Quantitative Finance

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Module – 03

Lecture - 16

So, now, we will start a new topic in the area of option, futures. So, it would be a quite lengthy one. So, obviously, few of the classes may be a week or six or seven classes – more than that; it will consider options, futures, how stock markets are there; based on which options and futures found out, how the options prices are calculated; what are the forward, futures, how the rates are calculated; how different swaps are done; and, how the option pricing are done. We will consider; and, this will cover a huge portion of this course. So, we will consider very briefly the concept, but I will urge and request the students who are taking the course to consider the book options, futures by John C Hull, which I considered as a very good book; reading which would definitely make the student quite well aware of overall concept of option, futures. And, it has got excellent problems, excellent concept which has been given along with simple concept of how simulations are utilized. So, it will definitely broaden the overall concept that student wants to have about this concept of option, futures and derivatives.

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So, we will cover and discuss about what are forwards, what are futures, what are options and types of traders, which are there. And, we will consider what is the difference between forward and futures; what are the intrinsic characteristics of forward, futures, how the forwards prices have found out; what is the value of forward and future. We will also consider options. What are options; what we mean by options; what is a call option; what is a put option; what are the different combinations of options you can have; and, what are the different type of traders; whether the traders are there to minimize whether the traders are there in the market to make extraordinary profit; we will consider that also. But, more it will be a feel of the conceptual frame work. And, I will definitely urge and request the students to go through the book in order to expand their knowledge or whatever concept, which I follow.

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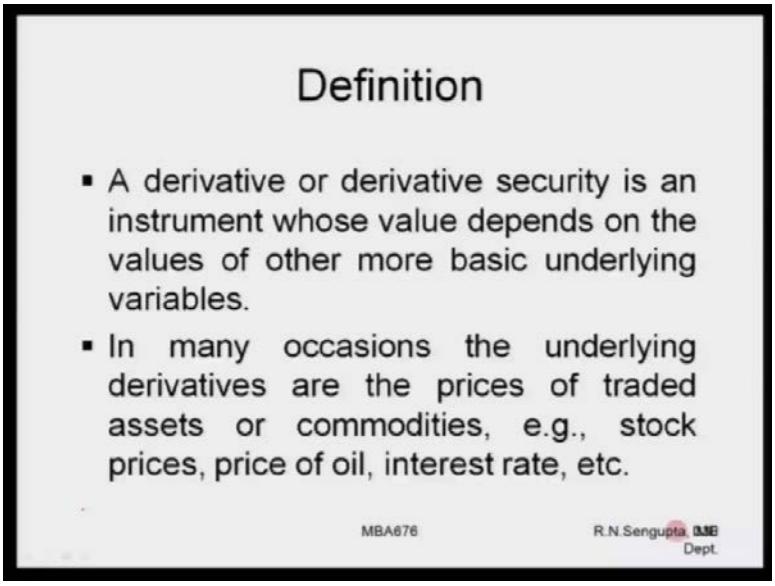

So, the derivatives basically are derived product or a derived security and is an instrument whose value depends on the values of other more basic underlying security of the variables. So, what you have is basically – consider there are two rooms; in one room, room 1 – you are basically having the actual product, which is being sold and bought. It can be stock; it can be any commodity; can be gold; it can be silver; whatever it is. And, in the other room, room number 2 – there is some derived product based on this principle product, which is being sold and bought in room number 1 such that the derived product also has a price.

So, if you remember; in the first class, we discussed that the demand and supply dictates the price. So, obviously, now, what you will also have is that, apart from the demand and supply of the derived product, they would be a derived so-called relationship between the primary product, which is being bought and sold in room number 1 with its price – the price of the derived product which is be sold in room number 2 would also be dictated. in many occasions, the underlying derivatives are the prices of the trade assets or commodities, example, stocks, price of oil, interest rate; it can be anything. So, derived products if you find out in the market or if you read the book; derived product can be options or future or forward based on rainfall also. It can be on the amount of wheat, which is being produced; it can be on the overall amount of cocoa, which is being produced; it can be priced on the cocoa, which is basically the primary product based on which you will have different derived products, which are being sold in so-called room number 1, which I mentioned.

