Quantitative Finance

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Module – 04

Lecture – 19

So, welcome back, let us consider continue our discussion about quantity finance and the area of option which we are studying. So, then the last class we ended at the stage where we want to discuss what is the basis risk? Basis risk in very general sense is the deference in the sport in the features, considering that the spot on the futures on a based in particular asset. And also the fact that the spot price which is their exactly mimics the demand and supply of the actual product buying and selling. And the futures which you trying to buy or sell futures forward whichever it is, consider I am basically trying to use interchange the word ever though the concept wise there may be same the actual reality different. Very using the forward and future is such a way the forward and future price which is capital let exactly mimic the sport in all sense. That means, is an exactly applicable was spot which is their in room one and is exact replica is there in room 2 where the forward and futures are been traded.

So, now if you can recollect a forward as the over the counter market and they are expectation date is finalize between the two parties, but in the futures you basically in traded in exchange. So, whenever you buy and sell that time duration and when you want to buy and sell is less definitely then the expression date what is not exactly equal to that. So, and if you can also recollect we have the use the simple S or the simple F whatever it is you can particular suffix for the suffix denote the time. So, with that notion with that concept, if you see the slide you says that suppose that F suffix 1 in the initial future price, when you go in to the market to the futures that is the time t equal to capital T 1.

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And F suffix 2 is the final futures price when you basically go out in the market at time t is equal to t 2, where t time t is equal to capital T 2 is greater than t 1, and S 2 is the final asset price of the price of the asset in room 1 at time t is equal to 2. So, now you basically of 2 rooms in room 1 technically you have S 1, which is not noted here and S 2 is the final asset price and in room 2, you have F 1 and F 2 which is the initial future price final future price depending on the time, you hedge the futures purchase of an asset by entering in to long futures contract. So, remember is a long hedge. So, if it is long; obviously, you are trying to buy. So, long sort would be planning to sell. So, if you are going long for that particular hedge, then you cost of the asset is like this you open the position and time t equal to 1 close at time t equal to 2. So, your cost of asset in room 1 is the asset price, which is S 2 minus would be the difference in the price which is happening in room number 2 at time deviation 1 and 2, which is basically the difference in the futures price F 1 and F 2.

So, if you expand this equation you will have F 1 which is the futures price, and time t is equal to 1 and the total quantum of basis would basically would the difference in the spot in the futures at any particular time. So, if you ask that what is the spot on the futures difference in the time t is equal to 2, 3, it will be basically S 3 and F 3 in the different of now this is only for 1 time period; that means, for the duration if you consider for 3 months, so obviously, this a duration for next 3 month, which is from the end of the 3 month to the end of the 6 month or for the beginning of the 4 month to end of 6 month. Again you will basically I have the corresponding basis risk, in that case it will be not F

one, but it will be suffix base on the time period when you trying to understand. So, meaning for the next time period of another 3 months, which is a beginning of 7 month to the end of 9 month. Again you have a basic risk where again the time unit would change accordingly, where you are trying to consider the day time is and what is the time duration of a total considering now remember, this is the long hedge. So obviously, the concept was sort hedge would be in similar line, but in the change in the different is the sign depending on the where you are trying to buy or sell.

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So, the long hedge for this example you considering F 1 is 32.5 at time t is equal to t 1, F 2 is equal to 32.6 as the time period, you will be t 2 and S 2 this part is 35.0 at time period is equal to 2, now you hedge the futures position of an asset by entering into long futures contract. So, the cost for the asset in could buy, you is 35 year paid minus of the difference of the future to the forward price, which is there is you calculate accordingly you will basically have 32.9 which can be calculated depending on the price the consistency.

So, we are actual price is 35, which is given year, and if you consider the basis this as for the problem this basis with this would be given by the value of F 2 and S 2 difference which is 0.4. So, higher the value is or the lower the value is basically give you some concept of what is the overall basis depending on the price difference between the knot of the futures at any particular time, which is the total cost in ((Refer Time: 06:07)) by the operator this would total cost means 32.5 plus 0.4 by the operator in order to operator 2 market where 1 market this part which is room 1 while are the market is futures, and

the other hand if you have only operator only in room number 1 with this part then you actual cost of trying to buy and sell would have been only 35.0.

