Quantitative Finance
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Module – 03
Lecture – 15

So, welcome back to this quantitative finance course.

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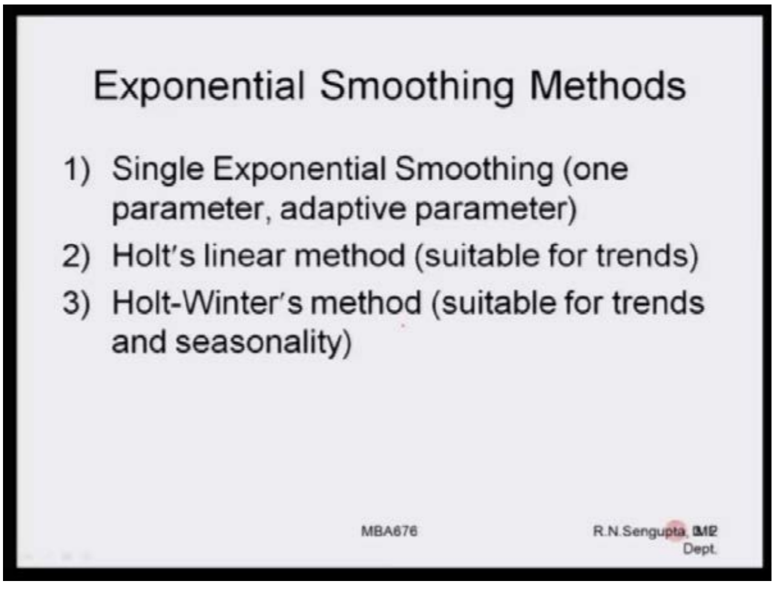

Now, exponential smoothing methods will consider the single exponential smoothing one, that is, one parameter or the adaptive method; will consider the Holt's linear method suitable for trends; and, you will consider the Holt-Winter's method, which is suitable for both trends as well as seasonality. So, seasonality will say there what can be the seasonality for products. So, it can be… Say for example, you are selling woolen cloths. So, during the winters, the sale of woolen cloths is the highest. Or, say for example, you are selling coolers or fridges. So, during the summers, it is the highest. Or, say for example, any product – it has the seasonality depending on the seasons. Or, it may have the trend also. So, you will try to basically bring those in our calculation for the smoothing methods using the Holt linear and so on and so forth method, which are there.

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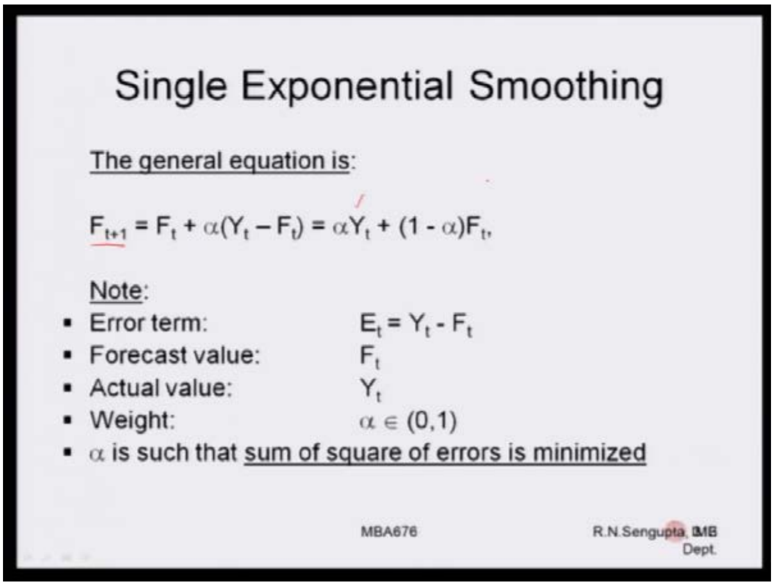

Now, consider the single – simple single exponential smoothing method. So, the equation if you look at it is very simple. F t plus 1, which is the forecasted value you are doing for the t-th plus 1 time period basically depends on two things: number 1 – it depends on the actual value Y t for the last case; but, you put a weightage as alpha. And also, it depends on what has been the forecasted value for the last stage and you basically put a weightage of 1 minus alpha. So, what you are doing is that, standing today, you are trying to basically weightage or give weightages to your actual value for the past and the predicted value for the past, because the actual value would give you actually what is happening. And, the overall methodology based on which you are trying to predict would also come out in the case, which is F t. So, say for example, you move one step forward now; the F t plus 1 would basically give some weightages of alpha 1 or 1 minus alpha 1 to the value, which is Y t plus 1 and F t plus 1. So, as you keep moving for what you are basically trying to be more and more weight… I would not say the word more and more; you are trying to basically give weightages to the past data; where, the past data basically consists of two parts: one is the actual one and one is basically predicted one.

Now, you may ask a question – why not make it much more – much more inclusive or past to past data. So, why not make… Say for example, F t plus 1 would basically give weightages to four such values. What are the four values? The predicted value and the actual value for t and predicted value and actual value for t minus 1. So, standing on t plus 1, you are basically giving some weights say for example, alpha 1, alpha 2, alpha 3 and alpha 4 for the predicted actual for t value – t-th time value and predicted and actual value for t minus 1. So, if you can basically go recursively, you can make it much more comprehensive; but, the only problem is that, what are the weightages based on which you are trying to find out these – give these values to the weightages of F t, Y t, F t minus 1, Y t minus 1 – obviously, will be a question which has to be answered by you such that the prediction value, which you do gives you the best result. So, in this equation, what you have? One is the error term. So, error term is basically the difference in the predicted and the actual value. The forecasted value, which is given by the symbol F; whatever the suffix… Suffix basically gives you the time period. And, one is the actual weight, which is basically y with that suffix at what time. And, the weights are basically alpha and 1 minus alpha, which is basically between all 0 and 1. So, obviously, you see the some of the weights add up to 1. Now, alpha is such that the sum of the squares is minimized.