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So, derivatives can be classified on broad two terms. One is the exchange traded, where there is basically a big room; people come and exchange the product and there is a trade. So, obviously, there is a demand and supply of that product and that exchange or the place where you trade is basically a place, where the demand and supply prices are dictated depending on the demand and supply. On that, other is basically over-the-counter. So, consider there are two players: player 1 and player 2. Player 1 has such a product, for which there is no demand; but, obviously, player 2 is there in the market,

which wants the product – financial product. So, obviously, they will sit across the table and decide what is the price at which they would be bought and sold. So; obviously, there is no such trading houses and there are no such other players, who are also there, who want to basically buy and sell that particular product. Derivatives can also be divided based on the concept of type of derivatives, which you have. You can have forward; you can have options; you can have futures; you can have swaps; you can have different type of options like callback options, American options, European options, so on and so forth.

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Now, use of derivatives is done primarily to hedge the risk. So, considering the prices are on upward trend and you want to buy at a lower price; obviously, you will go for a derivative. Consider prices are on increasing trend and you want to basically sell at the highest price; then also, you will go for a different type of derivatives. You can also go to the derivative to speculate or take a view on the future direction of the market and basically the sell or buy at such a price that you will get the maximum benefit. You can also go for a derived product based on say for example, lock on an arbitrage profit. So, say for example, you want to make some profit; and, that profit should be done in such a way that, you want to go into the derived product in order to basically make the maximum amount of profit.

You may also go into derivative market to change the type of risk you are facing. Say for example, you are buying and selling on some – based on some dollar to rupees exchange. So, now, you consider the dollar prices are increasing; which means that any fluctuation in the dollar price would have adverse effect on your overall trade. So, what you want to do is you want to basically convert that rupees to dollar prices into say for example, Indian to Yen. So, you are trying to basically change your overall risk from one currency to other. So, in that case also, you will go for a derived product or derivative. Other case can be – you are trading on interest rate; and, the interest rate is fluctuating and you have gone for a floating interest rate. So, floating interest rate is changing to a maximum possible extent; what you will do is that, you will convert your floating interest rate to fixed interest rate. Or else, it can be so possible that, your fixed interest rate is not at all good for you; you will like to go for a floating interest rate. There may be that, the nature of the liability or nature of the product or nature of the asset is such that you want to basically change your position from going for a positive to negative and negative to positive such that you need to go into derived; you need to go to a derivative markets such that those that derivative product, which you are going to buy and sell would basically fulfill your demand of what you want to meet.

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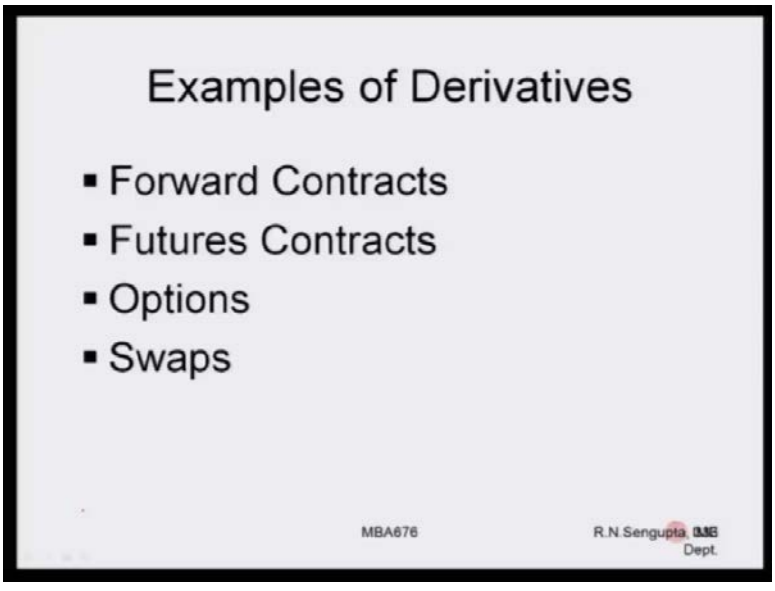

So, examples of derivative – as already mentioned, they are forward, futures, options and swaps.