So, now, the prices of buying and selling is now 32.5 plus 0.4; obviously, will be template say that the S, we did the food thing in trying to going to the delivery market that we should ((Refer Time: 06:48)), that is the prices are moving opposite direction in you would have maintain loss, but you are loss, in this case that you main concern is that any price fluctuation in opposite direction to what you thing would happen should be considering in such a way that you will different in go for a futures are forwards as a option in order to medicate your over all lose, because you remember. We are already saying for this course for the later part of the delivery is that you are main motivation is to reduce your risk, reduce your ((Refer Time: 07:17)) reduce of variance whatever is the best box, you have the loss.

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Now you consider the sort hedge, now you selling. So, hence be very, very well aware that the word short, suppose again the same fundamental principle F 1 and F 2 are the initial future price, and the final price and S 2 is the final asset price. So, you hedge on the future sells on the asset in that you going to sell by entering in to a short future position. And again if you calculate you will find out the difference the basis risk is coming due to the fact this different between this part on the future at time is equal to 2 or 2, and then you basically add or subtract F 1. Now remember why I am use of the odd of add or subtract F 1 is that depending on where you bottom. So, in a particular asset

and you are trying to buy and sell in the room 1 based on that you have gone for a contract in room 2.

So, only when you try to solve the problem for the sort, and the long we very careful what are the different type of variables are using, but also be very careful what are the actual units and the sign. So, if you are selling; obviously, input of money would come into an account here is positive, if you are if you are buying, so obviously amount of money will going out from you a pocket hence with the negative science.

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Gain for the short hedge you have 32.5 as at time t is equal to 1 is the futures price of the forward price a time 1, F 2 is 29.8 at time 2, S 2 is 35, and time 2 is equal to… Now you hedge the futures sending of asset by entering into a short futures contract. So, with the price realize by you is given by this 35, which is S 2 and the different between F 1 and F 2 hence if you find out the total value it basically comes out to be F 1 and the different of the spot of the and the and the futures at times t equal to 2, which basically gives you the basis assets. So, once you find out the total price, because you are operating in 2 different market 1 in room 1 which is the spot room 2 which is the derivative and the other hand. If you have only operated only in the spot market which is room not room, then the price realize by you from selling the asset would have been 35 as given by S 2 value. Hence you can understand, again I am repeating price fluctuation will happen, but what you a more concern is negative price fluctuation depending on what you thing we are do not go to you are going to buy or sell, such that you want to mitigate that overall wish by going to the derivative market.

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Now, let us extend this concept little bit more, where again the same thing is there futures prices are there or F 1 and F 2 and the final asset prices there, but there are 2 concept 1 is S 2, and 1 is S 2 star. Now let us consider very simply this concept, there is 1 particular product in room 1, and in exact replica of that in the futures of forward market is not there. So, what you will do you will search in room 1 a type of product which you can buy, and sell which as a price moment, which is intent in the primary product which you going to buy. And sell consider this is very simply goal, you are going to buy and sell at twenty 2 carats, but consider any futures and forwards for twenty 2 carats is not there in room 2.

So, what you will do you will search for a 24 carat gold part price, which is there in a room 1 and check that if 24 carat for as an exact replica room 2, then you will try to utilize, these 2 things sport for 22 and the forward and future for 22. So obviously, they may be a mismatch in the part hence it is been reflected as S 2 and S 2 star. So, now the cost of the asset would be of 2 friends; 1 would be this part which is S 2 which is you are buying and selling, and another would be the difference in this part in the futures for both this products in room 1 and room 2 technically you are… Now trying to find out the price different between the parts, and the price different between the forward considering this is the mismatch in this part in the forward in the future considering, there is no exact replica of that particular spot and the forward, and the future as mention in the example of 22 carat and 24 carat. So, what we are meaning is 22 carat is in room 1 in room 2, there is no such forward.

