Quantitative Finance

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Module – 03

Lecture – 14

So, welcome back to this quantitative finance course. So, till now and in the last class, which we ended, we discussed about different other portfolio optimization problem. When one is being that, you want to basically have the geometric mean; another case you have the safety first principle or else you maximize the return of the portfolio or try to basically maximize r l; r l means some cut-off over which you definitely want your portfolio turns to be and so on and so forth.

Now, today's class or lecture would be more to do with trying to understand different forecasting or simple averaging techniques. And, they are generally used in order to find out the average prices on a long run considering there is fluctuations and how you try to basically do these averaging prices without going into the details of the time series. This is not the time series part of the quantitative finance course, which we did mention during the discussion of the syllabus. We will come to the time series part later on.

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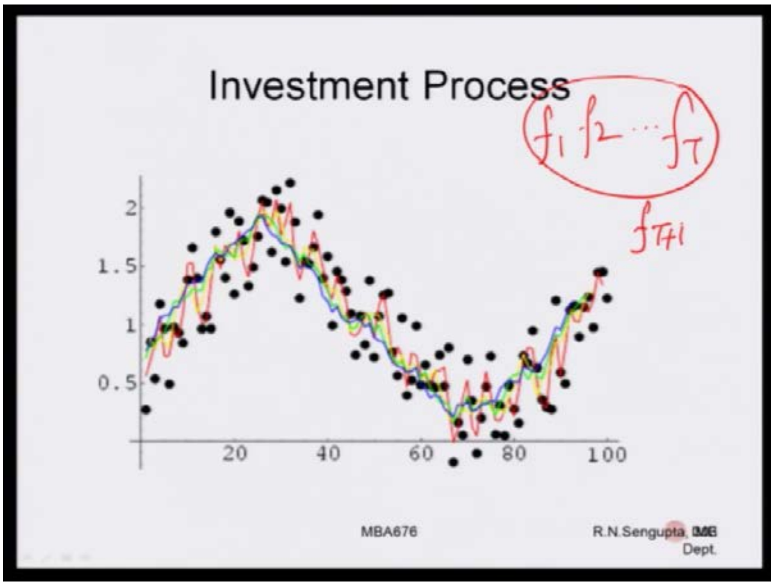

So, consider that you have this diagram, where the black dots are all the different type of actual values which you have. And, the green, blue or the red one are basically the best lines, which you are trying to basically fit in order to basically predict or forecast what would be the future values of the so-called prices or so-called trend analysis – trying to find out the trend analysis of different type of so-called time series values. Now, you should remember one thing that, along the x-axis, you have the times; that means, they are of unit intervals like it can be month, it can be days, it can be weeks, it can be seconds, whatever it is; along the y-axis, you basically have the values. So, what you actually would have is this. So, generally, you will have f 1, which will be the price of particular product at time period is equal to 1; f 2 – price of the same product of time is equal to 2 and so on and so forth. So, we have n – t number of such data points; they would be f 1, f 2, f 3, f 4, so on and so forth. So, what you want to do is that, you want to utilize the values of f 1 to f t in some way in order to basically predict the price of the value of that function nor of that value at f t plus 1. So, how it this done? We will see that in a very simple way trying to utilize the concept of averaging techniques.