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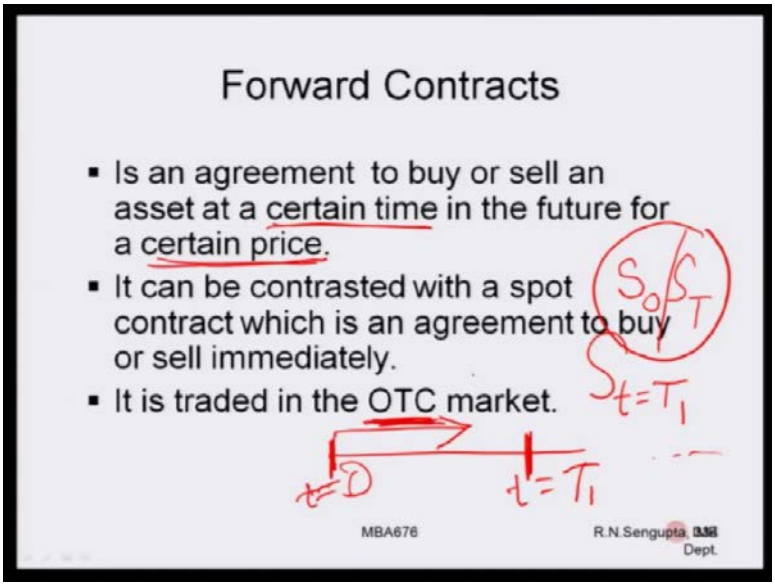

Now, forward contract is an agreement to buy or sell an asset at a certain time in the future for a certain price. So, remember this thing – certain time, certain price – these are important. It can be contrasted with a spot contract; spot means as of today based on the on the selling and buying. So, we will denote the spot with an S; where, the suffix, which you have – if it is 0, it means the spot as of now. If it is given by S suffix capital T; it means the spot at the time t is equal to capital T. So, what we will generally denote is this; which means the spot when the time frame is T 1. So, obviously, there would be a data; in the sense, if you are at this time, this is 0; and, if you are at this time, then this is 1; and, so on and so forth it will continue. So, obviously, you are measuring as you go along the time scale.

Now, these forwards are which are to be traded in the over-the-counter market; that means there are only two players; there is no such big market, where the people will come and buy those particular products. So, what you should remember is there is a certain time at it will be bought and sold; there is a certain price; and it should not be confused with the price of spot, which is S. And, obviously, it is over the counter product.

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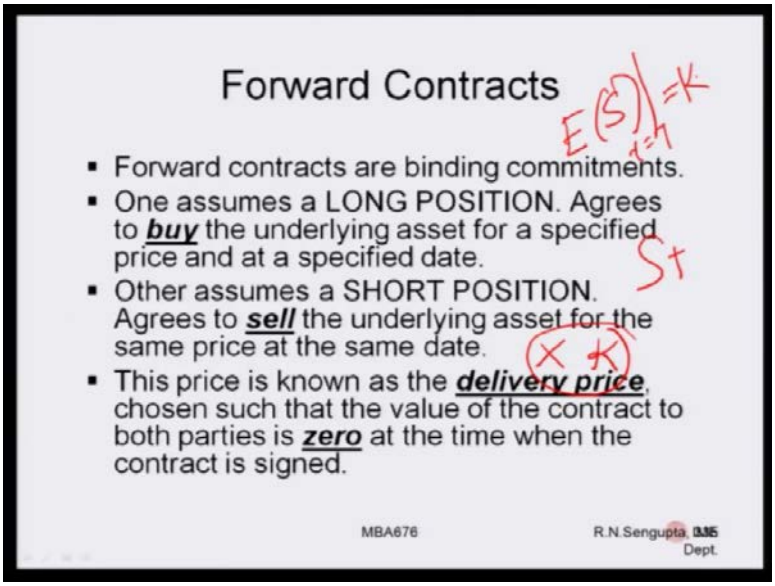

Forward contracts are binding commitments. So, once we decide on the forward contract, it is binding between these two players. One assumes there are two types of positions: long and short. So, let us make conceptually very simple distinction between what we mean by a long and short. Consider short means the word is s; short also means sell. So, if I am in a short position; that means, I am going to sell. And, if I am a long position; it means I am going to buy. So, obviously, for any contract, there would be a person of long position they would be a person of short position. It means that there are players in the market, who want to go into long and short position.

Other person as I mention is a short position; agrees to sell underlying asset for the same price at the same date. If you remember the certain price and certain price, which you are mentioning is that certain price and the certain date which has been dictated had been agreed up on both – by the player, who is at a long position and a short position. The price is known as a delivery price; which later on in many of the books, you will denote is as capital X or by capital K. This delivery price is chosen such that the value of the contract to both the parties when they sign the contract is 0. This is very important to understand. So, when I am trying to basically sign a contract with the other person; if I am going for a long or a short position; so, both should perceive that the price at which I am going to sign this contract, which is K or X is such that if the actual value of that spot on that day, which is when it expires S t; if that is exactly equal to the K value; then, neither of them makes a loss or a gain. How? Say for example, I try to basically sign a

contract of a value of K; so, which means that, when the time arrives, I can go into the actual market, buy that product and sell it to the other person at the value of K. So, if I am not willing to buy that product or if I am not willing to sell the product; which means that there is no difference between the value of S t and K such that neither of us – the buyer or the seller or the long contract position or the short contract position would make any profit or loss. Now, if I extend the concept further more; it would mean that, the expected value of S at the time t should be equal to K such that there is no difference.