So, if they were it what I have been match. So, what you do is that you check 24 carat and it as some forward and futures for 24 carat. So, now, this moment is there exact this ((Refer Time: 12:16)) would have been exact if they was something here. So, as it is not there, you will try to basically consider this product is futures, and forward price with come to the picture plus this spot prices of these 2 come. Hence we will consider, there are 2 different parts which are basically being denoted by S 2 and S 2 star should be very careful, when you are try to solve the problem where that the product, which is the there in the market spot and futures are exactly replica in each other, if not you will try to replace. Then actual product which want to buy and sell with this counterpart says that it as a part of futures, hence that price concept based on S 2, S 2 star or so on and so far with common to the picture along with the ((Refer Time: 12:59)) futures and forward and so in room number 2.

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Consider again a long hedge. So, whenever you consider the problem, you will be considering both the shorter, and the long consider the long hedge that price of the F 1 is given as 42.8, F 2 is given by 43.4. Now we are basically 2 parts as star and without the star the value a star is given by 42.7 without the star is 44. So, cost of the asset considering we are in the long position would now basically be considering the difference in this part, and also consider the futures and the forward. Hence the price concern to be 33.4, on the other hand if you operate only in the spot market. So obviously, we are operating only in the spot market. So, 22 carat which you have 24 carat

which you have clear. So, there is a forward year. So, if you not operating this are not operating this also.

So, you are only concern with the 22 carat this process given as 42 concern. So, in the first case on the second case 1 and case 2, you saw in the second case only operated on the spot market, and only consider in the product in case 1. We consider the product as it is along with that we considering the derivative for which another product is also there in this part, but there is some certain different between the actual product, which you need and the product based on a which you trying to buy and sell particular derivative. So, you are basically going in this diagonal fashion, but as you doing that the effect to the prices of the 24 carat gold would also have an effect in case of the derive.

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Now when you trying to do that you are very much concern about the ratio of the size of the position being taking on the future, where you want to buy or sell. So, that does not matter is basically size of the future contract, which would basically given you an expose a, hence such that the overall espouser based on which you are trying to do that ((Refer Time: 15:02)). Remember that when you read the book try to understand your optimum hedge ratio of the hedge ratio of trying to find out is done. In such a way that your overall risk is minimize, which means that you over all fluctuation in the prices minimized. Now consider this if delta S and delta S standard deviation on the rate of change of there is spot and the rate change of futures.

So, what you are concern is this and not this. So, you are concern about this you are concern about this not this. So, the standard deviation which are given sigma suffix S and sigma suffix F or technically this. So, be careful about that and correlation coefficient row is that, which is existing between this these rate of change again rho would have del S del f. So, I am interested more in the rate of change of this part on the rate of the change of the futures and what is the correlation coefficient rather than the actual change of part of this futures. So, what you are considering is the correlation coefficient existing during that time which is equal to the life of the hedge and it basically is the correlation position between delta S and delta F, where delta S is the change in the parts and delta F change in the forwards of F then for a sort forward the change in the value of the h position is give by this.

So, the first quantum is delta is this part rate of change. So, this is the total amount which is happening, and the second amount is that if you are basically trying to go for the sort futures, then obviously you trade basically go for a long and a sort position both these 2 room separately. So, initially what you will consider if you have looking any parts then obviously, there is no trading, but the moment you bring this part for the time first, and without going to the futures market, and you only trading would be 1 the spot market whether you are buying and selling. Next stage you consider the over amount of spots and you will basically futures of forward is market also. So, what you are trying to do is that buy and sell in this part plus doing a counter active argument a buying and selling in a room number 2, such that the profit function which we are trying to find out is… Now cumulative such thing of buy and selling part and buy and sell in the forward in the future market depending on whether you are going for a long or a sort position.