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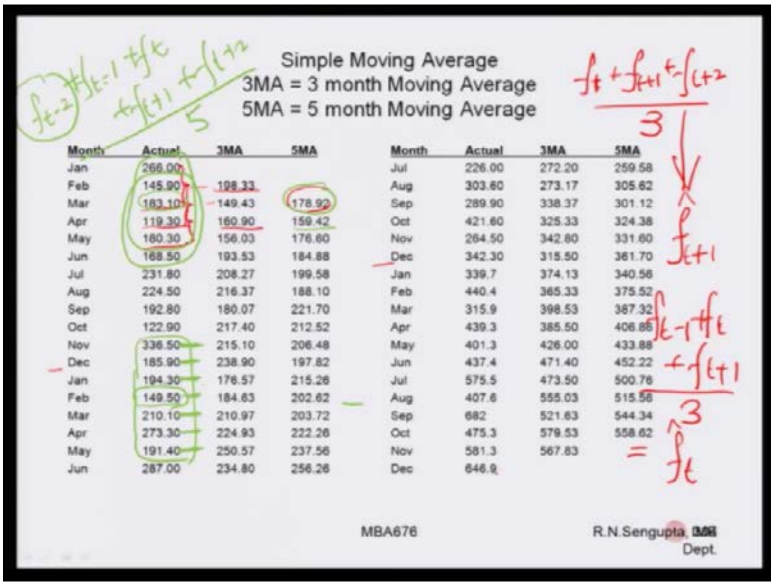

Let us go into directly into an example that will give you much more – more practical flavor of how you try to solve. Consider you have actual datas in values for month-wise it is given. So, what you have is month-wise values starting from January, February, March till December. And, these values are in either rupees or in dollars, in numbers, whatever it is; this does not matter. So, the values basically start at 266 for month of

January as you can see here. Then, it basically ends at 649, which is the December. So, what you actually have is January to December is 1 year, then January to December is second year and January to December is third year. So, three years datas are there. So, 3 into 12 – 36.

Now, let us consider the concept of odd moving averages. So, if you they are 3 month moving average, 5 month moving average; and, it can be made into 7 month, 9 month, 11 month, whatever it is. But, we will just simply consider the third and the fifth in order to understand how the odd moving averages are done. So, if you see, the actual 198 value, which is there is basically the averages of these three. Then, if you go to 149, it is basically the averages of these three values. And, as you move, what you are doing is that, you are trying to find out the average of f t; this is small t plus f t plus 1 plus f t plus 2 divided by 3. And, the value, which you get, would be the predicted value, which will denote by f hat for the time period t plus 1. So, in case rather than t, if you had say for example, f t minus 1 plus f t plus f t plus 1 divided by 3; then, it will basically give me the average value based on which you will try to predict for the value of f hat t. So, similarly, the value of 160.9 is basically the average of 183 plus 119.3 plus 180.3. Add them up; divide by 3. And, you basically proceed to step by step as you do that.

Now, consider the 5 month moving average. So, what you are now doing is for the 5 month, which is 178. So, let me use our different colors in order to make you understand it much better. For the value of 178.9, now, you are taking the averages of 5 values; the middle one being 183 and the two preceding and the two succeeding one once considered are these values. So, what you are doing? You are adding 5 values. So, technically, you would be adding f t minus 2 plus f t minus 1 plus f t plus f t plus 1 plus f t plus 2 divided by 5. So, f t minus 2 would be 266.00; f t minus 1 would be 149.90; f t would be 183.10 and so on and so forth.

So, now, if any proceed to 159, what you will do is that, you will just move 1 shift; that means, the f t minus 2 would now be 145.90. And, the last value would be now 168.5. So, you have 145.90, 183.10, 119.3 and 118.3 and 160.5. So, this will be 1, 2, 3, 4, 5. So, it correspondingly move. If you consider 7 values, it will be which I have not noted down, if you have the 7 values. Say for example, you want to find out the 7 value here; so, what you would do is that, you will take these three first; then, these three. So, this is the middle value. So, this would be f t minus 3, f t minus 2, f t minus 1. This is f t, this is f t plus 1, f t plus 2, f t plus 3. Add them up; how many values are there? 7 values; divide by 7. Similarly, you go for the ninth, eleventh, so on and so forth.