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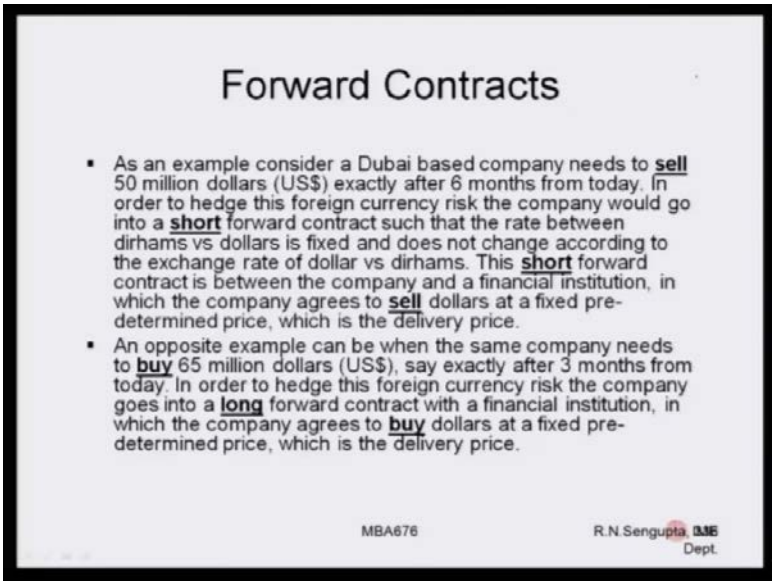

As an example, consider a Dubai based company needs to sell 50 million dollars exactly after 6 months from today. In order to hedge this foreign currency risk, the company would go into a short forward position such that the rate between dirhams versus dollars is fixed and does not change according to the exchange rate of dollars to dirhams. This short forward contract position is between the company and financial institution in which the company agrees to sell dollars at a fixed predetermined price, which is the delivery price or K or X. And, opposite example can be when the same company needs to buy dollars of say for example, 65 million dollar or the million dollar; but, the time duration is now three months. In order to hedge this foreign currency; hedge means such that there is no untoward fluctuation in the price. The company goes into a long forward contract with the financial institution in which the company agrees to buy dollars at a fixed price way at the delivery time.

So, in the first case, the company is going to go for a short position, sell. What is the time period? Time period is basically 6 months; capital T is 6. And, the amount of the delivery price would be dictated by both the seller and the buyer. In the second case, the actual amount needed is now 65 millions; in the first case, it was 50 millions. Now, the time frame is 3 months; but, the position of the player; the company, which you are talking about is basically to buy. So, initially it was sell, which is short. Now, it is buy, which is a long. So, obviously, the prices at which they would be done in those two cases are different. But, how to find out? We will come to that later. But, remember another thing – the prices or the delivery prices, which is X or K – once decided is fixed; it would not change. The time duration is also fixed and would not change. So, there would be two players: buyer and seller, long and short.

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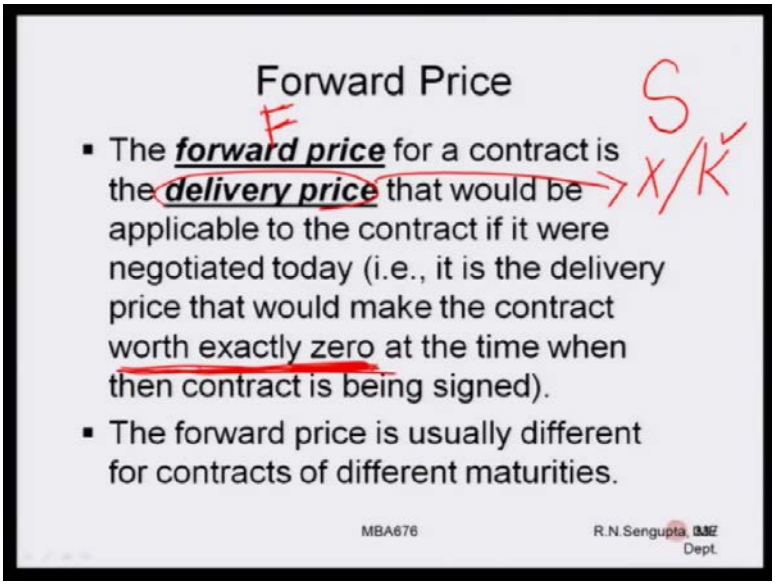