So, if we consider this for a sort position the overall life of the hedge the change in the values given by this, where hedge is the hedge ratio that is the number of such contract, which you want to find another long position, which is the just the opposite sort it is just the reverse. So, what I am buying now will basically be a last to the person and vice versa. So, if you try to minimize over all fluctuation, and if you check the book and then try to basically understand, then the overall problem formula should be try to find out is to find are all optimum hedge ratio of such that you overall risk is the minimize the maximum possible extend.

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Optimal Hedge Ratio

Proportion of the exposure that should optimally be hedged is

$$h^* = \rho \frac{\sigma_S}{\sigma_F}$$

where
- σ_S is the standard deviation of δS, the change in the spot price during the hedging period,
- σ_F is the standard deviation of δF, the change in the futures price during the hedging period
- ρ is the coefficient of correlation between δS and δF.

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So, once you find out the optimum hedge ratio comes out to be ratio, where in the denominator what you see is standard deviation of the rate of change of S divided by the standard deviation of the rate of change of the F multiplied by the correlation coefficient which is existing between del S and del F, whereas mention del S is the standard deviation of delta S with change in this spot price during the hedge period and delta F with the standard deviation of delta F, which is the change in the forward price, that is the change in the futures price are the forward price whatever it is during the hedge period. So, I mention in forward and future gone the confuse to mush of 1 that equal to concept wise they would try to give you the same answer ((Refer Time: 19:06)) even the practically over the counter, and not of the counter being trader at any point of time being basically trader as a expiration date these are fundamental different, but they would not have much our implication for are coefficient trying to understand in a very ((Refer Time: 19:21)) manner. And finally, as mentioned row is the correlation coefficient of the relation correlation, coefficient exiting between the rate of change of del S and del F.

Now, consider that we have a hedge using the index futures. So, now you are commodities not the practical product, which you are trying to buy which is basically index room, and want to basically have such an edge based on a index, such that on the part of this index which is the derive product and the forward and futures room. So, now, you want to basically hedge the risk of the portfolio, it is the number of contact that we should buy, and sell that if you find out it is given by this ratio that beta. You remember beta that particular portfolio, which is multiplied by the ratio of the portfolio value; these are also again have to basically understand the concept of rate of change.

So, you are basically the P which is the rate of change of the portfolio, where divided by the value of A, this the value of the asset underlying the futures contract. So, what you have is basically the change in the portfolio value in room 1 and change in the prices of that particular derivative which is there in room 2. So, again I am mentioning. So, if there is a actual index in room 1, if it is an exact replica mostly this changes would be based on in room 1 room two, but as you saw in the example of 22 carat, and 24 carat. If you have a actual index which does not have an exact replica room 2, then what you will do is that you will try to basically mimic the futures are a forward market with another product, such that it is count part is already then in room 1.

So, when you find out the rate of the change is very careful that the value of P of the value of A, which is given have to denoted on, in such a way that the rate of change in the prices mimics, you are actual change of the price the spot of the future maximum possible, such that you can find out optimal major issue.

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Now, let us consider extend is further that how you want to find out the row over hedge rollover basis risk. So, now consider if I remember in the last class, we have mentioned that for us partial buy and sell any point of time, but not for the futures, because this is the particular duration. And if you are duration in spot is for 4 months, and you are duration for the forward and future is 3 months; obviously, this in mismatch number 2 type of mismatch is could be which may be of a less degree of importance would be you are actual response is required for us, such a duration time which is an even number of time or duration for the future. So, say for example, you spot is needed in 6 months and a forward and futures is for 3 months, which means that you will basically have 2 such forward you join me rollover, and you have to minimize your allowance.

So, if you consider that in the time frame what you have the time frame is given, this is t time t 1, t 2, t 3, and so on and so for. So, in time t 1 when you start you sort of futures contract 1, and when time t 2 comes in immediately close that futures accounted, but are the same time again shoring a second contract. So, what you are doing is that you started closed, but moment you are trying to basically close in started again start with the second contract and continuous doing such that you will reach the time to maturity. So, as you trying to do that they would be basically since stage 1, stage 2, stage 3, stage 4, and so on and so for, you should be carefully. In order to understand that now the basically this is not for only 1 period. It is basically based on the rollover basis which is happening from time period 1 time period 2 from time period 2 to time period 3 and so on and so forth.