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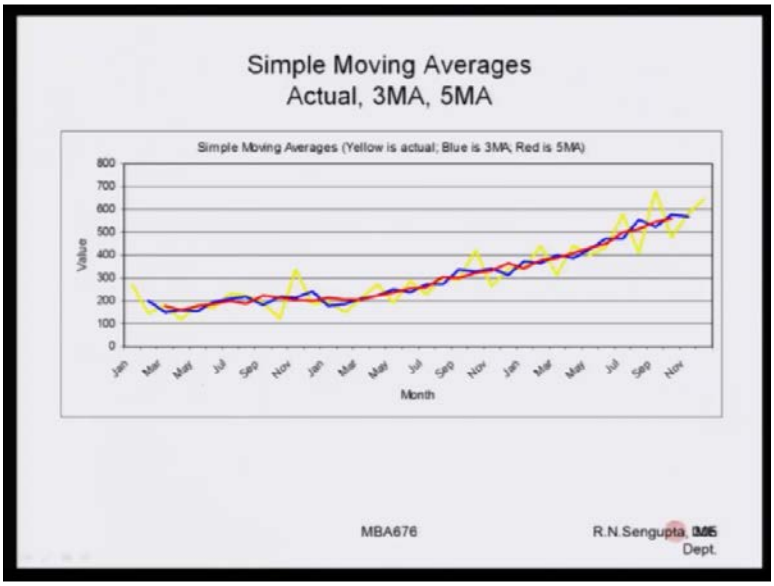

So, if I plot these values with time along with the x-axis from January to December or January to December or January to December, that is, 3 years; then, the yellow one is the actual one. So, you see there is a fluctuation in the yellow lines. So, I am not trying to basically market, because the color scheme then would be disturbed, then you cannot understand. The blue one is for the 3 month moving average. So, if you see the blue month, fluctuations are much less the yellow one, because what you are trying to do – you are trying to basically average out for the plus and minus fluctuation; and, divide by 3 would basically bring down the fluctuation to a quite get extent. Further on if you consider the red one, which is the 5 month one, you will see this is almost smoothened out. So, any positive, negative fluctuation, which you had in the actual value, which is the yellow one would not be shown or would not be apparent if you basically check the 3 month or later on on the fifth month or the 7 month moving average. But, what you are trying to find out is rather than predict the actual values; you are trying to basically find out the trend analysis based on which you are able to at least gorge and understand in which direction the prices are movement and how the process can be predicted using other methodologies; but, this moving value concept would give you a good data or a good hunch that, in which direction and what is the overall trend of that overall price movement or the value movement, whatever it is.

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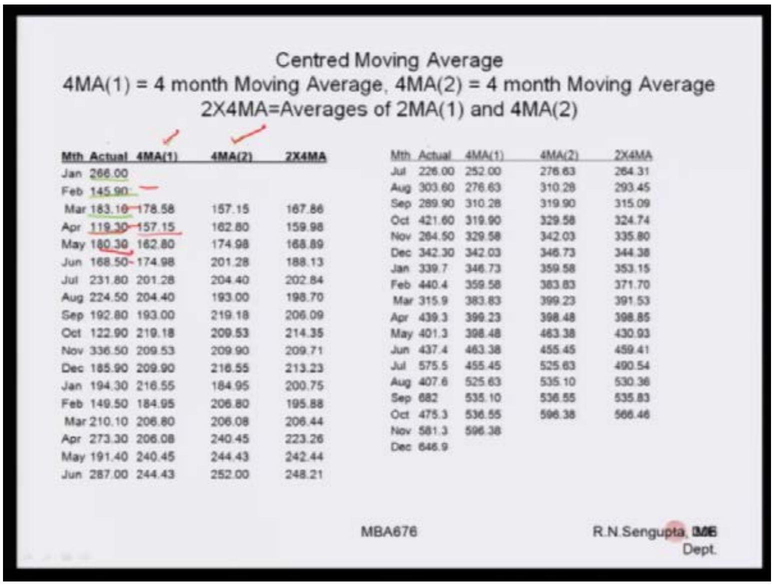