Now, let us pause here for two minutes. If you remember; the single index model we considered and we basically tried to say that how would you find out beta. What we did? We basically found out the square of the difference of the predicted and the actual; sum them up and basically differentiate or partially differentiate with respect to alpha and beta; put it to zero and find out those alpha hats and beta hats. So, if you remember of you can go back to your actual the slides or basically go back to my earlier lectures; I did mention that, you are trying to basically minimize the sum of those squares. Now, what is squares? Some of the squares is very simply the variances concept, which you are going to consider; because if you consider the variance concept, what you are trying to do? You are basically trying to do decrease the dispersion as low as possible.

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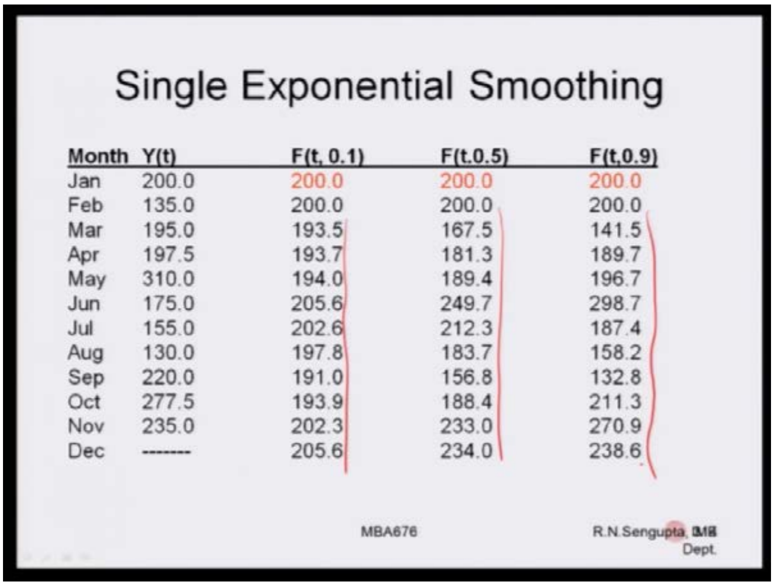

Month	Y(t)	F(t, 0.1)	F(t,0.5)	F(t,0.9)
Jan	200.0	200.0	200.0	200.0
Feb	135.0	200.0	200.0	200.0
Mar	195.0	193.5	167.5	141.5
Apr	197.5	193.7	181.3	189.7
May	310.0	194.0	189.4	196.7
Jun	175.0	205.6	249.7	298.7
Jul	155.0	202.6	212.3	187.4
Aug	130.0	197.8	183.7	158.2
Sep	220.0	191.0	156.8	132.8
Oct	277.5	193.9	188.4	211.3
Nov	235.0	202.3	233.0	270.9
Dec	-------	205.6	234.0	238.6

Now, consider the single exponential smoothing average. So, whatever values of alpha you take; I basically made a note of that. So, the first column is the months, time periods; second one are the actual values y. So, you have not the considered F. Now, the third, fourth, fifth are the F values, but considering some different values of alpha in each case. So, in one case, you consider alpha is 0.1 and other case you have considered the alpha is 0.5 and another case you consider the alpha is 0.9. So, obviously, 1 minus alpha would be found out accordingly. Now, just for our interest and just to kick start the whole process, I am considering the forecasted value for all these three cases, which is mark by rate are equal to the actual value. So, you can consider anything else also; but, basically, considering the actual value is basically you are considering from the past, where I am starting just the past value; the predicted and the actual value are exactly the same. The moment you consider that, you will basically try to find out a set of utility methods, which basically kick start the process based on which you can do your calculation. So, using this method, if you basically find out this third column, which you have, is basically single exponential smoothing average considering alpha value as 0.1. Similarly, this is the set of values for alpha is equal to 0.5. And, this is basically the single exponential smoothing values considering the alpha is equal to 0.9.

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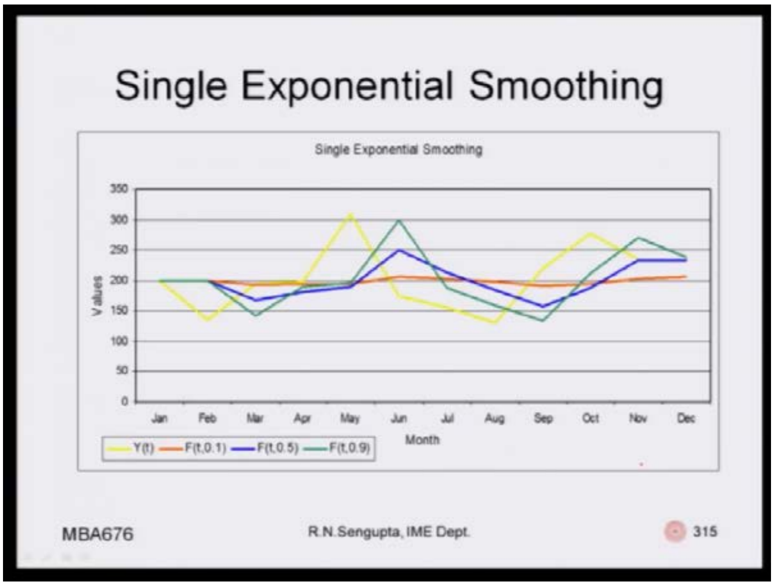

So, if you plot them; so, yellow one is the actual values; F, red, blue and green are the corresponding predicted or forecasted values considering alpha 0.1, alpha 0.9 and 0.5 and alpha 0.9. So, you can have different values of alpha also; you can find out. So, if you see; the trend is almost the same. What is the magnitude of the trend basically that is coming out from the fact? That what is the value of alpha you are putting? And, value of alpha basically means that what is the overall weightages you are trying to put on the predicted and the actual value for the past data – just past data. If you basically expand that; so, obviously, in the alpha 1, alpha 2, alpha 3, alpha 4 values, which you have will give you what are the weightages you are trying to put for the predicted and the actual value for the past two sets of datas; that means, F t and F t minus 1, Y t and Y t minus 1.