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Comparison between Forward and Future contracts

Forwards	Futures
1. Private contracts between two parties	1. Traded on an exchange
2. Not standardized	2. Standardized contract
3. Usually one specified delivery	3. Range of delivery dates
4. Settled at the end of the Contract	4. Settled daily
5. Delivery or final cash Settlement usually takes place	5. Contract usually closed out prior to maturity

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So, to give a not sell the compression between the forward and futures can be done in this way. So, privet contract between 2 part is under forward, so obviously, over the counter, it is not standardize is easily 1 specify delivery date is there settlement, and end of the contract only delivery a final case or the final product does takes place at for the futures that trade in a particular room. So, their futures traded the standardize contract with respect to the none standardized. Once you for the forward range or delivery dates are there, they are does not depend on the expire date, they are settlement daily as you say in the margin according on the to take care of a overall risk perception in many other player, and the contract usually or closed prior to the maturity, because actual no delivery of the actual product of the of the commodity is does take free between the 2 players.

Suppose that you will enter in a sort futures contract sell July silver for 5.20 per ounce on the Newyork commodity exchange. The size of the contract is for 5000 ounces the initial margin is given a 4 thousand and the enter margin given 3000, what is the change in the futures price that we need to margin call. So obviously, your actual value of 4000 given; that means, this overall values given the internal margin is given you will no per ounce value, you need this amount of contract. So, you will find out what is the difference of prices have been whether there is a margin called, if yes you again pull it back to the initial case, if know the continue ((Refer Time: 24:43)).
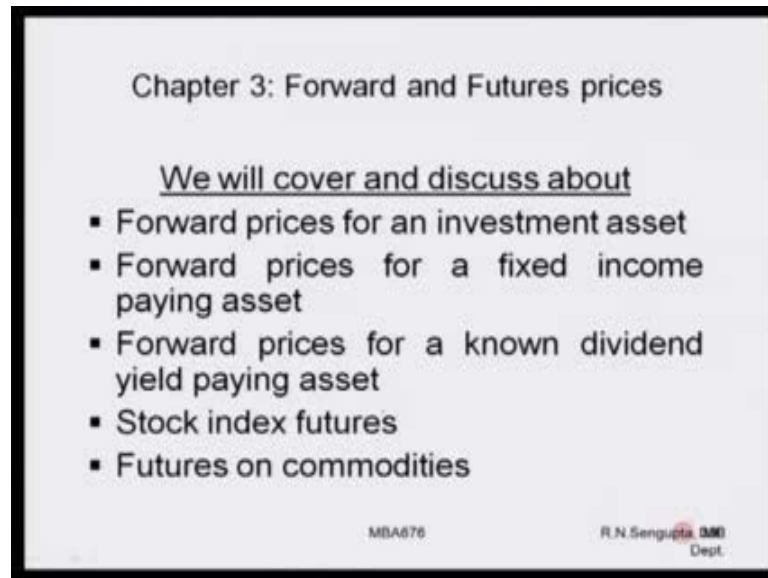
Example 2.2

Suppose the standard deviation of quarterly changes in the price of a commodity is $0.65, the standard deviation of quarterly changes in a futures price on the commodity is $0.81, and the coefficient of correlation between the two changes is 0.8. What is the optimal hedge ratio for a three month contract? What does it mean.

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We will consider as a example 2. Suppose at the standard deviation of quarterly change in the price of a commodities given the standard deviation quarterly change in the futures prices also given the coefficient correlation existing between these 2 changing, which is 2 is given. Then you can find out the optimum hedge ratio depending on the formula which you value is constraint.

So, will end of this class today, but before I wind up will discuss that in the next session will cover, and discuss the forward price of the investment, forward price for a fixed income, forward price for a dividend for paying income, and how it is stock index are also calculate.

Thank you all and we will see you in the next class.