The forward price for a contract is the delivery price. So, now, we are again encountering different concepts. Initially, it was the delivery price. Now, we are considering a new concept, which is the forward price. And, we will see later on that the forward price is denoted by capital F; like spot is denoted by capital S, forward is denoted by capital F. The forward price and the delivery price… Again to repeat – delivery price would be denoted by X or K; but, generally, we will denote it by K. The forward price for a contract is the delivery price that would be applicable to the contract if it were negotiated today; that is, it is the delivery price that would make the contract worth exactly 0 at the time when the contract is being signed. So, remember this. This concept is important. Net

worth of the contract should be 0 between both the players perceivably. The forward price is usually different for contracts of different maturities. And, when the contract is being signed, then the price at which the delivery price is fixed; but, the moment the clock starts ticking as you go down the contract, the S price, which is this spot also changes; similarly F also changes depending on the demand and supply. So, S is changing on the demand and supply of the primary product in room 1; F is being dictated by demand and supply in room 2 plus the derived product based on which the functional form of the prices are being dictated between the primary product and the secondary product.

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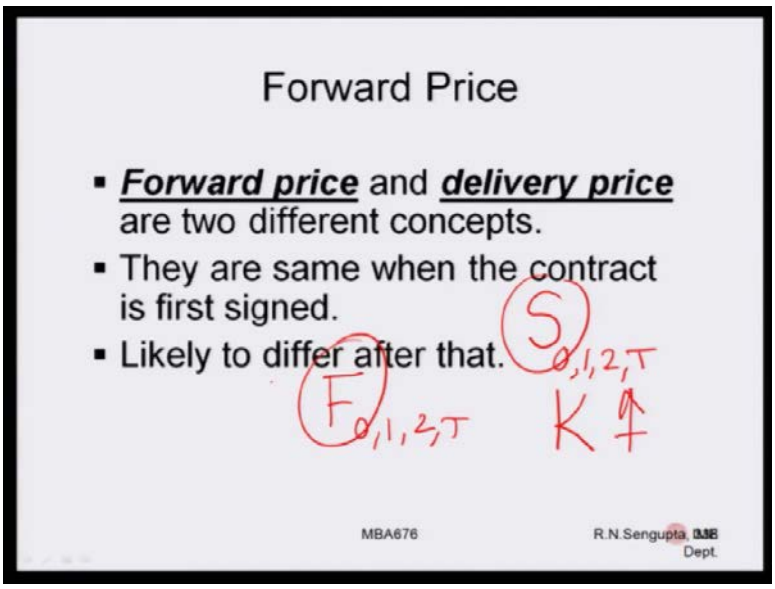

So, forward price and the delivery price are two different concepts, which I mentioned. They are same when the contract is signed. Another important thing – when I sign the contract, the forward and the delivery price are exactly the same; and, they are likely to change as time goes by. So, the variable, which I have is S with suffix 0 or 1 or 2 or T depending on when we are at what time frame. F again with 0, 1, 2, T – whatever a time frame is that, when we are… But, the value of K, which you have, is fixed. So, whenever I sign the product; this K becomes fixed; these values of S and F are changing.

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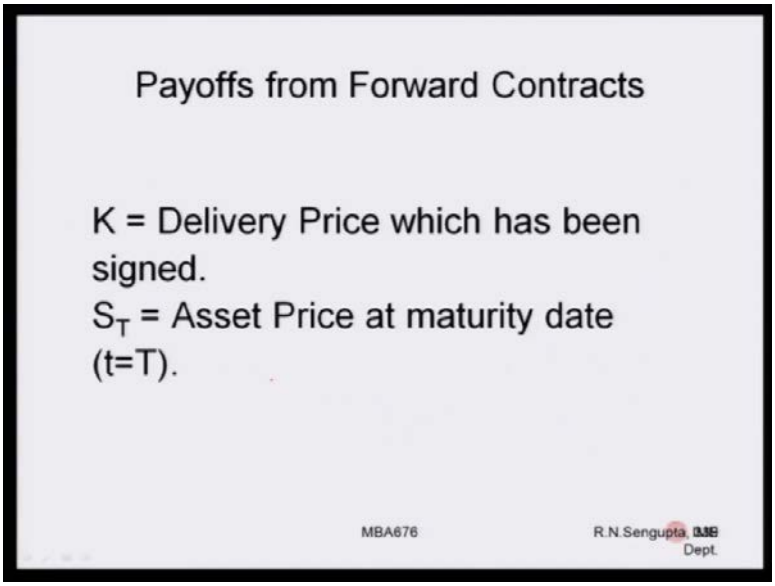

So, K is the delivery price, which has been signed. S T is the asset price at maturity date t.