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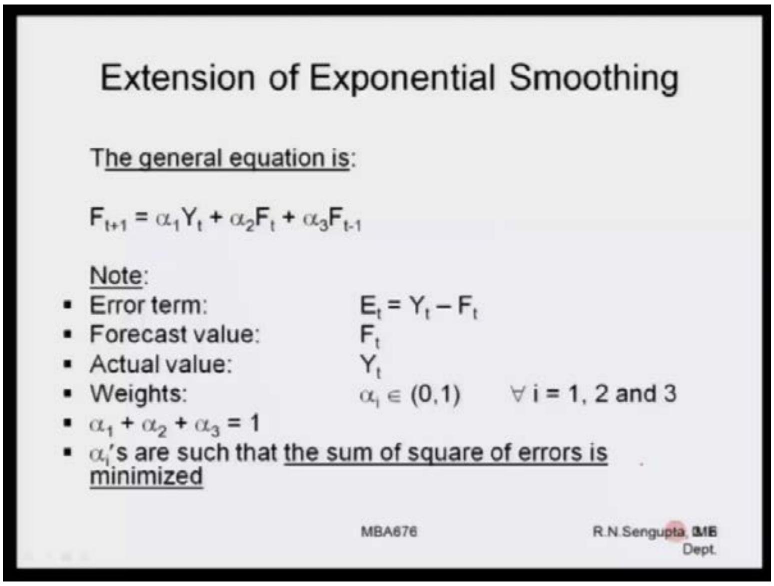

Now, the extension of the exponential smoothing average is now basically…. Now, we are slowly considering the general one. If you see the equation, what you have? You are putting a weightages of alpha 1 to the actual value for that past data – just past data, which is Y t. And, you are putting – trying to put weightages of alpha 2 and alpha 3 for the forecasted value for the past data – immediate past data, which is F t and back to back data, which is two steps back, which is basically F t minus 1. So, you can extend that also as F t minus 2, F t minus 3, and so on and so forth. But, we should basically be rational to what extent we will try to put another weightages such that the weightages are add up to 1; and obviously, the forecasted value are in such a way that, they give us quite a huge amount of information that what would be the forecasted value in the future.

So, note again – error term is given by the difference of Y t and F t; forecasted value again as such is given by the symbol of F; actual value is given by the symbol of Y. Weights as you know is again given by the values of alpha. In this case, you have basically alpha 1, alpha 2, alpha 3, alpha 4, alpha 5, whatever it is. Some of the weights should add up to 1. And, alpha should be put in such a way again the squares of the sums of the weights – basically of the weights considering that you are trying to minimize should basically give us the case, where the variance is minimized. So, what you are trying to do? You are basically trying to find out the difference of the predicted and the actual. And, when you are trying to find out the difference of the predicted and the actual; this alpha 1, alpha 2, alpha 3 come as unknown. So, you have basically three equations; differentiate that three time; once with alpha 1; once with alpha 2; once with alpha 3; put it to 0. These three equations will give you the values of alpha 1 hat, alpha 2 hat, alpha 3 hat; based on that, you will predict and basically forecast for the future values.

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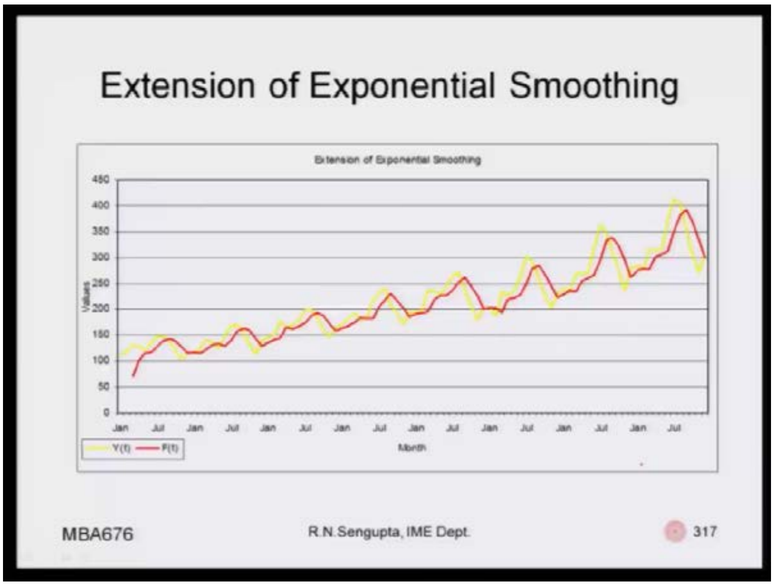

So, extension of the exponential smoothing average – for the same data set, we are considering January and to February; and, the yellow and the red ones are basically the actual; and, the red one is the predict. So, obviously, you see there is a huge amount of overlap; but, only that, there is a shift; that means, some delta shift happening. If you basically shift or pull down the red one; it will exactly merge the yellow one; but, there is a shift because you are basically trying to utilize just the past data in order to predict the futures. So, obviously, there would be some lag, which is happening. So, this lag will increase or decrease depending on how many such data points you are taking from the past or what is the average values of alpha you are trying to take in order to predict.