Now, let us go to the even numbers. So, again we consider the same data set – January to December, again January to December and January to December. So, there are basically 3 months. Now, if you see the second column; again is the actual value; here also is the actual value. But, now, the even number of movements is a little bit different. So, they will see it is 4 month 1 and 4 month 2. So, what you are doing is that, say for example, if you see the values; so, if you take the 4 month average; then, it will be either 1, 2, 3, 4; but, in this case, when you are trying to find out the 178.58 value; there would be two values before that. So, what you are doing is f t minus 2, f t minus 1, f t and f t plus 1. So, obviously, the skewness is more towards the upper values or towards the previous values. If you consider the 4 month second average, which is here; rather than taking, putting more weightages to the prior values, you will now basically put more weightage to the future one; which is, now, you will consider – let we take a different color – you will consider this as first, this as second, this as third and this as fourth. So, now, your shift has been more from the left; that means, left was this part, on to the right. So, once you have that, you find out the average is of this average. So, once you have the averages of this average, what you are trying to do is that, you are trying to basically equally weight any positive and negative movements. We will come to that concept within two minutes. And, that will give you a very good concept that, how the concept of normality would also be brought into the picture.

So, if you proceed in this one – if you have a 6 months one; so, what you will do is that, in the 6 month one, you will basically take one extra reading prior; and the second case, you take one extra reading post; add them up, divide by 2 and basically you will have the actual averages value accordingly. Similarly, if you find out the 160, 157 value; then, obviously, you are taking that 119.3 definitive value would be there. But, in the first case, 145.90 and 183.10 plus 180.3. So, what you will basically have? These four values – 1, 2, 3, 4, which is f t minus 2, f t minus 1, t, t plus 1; that is, for the 4 month moving average 1. When you go to 4 month moving average is 2; so, rather than take 145.9, you will just shift one step forward; which means f t minus 1, f t, f t plus 1 and f t plus 2 such that once you add up, divide by 2, they would be smoothened out. So, the more you do such averages, it will basically smoothen out considering any positive, negative fluctuation; both for the past data more down the line and the future data more down the line, you are basically try to give less weightages to such extreme such values, which are there from the center or else from the state from where you are trying to find out the averaging technique.

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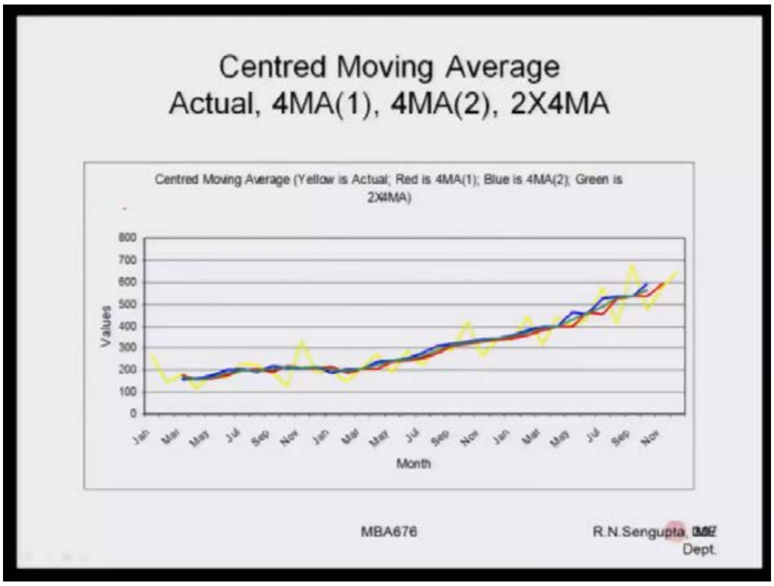

So, if I basically plot this graph for the case of the even month moving average – 4 month 1, 4 month 2 and 2 into 4 month average; again yellow is the actual one, the red and the blue one are the 4 month average 1 and 4 month average 2. So, obviously, you see there is some fluctuation, but this fluctuations are happening in such a way that, if you add up both these values, which is basically the red and the blue one; you will get a much more smoothed line, which is the green one, which will give you a much better prediction or forecasting for the overall fluctuation, which is happening for the actual data.

