consider an amount A is invested for T number of years at r one per in annum interest rate continuous compounded. Equivalently you could also invest that r two interest rate per annum, but m number of times continuous compounded and the third equivalents is in investing in r

three interest rate per annum being compounded n number of time. So, do not confuse this n and the n which we did. So, now, will basically replace that n initially which you consider with time period t. So, now time would always be consider by t and the number of compounding would be denoted by m 1, m 2, m 3 or n 1, n 2, n 3 whatever it is.

Now, if you consider the first instead of in continuous compounding the formula would be very simple A - principle amount into e to the power r 1 T. So, this means the total time duration which you have which is T, so in that time period the overall amount is this. Now if you are doing it if I added different interest rate r 2 per annum, but compounded m number of times, so this is the formula exactly the same, so as m tends to infinity, you can get back this. And in third case, you consider that it is being done n number of times. So, in this case that again the formula remains the same where m gets replace by n. So, in the case again in the n tends to infinity you will get this formula. So, what you are actually interested to know is that if the compounding numbers are given and if the continuous interest rate r 1 is given then you want to find out what is r two or what is r three depending on the number of compounding is very specifically mentioned and also remember that the time duration which have just drawn which is T is given in all these three cases. If they are different then also the calculation can be done without any much problem.

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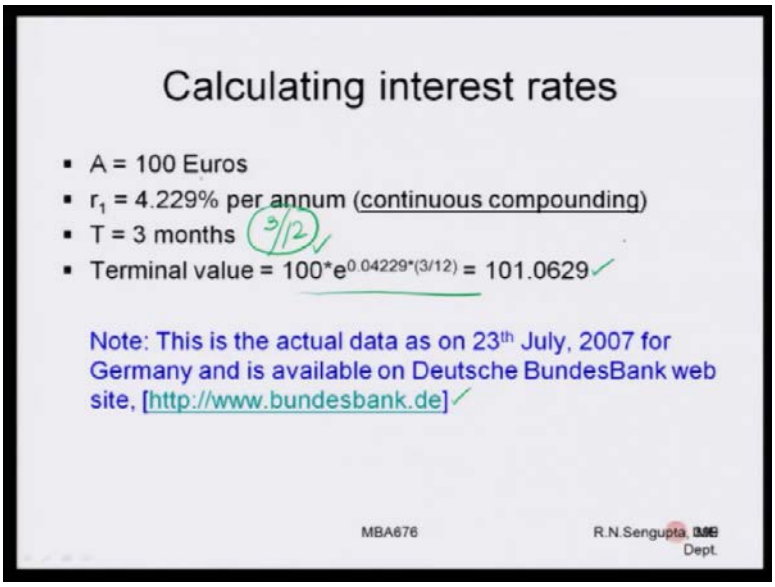

So, this is the actual data on taken 23rd July, 2007 from Germany and is available on Deutsche Bundes Bank website, the website is given. And you consider A to the hundred year which is r 1 which is continuous compounding interest rate is given by 4.229

percentage per annum time duration is 3, so obviously, you the time duration is 3. As I mentioned immediately convert into a earlier scale for this three by twelve the terminal value would be given. The principle amount A e to the power 0.04229 which is the interest rate per annum into 3 by 12, this is the time duration which is given by 101.0629.

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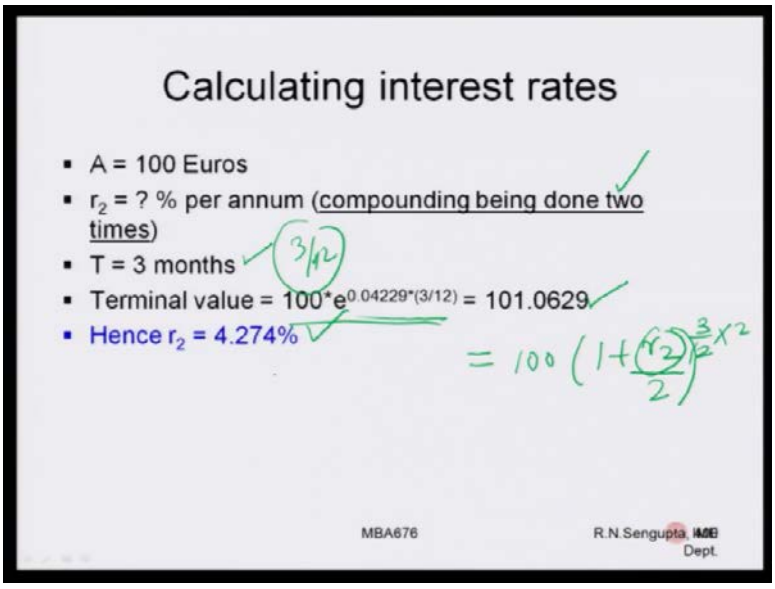