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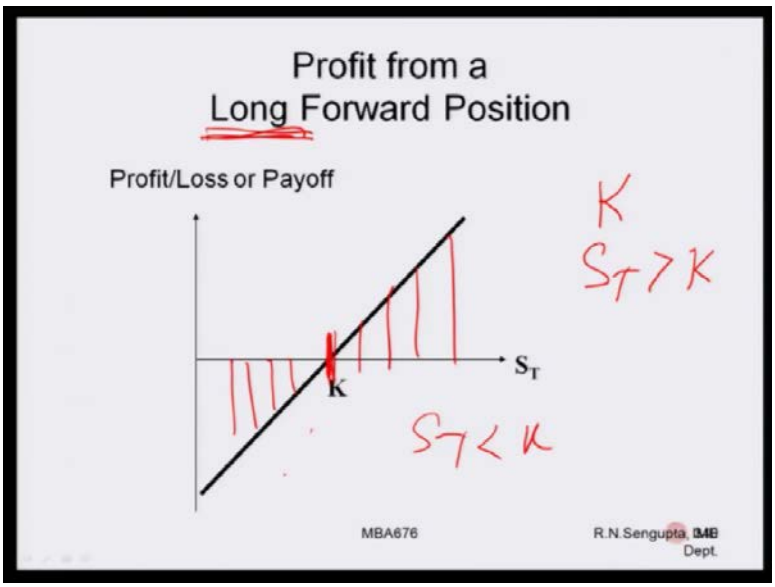

Now, let us consider the profit function for the long forward position; remember that. This is the long forward; long means the person, who has bought; short would be the person who has sold. So, the profit or loss payoff for the person who is at long forward position is very simply given by this. So, why it is this diagram? Consider I have signed the product at K. Now, if it is long; which means what I will buy. So, what I need to do is

like this – simply this. If the prices are greater than K; what I will do? I will immediately buy it from the other party, who is in a short position at K price and sell it in the market at the S T price, which is greater than K. So, I will basically make this profit, whichever the price difference is. But, on the other hand, if the delivery price is K; what I need to do? Basically, I have to buy it from the market at a price, which is less than K because S T is less than K in this region. And, I basically have to sell it to that person at the Y price of K such that I basically make a loss. Now, if you ask; seeing this picture for the long position, what would happen to the short position? It would be just the reverse, because you add up both of them; it will be a zero's total profit.

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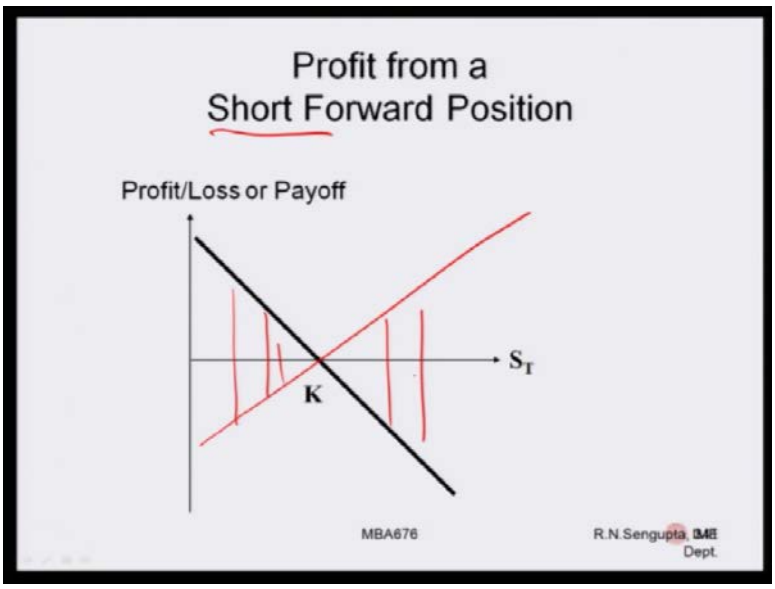

Profit and loss for the short position is just the opposite. So, in the initial case, it was this; now, it is this. So, if we add up both of them, they add up to same. So, this is at the different position. In the long position, when S T was greater than K, there was a profit; and, when S T was less than K, it was a loss for the long position. Now, in the short position, if one is making a profit; obviously, other will make a loss. So, in the case, when the long is making a profit, short will make a loss; and, in the second case – when long is making a loss, the short will now basically make a profit as shown in this diagram.