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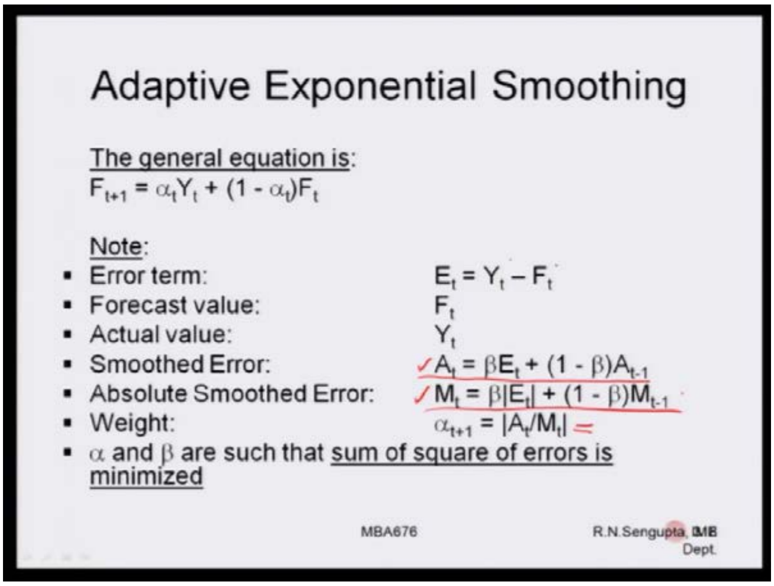

Now, for the first time, we will consider the adaptive exponential smoothing method, where the alphas, which you have found out – you have basically predicted or trying to find out would basically be adjusting as you proceed. How? The answer is that – why if we should do? Point number 2 is that how should we do that? To answer the question – why we should do that? Because as we go forward and back and take the data from the backward or the past data; obviously, there would be change in the trend, change the seasonality or change in the actual values. So, in order to basically have a much better knowledge about how the F and the t values are changing; obviously, you will try to put more or less weightages on Y and F depending on how good or bad these predictions are coming out to be. So, obviously, the alpha value should change; but, keeping the fact true that, the weights of the alpha should always add up to 1.

So, again, let us consider the equation; what you have is now alphas are not fixed. They have given you the suffix of t; which means that, they would change – keep changing as you proceed. So, there are terms again is given by the symbol – variable E – the forecasted value by the variable F; actual values by the variable Y. But, now, there are basically two smoothening errors. One is the smoothing error. So, one is the absolute smoothing errors. So, in one case, you will consider the values of betas. So, betas would be basically changing depending on the differences of the actual smoothened errors and absolute smoothing errors. And, as they keep changing, the alpha values will also be basically predicted. So, what you are trying to find out that, if smoothed errors and absolute smoothed errors are exactly equal to 1; then, the values of alpha would basically come out to be 1. But, how you do that? So, consider that this value.

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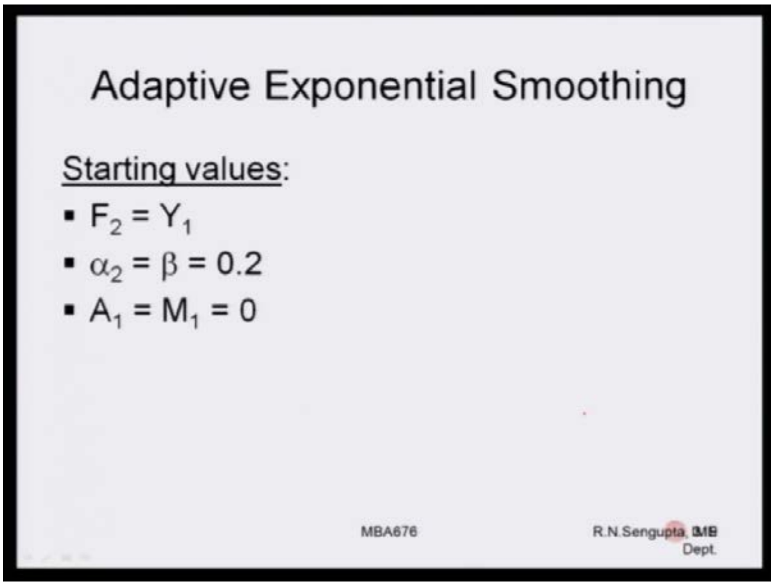

I take – assume a value of alpha 2 and beta as 0.2. And, I am considering the starting values of smoothened errors; both the absolute case as well as the smoothed errors as A 1 and M 1 as 0. And, you also consider the actual and the predicted values, which is F 2 and Y 1 are exactly the same.

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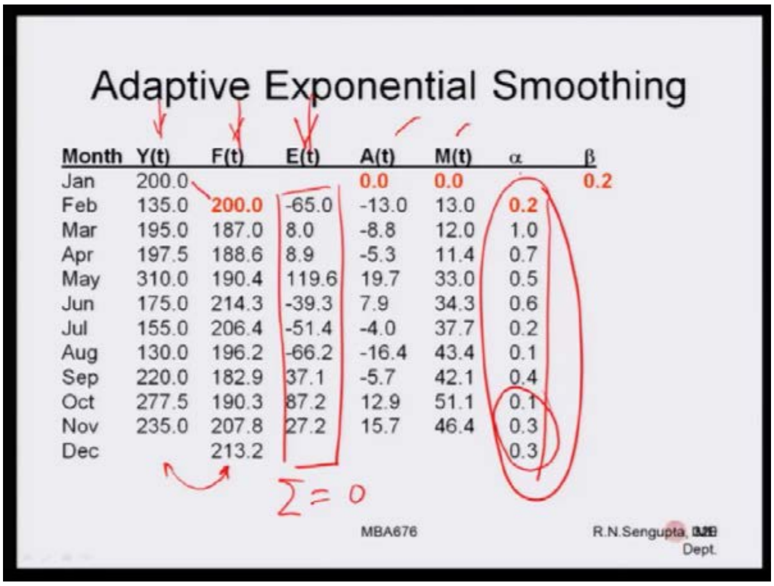