Now we consider add two as a interest rate for the same problem, but does been compounded being done two times here that is semi-annual compounded consider again time duration is three which is 3 by 12. And if you equate this problem what you have is this value which you have initially you find out is 101.0629. So, would basically equated to this equation which would be have is hundred one does this r 2 value which want to find out how many number of times twice and what is the duration is 3 by 12 multiplied by 2 find out that two short to be 4.274 percentage per annum semi-annual compounded.

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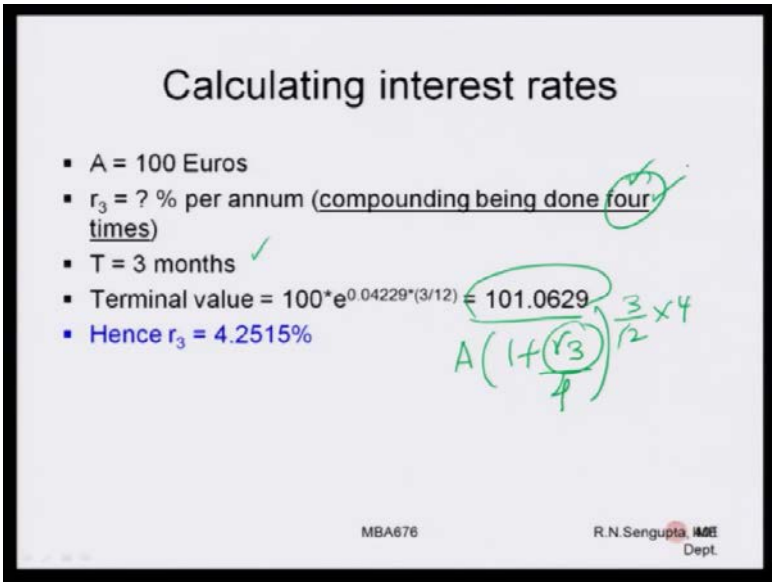

When considering you are doing it four number of times with the same thing a which is hundred into one plus r three which we want to find out how many times compounding four multiplied by time duration we may say same 3 by 12 into 4 is number of compounding equal it to 101.0629 which you very find out which is 4.2515 percentage per annum compounding number of times here.

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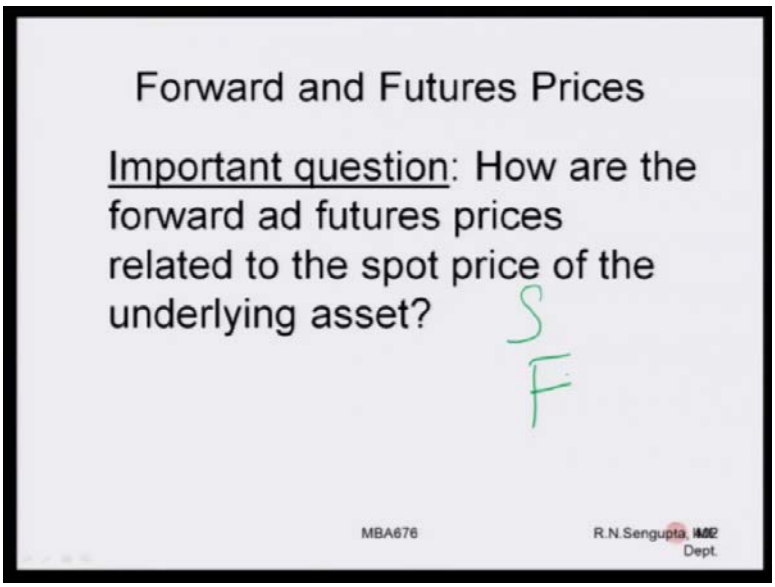