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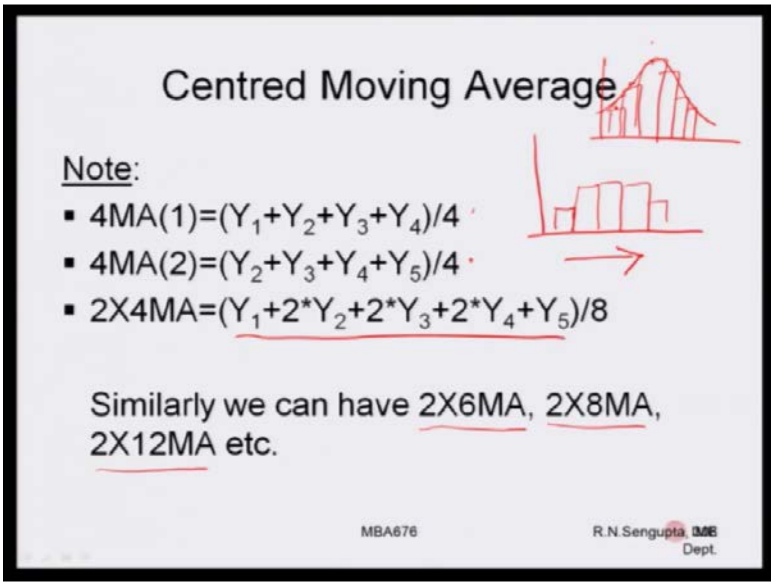

Now, consider the 4 month 1, 4 month 2 and an average technique. So, you say for example, you are trying to find out the averages of 4 units at a time. So, have a very careful look at this. You take the first value; consider Y 1 is the first value; first value, second value, third value, fourth value divided by 4. So, what you are doing is that, you are giving equal weightages of one-fourth of probabilities of one-fourth of each of the values, which is Y 1, Y 2, Y 3, Y 4. Now, if you go to the 4 month average 2, you see there is a slight shift onto the right; that means, you are going more towards the future and basically subtracting one value from the past. So, as you go one step forward, you delete one of the back values and then basically include the front value, which is exactly what is happening here. You take Y 2, Y 3, Y 4. So, as you have subtracted Y 1, you take Y 5. So, again you add them, divide by 4.

Now, in the next step, this total process is not complete. In the next step, for the moving average even, what you do is that, you add up 4MA, which is moving average 4 step 1 and then you add up with moving average 4 step 2, this is step 1, step 2; add them, divide by 2. So, once you divide by 2, what you actually have here – dividing by 8. But, look at a very interesting value; Y 1 is being only added once. So, it will be Y 1 divided by 8;

that means, you are putting one-eighth of its probability to the Y 1 value. Now, consider Y 2; Y 2 has happened twice. So, what you basically do is that, you are trying to give one-fourth of the weightages to Y 2; that means, which is 2 by 8. Similarly, if you find out Y 3, it is twice Y 4, it is twice. Overall probability for Y 3 and Y 4 is 2 by 8, which is one-fourth. And, again at the last end, when you add up Y 5 for the step 2, you divide by one-eighth. So, the overall probability is one-eighth. Similarly, you can have twice into 6 month moving average, twice into 8 month moving average, twice into 12 month moving average depending on what you do.

Now, what is important is these; not Y's, but the basically of the coefficient. See if you look at Y 1, the coefficient is 1 by 8-th; if you look at Y 2, the coefficient is 2 by 8-th; if you look at Y 3, the coefficient is 2 by 8-th; Y 4 – again coefficient is 2 by 8-th; and, Y 5 is 1 by 8-th. Now, if you draw a histogram, where along the y-axis, you basically plot the Y values; and, along the y-axis, you basically plot the overall relative frequency. If you plot the relative frequency Y 1, it is 1 by 8; that means, out of the 8 such readings, you are basically putting one-eighth or 1 by 8 into 100 overall weightages percentage-wise on Y 1. For Y 2, Y 3, Y 4, it is just the double. And, again Y 5 is exactly equal to Y 1. And, if you continue doing it, you will see a very interesting trend.