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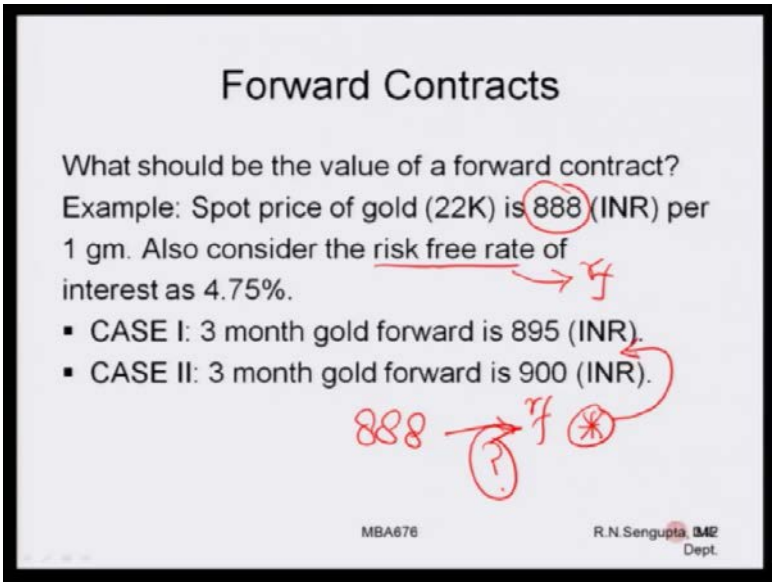

Now, what should be the value of the forward contract? Example – spot price for gold – 22 carat is 888 Indian rupees for 1 gram. And, consider the risk-free interest rate. Let us for our time being, consider the risk-free rate as is given. In case 1, you have a 3 month gold forward with 895; in the second case, we have the 3 month gold forward for 900. Considering the forward contract, where it is at the long position and the short position; and, we all know that, short means S means sell; long means buy. So, let us try to understand this problem rather than solve it because understanding would give us a better flavor in how you would try to solve different varieties of problems for forward, futures options and different types of options and swaps.

So, again repeating the same problem, consider spot price of gold for 22 carats is 888 Indian rupees per 1 gram. Also consider the risk-free interest rate is given. So, here I will like to basically highlight one fact. So, the risk-free interest rate, which is there is exactly this r f, which we have been talking about in portfolio management and in different concepts of other areas of discussions till now. So, we will again encounter that and find out how risk-free interest rate is utilized in options, futures and forwards contracts to find out the prices and necessary details as required. So, consider the risk-free interest rate is 4.75 percentage. So, now, make a very important note – what does this percentage means; it means per annum. And, technically, this risk-free interest rate is continuous compounded. So, remember these two concepts. One is the interest rate, which is given should be calculated on a per annum basis and should be continuous compounded. So,

that would give you a lot of information that how you do the calculations. And, if risk-free interest rate or interest rate, are given in other terms, we will convert them in the concept of per annum continuous compounded case.

So, in case 1 – consider the 3 month forward contract is 895; and, case 2 – the 3 month gold forward is 900. So, 888 is the price as of today. And, later on, that price would obviously increase depending on the interest rate. So, if you find out the 888, price increases; if it is in between 895 and 900, which will come out to be true; then, in one case, a person would make a profit if he is in a short position; and, in another case, the person will make a profit is in a long position; you can easily calculate. So, consider 888 is there. So, this will increase based on the risk-free interest rate is there. How you do the calculation? I am going to come to that later on. So, hence, the question mark. So, this 888 becomes some value; consider it is a star value; whatever it is in numeric terms. So, if this value is in between 895 and 900. So, in that case, somebody will make profit, somebody will make a loss.