Now, important question is this how are the forward in the futures was it related this the spot price this is the main question because we have already going to sign up contract f are a forward in a future and our main concept is to find out that what is the actual relationship between this spot and the futures which is capital F. So, we did mention

about that will try to find it out and this is the stage where we will really try to calculate how the values are illustrate and how the calculate.

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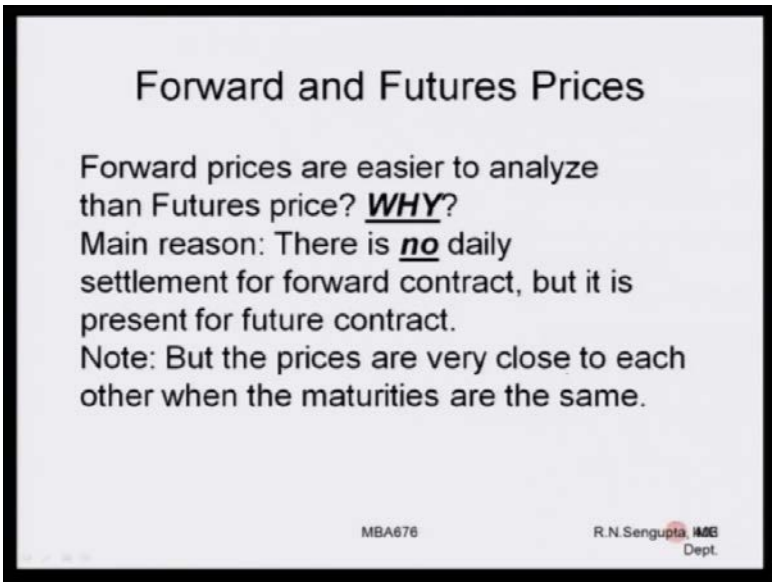

So, forward prices are easy to analyze than futures price the question is why main reason there is no daily settlement for forward contract, but is present in the futures contract so; obviously, forward prices are easy to find out because one is given and find out given the spot you can find out the forward price note, but their prices are very close to each other, hence the maturity time arrives. And if the about the same then the forward and futures are same ends to find out the futures price we use the help of the forward prices and will see that how it is done.

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We will analyze forward prices and that too mainly for investment assets and not for the consumption assets this is one of the assumptions which I want to basically highlight.

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Now consider for our problem the forward prices to calculation this forwards are given by symbol f the spots are given by symbol s the risk free interest rate is r with suffix f or not f it is does not matter, but it should be mention that is risk free interest rate continuously compounding the compounding interest rate are given something else we convert into continuous compounding sense considering the formula we have just is discuss t is the time period and number of years if it is months we converted into number of years and the suffix are the prices at which they are found out if it is zero means this is the price as of today.

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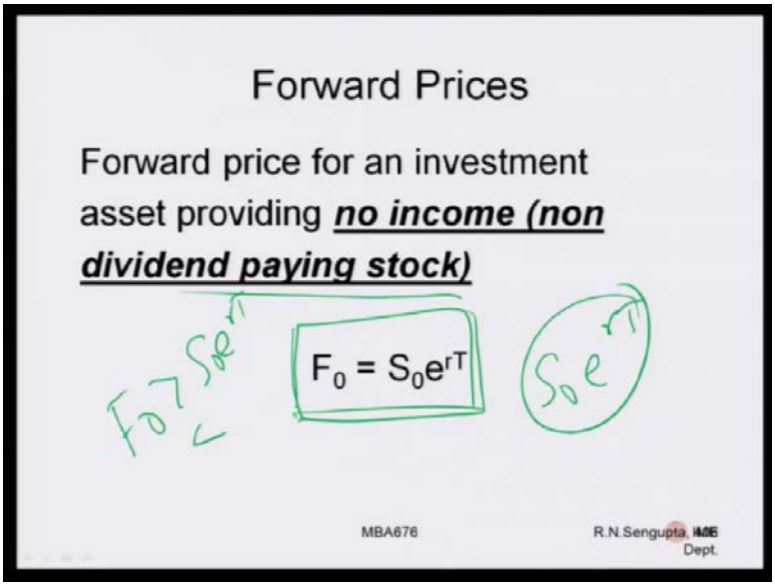