Consider that you take say for example, rather than 6, you take 8 and do 2-stepwise averages or 3-stepwise averages. So, if you do that, in the long run, what you are actually aiming is to put the weightages depending on a very simple histogram. Now, keep doing it. The more you do, more smooth then the histogram would be in the sense that, if you plot the mid points of the histogram, they will slowly turn out to be the normal distribution; which means that you are considering a priori the intrinsic distribution based on which you are trying to find out is normal such that the average value or the middle value, where you are trying to find out the averages; that is, the mean or the median or the mode for the normal distribution. And, more further down, that means more to the back or move to the front you go, you are basically giving less and less weightages, less and less linearity weightages to the overall values, which you are considering, which is right, because as you are standing today, you will try to basically give the maximum weightages for any past value, any futures value. Obviously, you will consider them, but consider them to a less probability sense.

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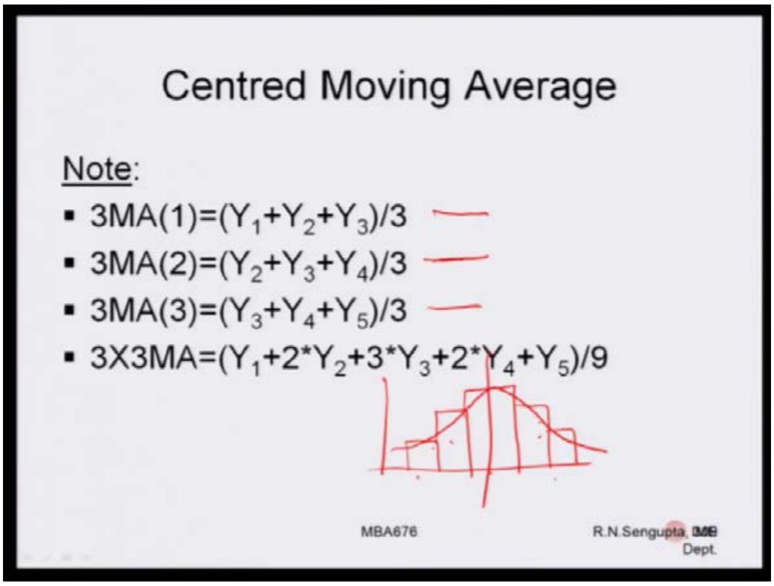

Now, consider the Y 3. But, we are basically considering the Y 3 in such a way that, we are also taking the averages of these also. Very simple; just you are trying to now extend this concept, where the concept of probability using the histogram will come out very smoothly. Consider the… When you are trying to find out Y 3, you add up Y 1, Y 2, Y 3; the step 1; you divide by 3. Obviously, there are three values; divide by 3. So, if I ask you a question, what is the corresponding probabilities for Y 1, Y 2, Y 3? You will immediately say the probabilities are one-third; or, means out of the 3, you are putting one-third or 33.33 percent weightages on Y 1; similarly, on Y 2; similarly, on Y 3. Now, move one step forward; that means, you delete Y 1 and include Y 4. So, if you see in the step 2, what you have is Y 2, Y 3, Y 4 divided by 3. So, now, you are putting for this step 2, 33.33 percentage on Y 2, Y 3 and Y 4 respectively.