Month	Y(t)	F(t)	E(t)	A(t)	M(t)	α	β
Jan	200.0			0.0	0.0		0.2
Feb	135.0	200.0	-65.0	-13.0	13.0	0.2	
Mar	195.0	187.0	8.0	-8.8	12.0	1.0	
Apr	197.5	188.6	8.9	-5.3	11.4	0.7	
May	310.0	190.4	119.6	19.7	33.0	0.5	
Jun	175.0	214.3	-39.3	7.9	34.3	0.6	
Jul	155.0	206.4	-51.4	-4.0	37.7	0.2	
Aug	130.0	196.2	-66.2	-16.4	43.4	0.1	
Sep	220.0	182.9	37.1	-5.7	42.1	0.4	
Oct	277.5	190.3	87.2	12.9	51.1	0.1	
Nov	235.0	207.8	27.2	15.7	46.4	0.3	
Dec		213.2				0.3	

So, let us consider these are the values. So, in the first column, you have the date and the month; second column is their data; and, in the first row, you are considering some values, which are predefined or prefixed. This is I am just taking as an example. It will basically depend on actual fact that, you will find out that, optimum values are the overall variance is minimized; again, the same concept. So, you consider A 1, M 1 as 0 and beta is 0.2. So, once you consider A 1, M 1 and beta values as given in the red; and, also you consider the values of F 2 as equal to Y 1, which is this one; you can basically start off your whole process.

So, once you start off the whole process; what you do is that, you basically need to follow this procedure. So, if Y t and F t are given, which you have already found out; and, if Y and F as considered as 200 in both the cases are given; A 1, M 1, beta are given; then, you recursively use this formula and go step by step. As you recursively use this formula and go step by step, what you get are these values. So, these are the errors, which is the fourth column. And, these are the forecasted value, which is given in the third column. And, this is the predicted values, which you have given in the second column. So, these A 1 and M 1 are basically the overall values of the smoothed errors, which you are trying to find out. And, the ratio of them if it is exactly equal to 1; obviously, will give a better prediction for alpha or whatever the alpha you want to find out.

Now, you see the alpha values are changing; which means that, as you go, in the first case, you give a percentage of 100 to alpha; then, you will give a value of 0.7 and so on and so forth. So, as you go down this value, you will see that, in the long run, the alpha value will slowly stabilize to some value as 0.23 or 0.3 or 0.75, whatever it is. So, overall trend would be such that, two important things. In the general and the long run, the average of this error should be 0; which should be true as we have found out in the single index model. And, as the trend counts out to be the 0 – the errors; which means the actual difference between the predicted and the actual value should definitely be equal; both the values are equal. If that is true; then, the value of alpha, which you found out in the long run should also basically stabilize to a fixed value alpha. So, based on that value of alpha, you will basically try to utilize in the future.

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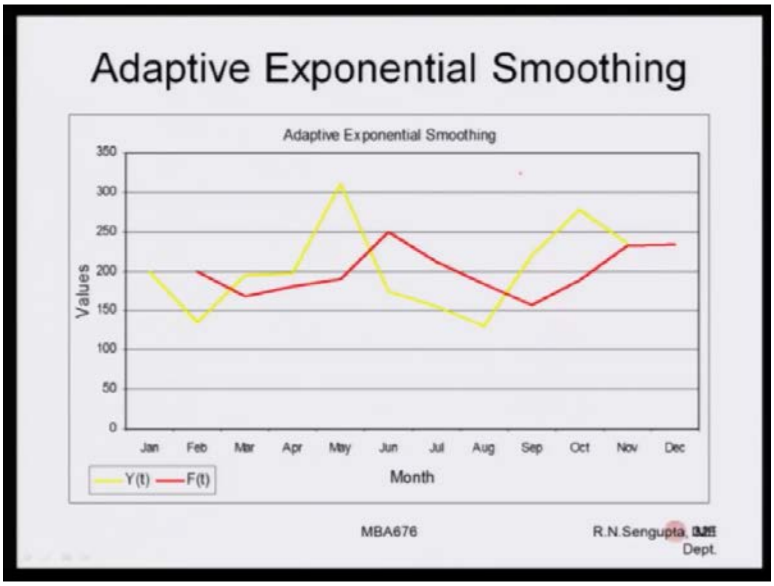

So, again for the adaptive smoothing values, we take some Y t and F t; based on that, we find out. So, basically, they do not exactly replicate each other; obviously, do not be surprised by this whole amount of difference which is; but, what is important to know is that, as you proceed more down the line and as your overall model learns the value of alpha, it is not a self learning process; basically, you are trying to adjust your value depending on how the errors are changing, how the predicted and the actual values are changing based on the smoothing errors. So, as the smoothing errors basically turns out to be 0 in the long run, the alpha value would definitely come out to be fixed. And, the difference between the Y t and F t would also come out to be basically 0 such that the overall average value comes out to be 0 – expected value of that.

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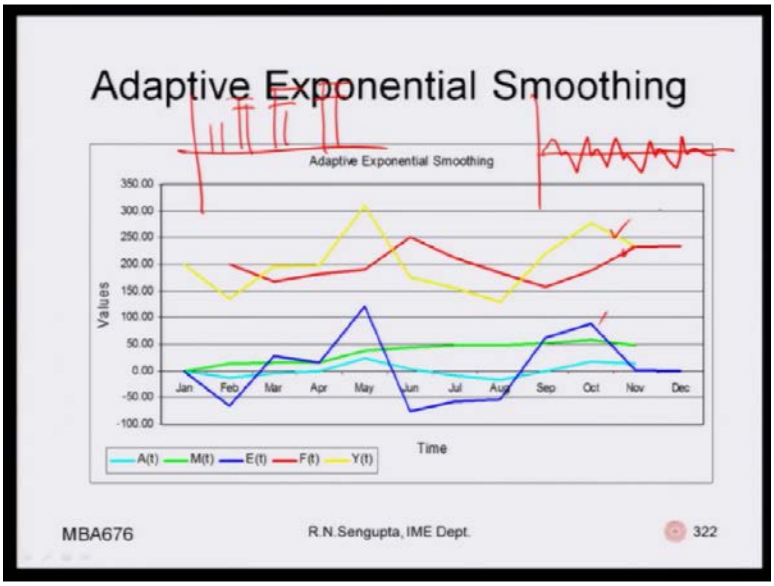

Now, in order to make our understanding much more clear; in this graph, which is different from the last previous slide, which I have shown; that apart from the yellow one and the blue one, which is with the predicted… – yellow and the red one, which is basically the forecasted and the actual value. I am also plotting what is the error, which is blue one; the smooth and the other error, which is the green and the bluish green one. So, if you see the dark blue one, there is a fluctuation; obviously, they would be a fluctuation. But, as you add up all the different type of averages of the errors, which happens; so, obviously, in the long run, there would be prediction errors. But, if you add up all the values in the long run, they should definitely be 0. And, if you compare the predicted and the actual; so, there would be differences. But, in the long run, the difference which you have in the predicted and the actual, which is this and this one will slowly turn out to be 0.