Now the forward price for the investment as providing no income, so it is given by this formula which is F 0 is equal to S 0 e to the power r T, why it is because say for example, I want to buy a forward. And another case, I do not want to buy forward I have that money, and at the end of the contract the expires, the amount of money which you have which I will deposit in the bank should exactly match the actual forward price because if it is not matching then some set of person will go to the bank if the interest rate of the bank is higher and make extra profit or some short of person will go and to the forward market and make extra money the risk free interest rate slow. So, if in case in order to ((Refer Time: 16:42)) that does the market is exactly symmetry and is equilibrium we will consider two cases; either I go for a forward position with the certain price as of today which is S 0 and the forward price increases at what rate I will come to that later. Another person does the same thing, but it does not go to the forward pa market he basically takes that amount of money which as you and goes in the bank and deposit.

So, this is kept as a fix deposit and amount of money which the second person will get when the fix deposit expires at the exact time duration when the future of the forward expire is equal to S 0 into e to the power r T, which is the interest rate. So, now, if this no profit and if known no loss the total amount of money which you will get from the bank of the person two will get from the bank exactly match the forward contract. In case, if is not in case F zero 1 was greater than S 0 e to the power r T are less than equal to S 0 into e to the bar t then somebody would make a loss and somebody would make a profit. Hence the concept of hedging in order to minimize the risk would not come into the

picture so obviously it mean that this no dividend paying stock value of the forwards would be F 0 is equal to S 0 into the e to the power r T.

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Consider this, we have taken the actual data on 13th of July, 2007, from Bank of England these are what they are S 0 is given as the UK pound, value is 327.518 then in continuous compounding interest rate is given 5.75 per annum continuous compounding; time duration is 6 which is 6 by 12 in the USD scale, and then you find out the forward decrease S 0 e to the bar r T, r T is given which 0.0575, where T is basically 6 by 12.

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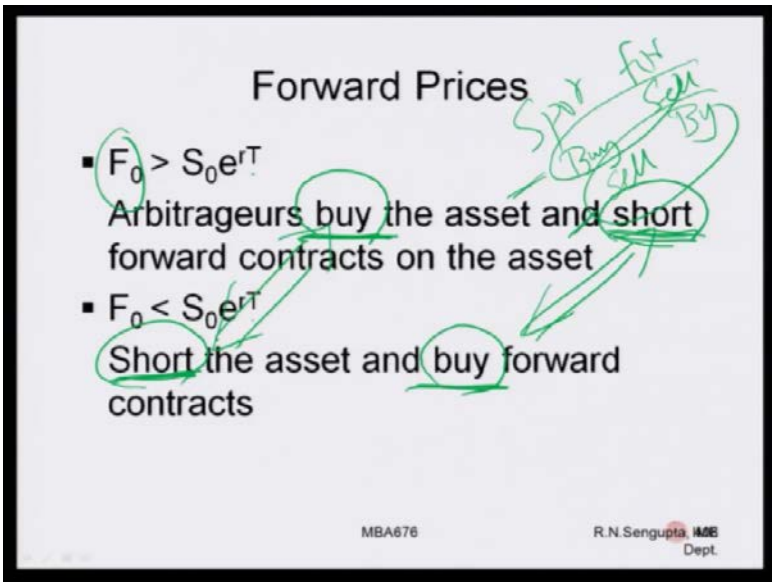

So, now, is f zero is greater than S 0 is the bar r t arbitrageurs buy the asset and short the

forward contract and the asset in the make in profit you will understand why I said that if this price was more than obviously, will buy the asset and short the forward later on such that on this bought for will basically get difference of the profit based on the price is difference in the asset and the short forward. In case, if just the reverse position rather than by the asset which is long will now will short asset and rather than go for the short for the forward contract we go for a long buy forward. So, what you are trying to do they there is a spot and this is the forward.