Again, move one step forward. Delete Y 2, include Y 5. Add them up. What you are adding? Y 3, Y 4, Y 5 divided by 3. So, if you find out, again the overall percentage, which you are putting on the weights for Y 3, Y 4, Y 5 are all equal and all equal to 33.13. But, the interesting fact is that, now you have three terms: step 1 term, step 2 term, step 3 term. So, if you want to find out the average, what you will do? Add them up, divide by 3. So, exactly, that is what I do. So, once you do that, you find out the weightages, which is coming out for Y 1 is one-ninth. Weightages is coming out for Y 2 is 2 by 9-th; weightages, which you are giving for Y 3 is 3 by 9-th; weightages for Y 4 is 2 by 9-th, and weightages for Y 5 is 1 by 9-th. So, if you plot the histogram again, along the x-axis, you basically have the y values along the y-axis. You have the corresponding relative frequency or the probabilities; then, you find out Y 1 has a height of one-ninth; Y 2 has a height of two-ninth; Y 3 has a height of three-ninth; Y 4 has a height of two-ninth; Y 5 has a height of one-ninth. So, if you continue doing this, slowly again as mentioned, you will slowly replicate the normal distribution, which will basically go into the intrinsic assumption, which is absolutely right in the case that, the more you are centrally located, obviously, you will get more weightages to those values if you have the past data in front of you.

And, for any abarasions in the sense that, you move backward and forward, you will basically give less weightages. So, further you go down from the actual state, where you are in, you will give less and less weightages; which means that, if you are standing on Y 3, the overall weightages you are putting for Y 1 and Y 5 is one-ninth. But, closer you come to Y 3, the weightages you are putting on Y 2 and Y 4 has increased from one-ninth to two-ninth. So, obviously, when you are at the position Y 3, you will give the overall weightages to the maximum, which is 3 by 9-th.

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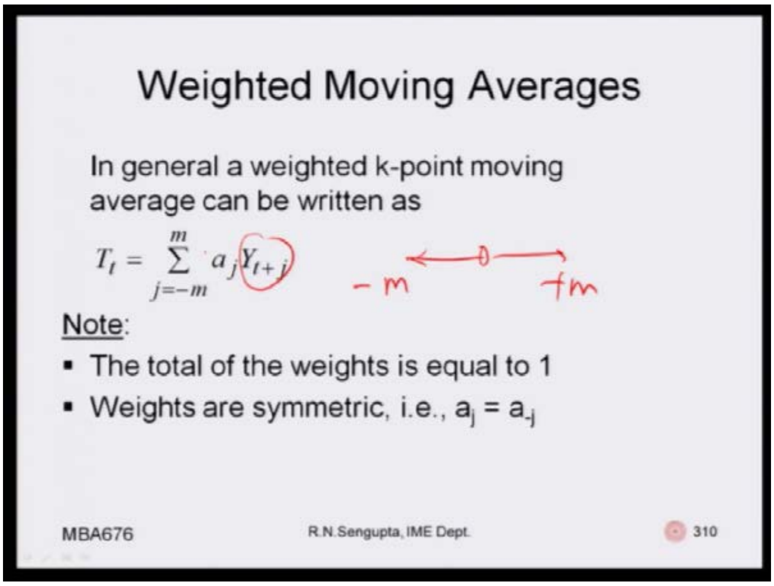

So, in general, a weighted k moving averages can be written by this. So, now, we are trying to give some weightages – weightages on the probability. So, if we have say for example, standing at zero-th level and you go to minus m on to the left and plus m to the right, what you do is that, you add up all these y values. But, when you are adding them up, you are putting weightages. So, from where do I find out the weightages? As just discussed in the last slide, the values of one-ninth, two-ninth, again three-ninth, again two-ninth and one-ninth corresponding to Y 1, Y 2, Y 3, Y 4, Y 5 are the corresponding weightages, which you have. And, obviously, remember another thing – if you go back, if you add up all the relative frequencies of the probabilities, what you have? One-ninth plus two-ninth plus three-ninth plus two-ninth plus one-ninth. It comes out to be basically 9 by 9, which is 1, which is intrinsically the corresponding probability such that when you add up the probability, it should definitely add up to 1. Remember – the total weights equal to 1 and weights are symmetric. So, if it is symmetric, it basically gives you an intrinsic feel that, the overall distribution you are going to consider for this moving averages is basically the probability distribution, which is normal.