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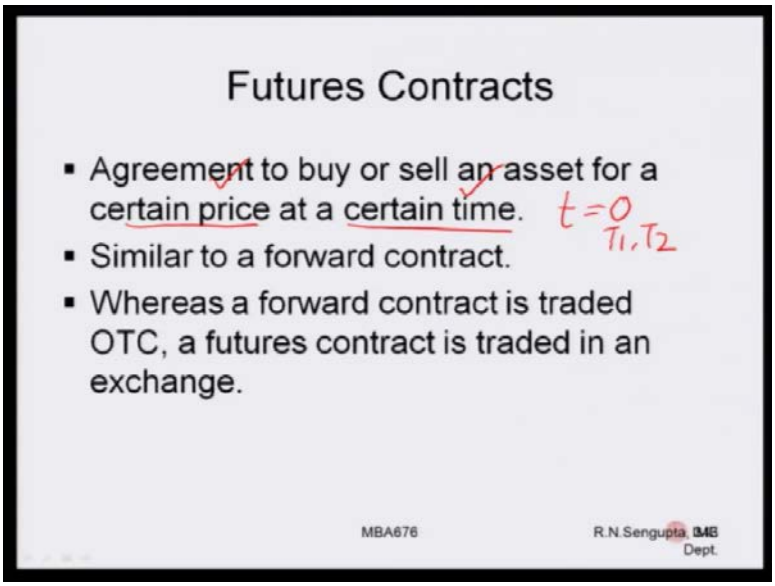

Now, future contract is an agreement to buy or sell an asset for a certain price at certain time. It is almost exactly similar to the forward contract conceptually wise; but, there are lot of difference in that. Whereas, a forward contract is traded over-the-counter; that means, between two persons over-the-counter such that there is no other such demand and supply. A future contract is traded in an exchange such that there is a whole body of

buyers and sellers and there is a particular demand and supply for that; based on which the price and the quantity is decided.

Now, another concept is that the time concept and the price concept would also become important for the case of futures and forwards. And, what is the subtle difference between them with respect to a forward, with respect to a futures. These two important concepts of price and time would become evident as we proceed. Now, remember time as I have mentioned will always measure with small t; and when small t is equal to 0, it means as of now, when the clock starts and as it progresses, T 1, T 2, T 3, so on and so forth; that means, small t value takes those capital T values. Then, you will need to find out what is the forward and future prices based on that, and how a person can make a gain and loss depending on whatever the situation is.

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Now, exchanges where different types of derivatives are traded; so, you can have a look when you go to the URL; on 9th July 2007, Chicago Mercantile Exchange, that is, the CME and Chicago Board of Trades – CBOT merge to form the CME; and, you can find out the details at CME.com. Similarly, of the New York Merchantile Exchange, Multi Commodity Exchange of India is a very big market here. Then, you have the London Bullion Market Association, where they basically trade in commodities. You have the Dubai Multi Commodity Exchange, Bombay Stock Exchange; obviously, BSE and NSE have also started in different type of derivatives. And, you can check their sites; check

what are the information; based on which, you can learn a lot of things. But, this is a small set of different type of derivative markets, which are there in some of the countries starting Dubai, India, then USA; obviously, there is in London, in Japan, so on and so forth; you can check them if you are interested.

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The NSE, which is the National Stock Exchange commenced trading in index futures on 12th June 2000. The index futures contracts are based on the popular market bench mark, which is the S&P – Standard and Poor's, Nifty index. S&P, Nifty future contracts have the maximum 3-month period. So, they would be traded for the duration of 3 months. So, if you remember that time period, which I said is small t. So, the small t can take values between 0 to capital T. Capital T; where, capital T is 3 months or 3 by 12 years if you are converting that in the concept of years. Why I mentioned about the concept of years? If you remember, just few minutes back, I mentioned the risk-free interest rate is per annum continuously compounded.

So, this word per annum means it has to be on a datum or a unit scale of time as here. That is why whenever you encounter any time period, you immediately convert into a yearly concept. So, maximum 3-month trading cycle that near month 1 and the next month 2 and the for month being 3. A new contract is introduced on the trading day following the expiry of near month contract. The new contract will be introduce for a time period of 3 months. This way at any point of time, there would be 3 contracts; that

means, one will expire, one is in between, one has just started; one near month, one mid month and one for month as I just discussed. S&P, Nifty futures contracts expire on the last Thursday of the expiring month. If the last Thursday is a trading day; that contract expires on the previous trading day.

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So, future contracts – in future contracts, the exact date of delivery is not specified. So, this is the first important point we encounter. It is traded in the exchange and if the contract for delivery of goods, then the quality of goods, each contract amount, etcetera are very specifically specified. So, with this, I will close today's class. And, starting the next session, we will basically discuss the subtle difference between forward and futures and what are the implications of risk-free interest rate and the time and the price in trying to find out the prices of the futures and forward contract.

Thank you.