So, one thing is will buy and another case you will sell and in the next instant if the situation is different you will sell the spot and the buy the forward. So, depending on the price movement of whether f zero is greater than equal to this always then this equal to c this you take either this position in order to make external profit. So, again I am repeating in one case you will buy that asset and short the particular forward in the next instant when the signs are divers you would basically short the particular asset and go long for that particular forward contract. So, these are different.

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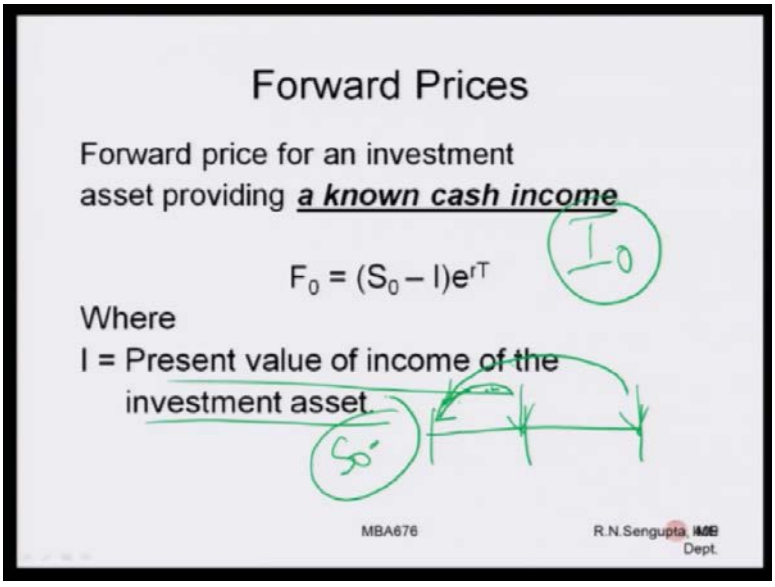

A forward price for an investment asset providing known cash income that means, in that duration of time the total amount of money which you will basically by is you will get is different. So, now there are incomes, which is happening in between. So, this again if you consider should have been S 0 e to the power r T, but now S 0 has been reduce what is the amount of deduction is the total present value of all the income which I am getting in this time duration. So, this is S 0 would be deduce by the amount of I or I 0whatever you denote I 0 means the present value of those intermittent payments which is

happening at time T is equal to zero, it is the present value of all the income of all the investment assets. So, now consider you have a time division is this and there are two independent payments, so point one, point two.

So, what you will do is that I would try to find out the present value of this present value of this add them up and that value would basically become I 0. So, if I have S 0 which is spot price and also the intimating actual values of the investment as of today. So, I will basically minus S 0 and from S 0 I will take out the these two values shows in the actual value based on which you will try to forward would take into consideration any payments which will come in the future because I have converted those payments into the present value which is I naught or I.

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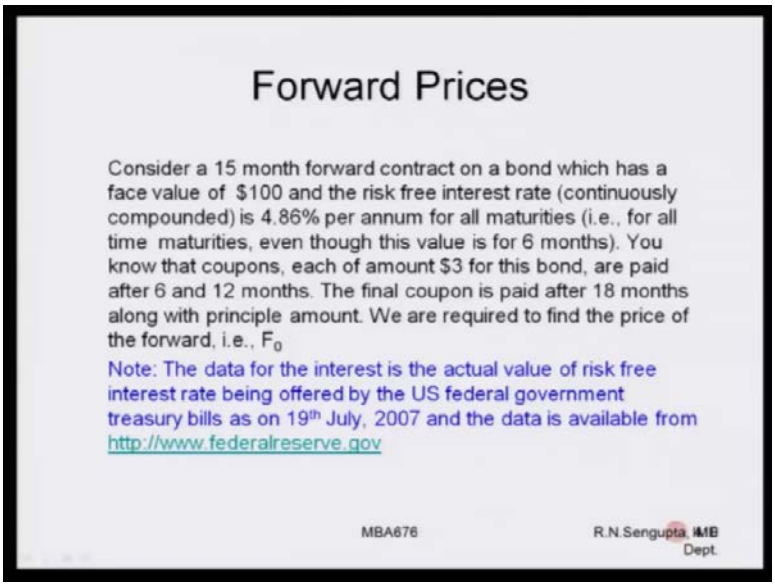