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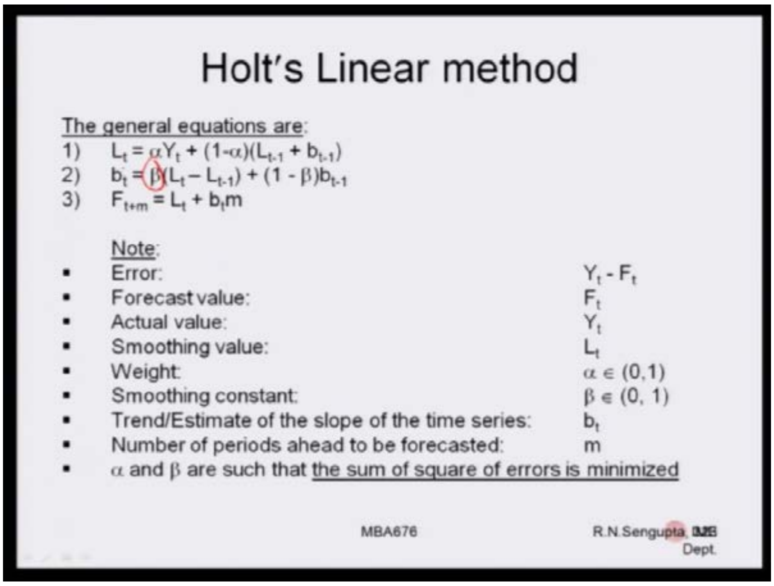

Now, consider the other method is the Holt's Linear method. The general equation just does not get complicated, but gives you much more better picture of the reality; everything remains the same. So, now, you basically have other variables also into the picture; error term remains as it is; which is the difference between the predicted and the actual. The forecasted concept is also same F; the actual value is also same was Y. Now, you are trying to take a smoothening value, which is L; this smoothing value will basically smoothen out any untoward fluctuation – both on the negative and the positive side. And now, you have basically two different types of constant: one is the smoothening weights, which is alpha as it is; and, another you have basically the smoothening constant, which will basically give you the weightages, which you are trying to put on the value of L, which is here.

Then, the trend or estimate of the slope of the time series, which you are trying to find out, is b suffix t, which is also changing. So, do not confuse this b suffix t as with the concept of beta or m, which we considered earlier. So, considered is some different type of trend or estimate. The number of time periods ahead to be forecasted; so, say for example, if I am standing today; I want to predict it three months down the line; then, the m value would be 3. If I am standing on January and try to basically predict for December; so, obviously, I will consider m accordingly. If I am standing on today and try to predict for tomorrow and there is only one time difference happening; so, m would be 1. So, corresponding to that, you find out.

Now, this alpha and beta are such that the sum of the squares of the errors is minimized. So, if you have a loop – detailed loop at this equation 1 to 3, which is written there; the values of alpha and beta have to be predicted. So, again you will ask the question – how do you predict? The question is again simple. Start with the arbitrary value and proceed accordingly. So, when you are proceeding, what you are finding out is basically finding out the smoothing values and finding out the smoothening constant and the weights. So, as you proceed, the end result would be the errors. Once you find out the errors, what you do? Simply again square the error, sum them up, differentiate. But, what you will differentiate with that with? You will differentiate with respect to the unknowns, which we have – partial differential. What are the unknowns? They are the alpha and the beta values. Once you equate them to 0, you find out the minimum values – the values of alpha and beta such that the overall sum of the squares of the errors turn out to be as low as possible, which is basically the simple concept of OLS, which is again the simple concept that you are trying to basically minimize the overall variance of the dispersion.

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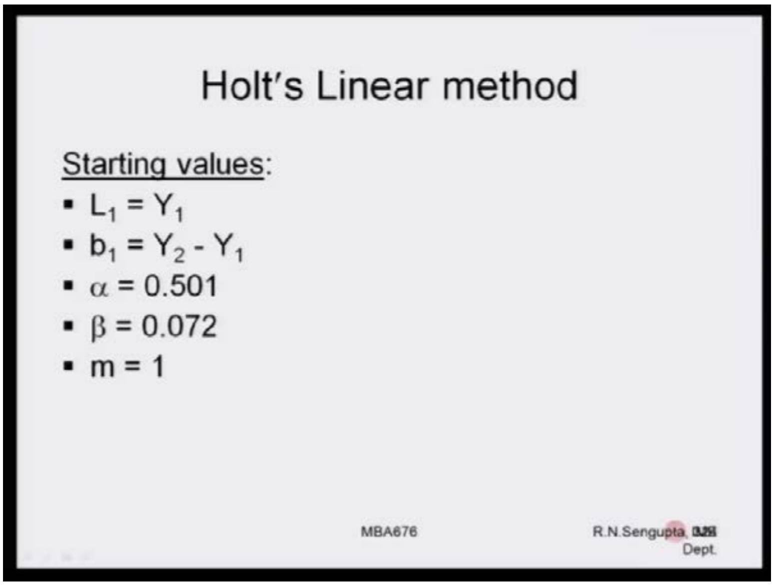

Again for this Holt Linear method, I have again taken arbitrary values. Whether smoothening values and Y 1 are taken out to be equal; I take b 1 as the difference between Y 2 and Y 1. It can be any other values. And, I take the value of alpha, beta and m as given; which is 0.501, 0.072; and, m is equal to 1. This is just arbitrarily in order to basically show you how the overall process works.