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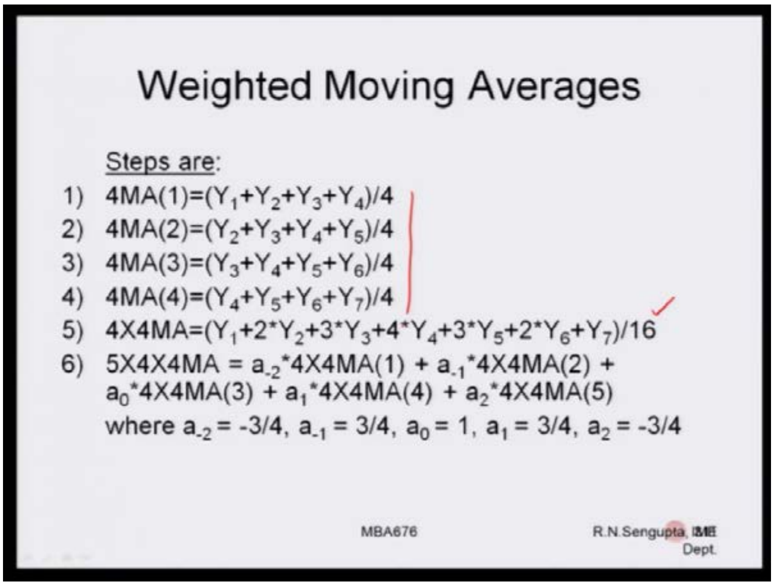

So, consider the moving averages and steps. I am basically now expanding them; considering the 4 month moving averages, but I am taking basically 4 different steps. For the first one, it is Y 1, Y 2, Y 3, Y 4 divided by 4, because there are four values. For the second step, you basically delete Y 1, include Y 5. Again, it is Y 2, Y 3, Y 4, Y 5 added up, divided by 4. And, the next step, which is the third point, which is written here. Now, we have again moved one step forward; that means, initially, I have deleted Y 1. Now, you are deleting Y 2. So, as you delete 2, obviously, values you basically will take two values, which are basically Y 6 and Y 5 are added.

So, now, in the third step, values are Y 3, Y 4, Y 5, Y 6 added up and divide by 4. Similarly, for the next step, it is Y 4, Y 5, Y 6, Y 7 divide by 4. Now, how many such times do you take Y 4 – average of 4 times. So, what you will do? You will add them up, divide by 4; so, that means, the overall denominator will become now 4 into 4 – 16, which is exactly what is it. But, now, it will become more and more apparent – what are the weightages. Y 1 is only appearing once. So, overall weightages – 1 by 16; Y 2 is appearing twice; so, it is 2 by 16. Y 3 is 3 by 16; Y 4 is 4 by 16. Again, it starts decreasing; 3 by 16, 2 by 16, 1 by 16. Add up all the probabilities – 1 plus 2 plus 3 plus 4 plus 3 plus 2 plus 1 divided by 16 comes out to be 1 as mentioned in the last slide, where the weights add up to 1.

And also, see is the histogram – very simply. If you draw, the histogram has the heights. So, 1 by 16 in the first left-most corner and the right-most corner is symmetric; again height of 1 by 16. And, in between, you have heights of 2 by 16 here, 2 by 16 there; 3 by 16 here, 3 by 16 there. And, the middle value is the highest, which is basically mean median mode, whatever you say for the normal distribution; and the weightage is 4 by 16. Similarly, you can have basically 5 method for other different time moving averages, which you have basically much more intricate, much more nice depending on what you think would be the overall weightages. Assume – basically go further down in the back or further down basically in the front; which means standing today, you are trying to take values from the past and from the future. And, depending on that, you will basically give weightages accordingly.