Consider at ten month. So, note the data for the interest rate in the actual risk free interest rate has been offered by the being offered by the US federal treasury bills as on nineteenth July at 2007. These are actual data have been taken from the federal reserve dot gov site considering a fifteen month forward contract on a bond which has a face value of thousand and the risk free interest rate is given by four point six per annum for all maturities. And for that is for all time maturities even though this value is for six months you know that the coupons; that means, the amount of put small amount tokens which you are going to get back, each amount you just three for this bond are paid after six months, after twelve months, the final coupon payment is meant after 18 months which is twelve plus six one and half years from now along with the principle amount of you are requested to find out the price of the forward.

So, if you look at it carefully three dollars would be paid after six months, three dollars would be paid after twelve months, and after eighteen months you will be paid, the three dollars plus the principle amount which you have already paid. So, basically there are three different payments in the first instant is three, second instant is three, third instant is three plus the principle amount which would be getting back based on the interest rate whichever it is there.

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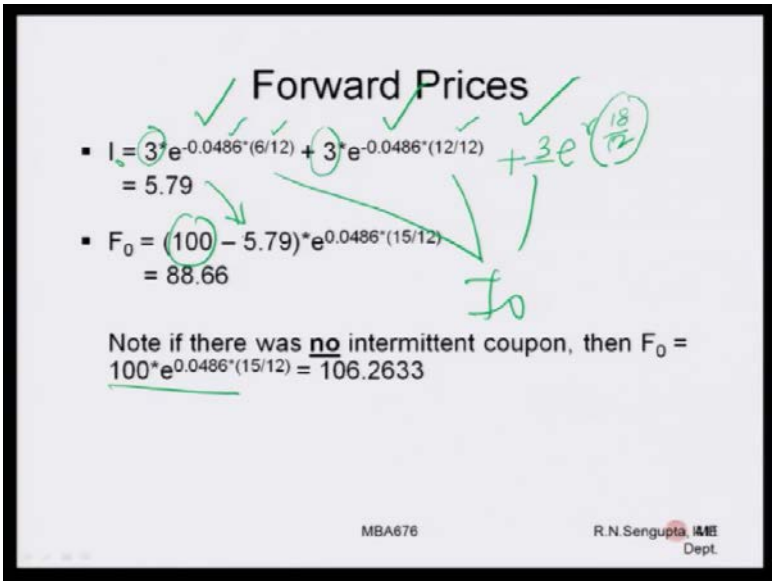

So, in the first instant, if you get, if there was no intimate, so in the case what we are considering in the first instant we will consider that this the amount which you getting back after six month which six by twelve this interest rate second again you getting three dollars after twelve, and technically which we may consider also, but in this problem I am not considering is the actual amount of three which you are getting e to the power r whatever it is after six months which this amount of I 0 which you found out will be subtracted from S 0 and in order to calculate the values. So, note if there is no intermittent payment, the value of the forward would be this that means, this hundred multiplied by e to the power r T, but now as independent payment are being made which is this, this, this, you covert everything to I 0 subtract that from F and I try to find it out.

So, we will continue the discussion the next class with this where will stop here and we will discuss how the forward and futures price would be calculated. So, initially we consider no coupons being paid then coupons are being paid then we will slowly expand that where dividends are being paid for commodities such that will get a better understanding of the forward and futures market prices are calculated based on the interest rate.

Thank you.