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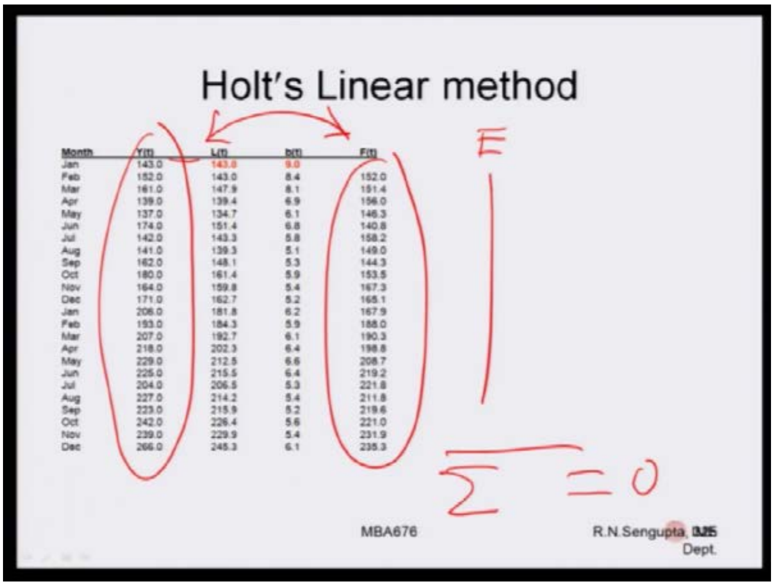

So, again I have the January to December data. So, basically, there are two years, which is 24; and, you have basically the Y t value, which is given in the second column. And, similarly, considering that, Y t and L t are same; which means that 143 values are the same here and the b t value is given as we consider; if you basically proceed in this direction considering some value, the actual predicted value comes out to be this; your actual value comes out to be this. If you find out these Differences, you will basically find out the errors. If you want to double check how good or bad you are starting values of L t and b t are; what you can do is that, add up these weights of these errors. If these errors comes out to be 0, these starting values are good; if they are not, it will be change these values in order to find out what is the best values at which you should start such that the overall average of this error should be equal to 0.

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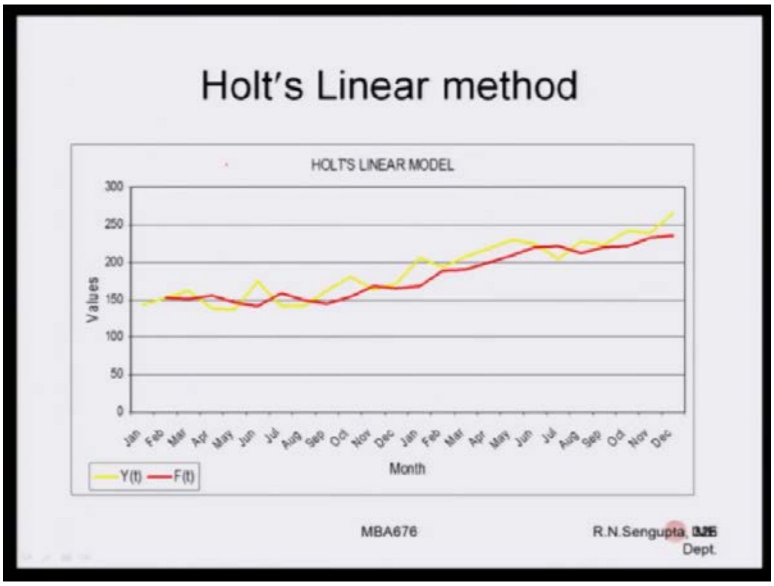

So, considering the Holt Linear method, again I plot the values of Y t and F t. Now, you find out there are lot of simulative of the trends. So, what you have been able to do is that, the more so-called complicated or more sophisticated the model you have made – you have been able to make; you are able to predict and forecast to a maximum possible extent such that any fluctuations are already taken care by you such that the smoothening value is the weightages, whatever they are – they are in a way trying to basically find out any abrasions in the overall fluctuation such that those abrasions, fluctuations are subsumed in the process such that the prediction and forecast is really good. So, obviously, you can find out the differences of the Y t and F t, which is the errors; and, basically plot the errors you will find out in the long run, the average value should definitely be 0.

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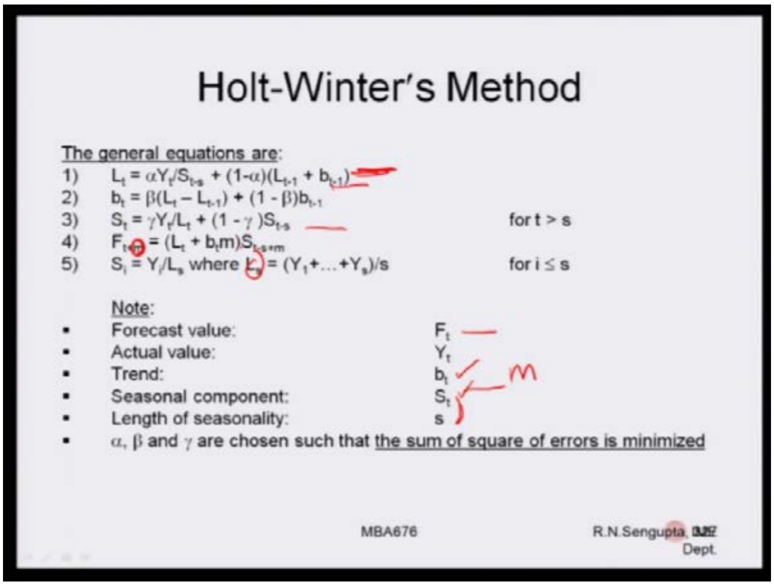

Now, going one step forward; the whole winter method basically considers that you are considering both the concept of seasonality and trend. Now, our question would be – how do we bring into the picture. Again, if you see the variables, forecasted value is F t, actual value is Y t; t is the suffix of the time. The trend is given by b as we have considered the Holt Linear method. And, there is a seasonality component also in the length of the seasonality. So, now, there are basically more than two concepts: one – you have basically the trends; and, for the trend, you basically have m as in the last slide. Another – you have the seasonality component and what is the time frame of the seasonality components. Say for example, if you consider the seasonality component for woolen garments to be 12 months; but, you want to basically predict it for 7 months; so, in this case, the seasonality time period S would be smallest would be 12. And, the m factor would be 7.

Say for example, it can be also the case whether S factor is 12; what you want to basically predict for 13 month; then, in that case, m would be 13. So, again the values of alpha, beta, gamma or whatever variables you are considering as parameters in order to predict and make a model; you will try to basically make the model accordingly such that again the same thing – some of the square of the errors would basically minimize; you basically minimize with respect to all the variables, which are their parameters – alpha, beta, gamma; put them to 0. Number of equations are there will basically find out the minimum of them such that the overall square of the errors is 0. So, again this is the same thing. You go step by step; find out basically the value of L; L is all already given here.

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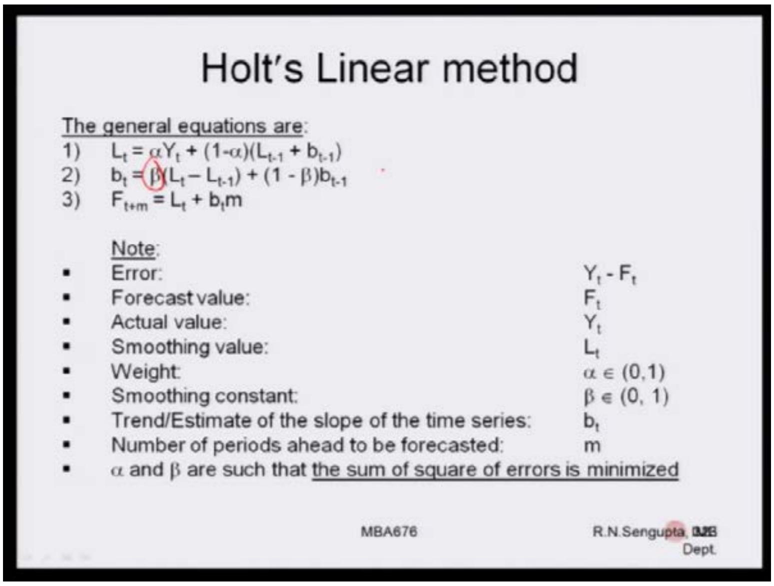

If you go back to the last slide, the L is basically the smoothening value, which you are trying to find out this L, which is there. This is being continued in the Holt Linear Winter method also. Then, b value with the suffix is the trend value; S is basically the seasonality component; F is basically the forecasted value. But, if you look it here, this is what is interesting. This is the m; this m value is exactly the m; standing today, how long you want to predict. So, this value of m is here. And, this value of S is basically the length of the seasonality you are trying to consider.

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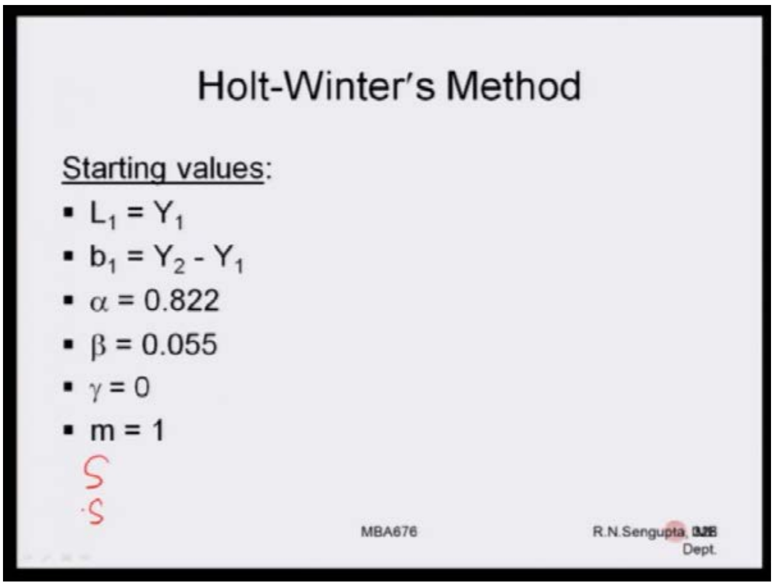

So, now, if we consider these values of L is equal to 1, beta is equal to Y 2 minus Y 1; alpha is equal to 0.82; beta is equal to 0.05; gamma is equal to 0; and, m is equal to 1. And, considering some S and m; and, small s and capital S values – again you can utilize the same data set in order to basically predict for the future and find out how good or bad your seasonality analysis is such that your prediction is done to the maximum possible extent. Having said that, this is not the end of the picture; we are just considering very simple trend analysis. And, that equations which are given there are available in any of the books. So, what I am urging the students would be – who are taking this course; and, which I will definitely put up in the assignments, which are to be done by the students by themselves – they would not be graded. So, obviously, there would be some questions later on in the midterm and the final term. What I will strongly urge the students to use this equation and I assume any values of alpha, beta, gamma, m and s; and, do it on an excel sheet such that they will get a hang of the picture how the whole process works.

So, with this, I will stop the forecasting method and start the in the next session the concept of option, futures; and, how option, futures are being heavily utilized in order to basically make your portfolio such that the concept of quantitative finance can be further into the case of option, futures, derivatives; forward, futures over and above the stocks, which you have already considered.

Thank you.