Quantitative Finance

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Module – 03

Lecture – 13

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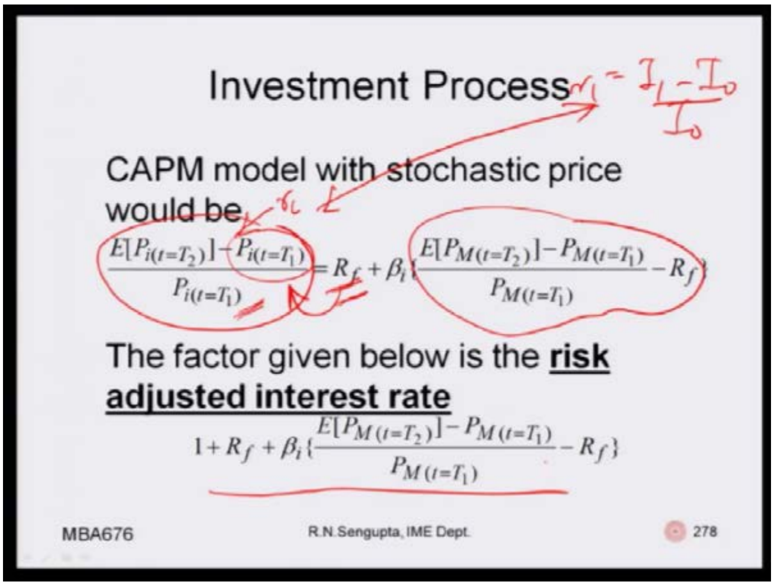

CAPM model with stochastic price, so; obviously, prices are stochastic means they are non deterministic and a time chapter also come. So, this is the certain different between, what is the probabilistic and what is stochastic and probabilistic is basically simply that it is non deterministic, but the values, which are there in front of first like when you do when you role that I if the values can come out to be1, 2, 3, 4, 5, 6.

But, in the stochastic sense of time factorial also come into the picture in that sense in the time would also pay clear major role in time to find out, what is the outcome depending on, what type of decisions, which we are whether their stochastic are probabilistic. So, CAPM model, which stochastic lower price would mean if you look at this again is exactly the same thing, here you had basically the expected value of the price at time t is equal to 2, which is some time down the future.

And then, the next term, which you have on in the numerate basically the prices of today. So, what we are trying to find out is basically you trying to find out in some way the or if you remember arrive was basically equal to I 1 minus I 0 by I 0. So, exactly the same thing, which is been done here and on the right hand side again you have risk interested. So, this risk interested was brought here it will with access return or that particular portfolio over the risk interested.

So, just for to avoid confusion the values whether I use the symbol small r or capital R they are interchangeable. So, in this case if are trying to find out r i small r i; obviously, this should also be small r I, if I am using the concept of capital R i then; obviously, this should also be capital R f. And on right hand side again you see the same thing beta is there, which is basically a proxy of the risk and again in the numerated and the denominator is same values are there this is the excess return of a particular market overall number of risk we interested.

So, we are trying to basically r simply between that considering the stochastic price and try to basically analyze the price change with the market with respect to the portfolio of the particular index. The factor given below, which you have here is basically the risk adjusted interested, so because that would basically we adjusted in such a way there in increase and decrease the prices would be mimic by utilizing this concept.

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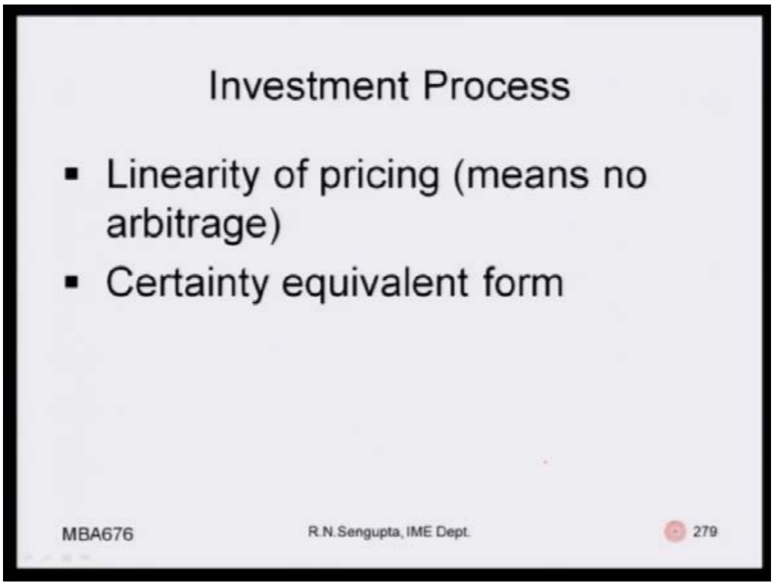

So, when we have investment in the concept of investment of the portfolio if you are linearity to the pricing it means there is no arbitrage, what is arbitrages how you make profit in a arbitrage, why arbitrage is used rather than basically trying to bring a parading in your risk return profile are trying to basically reduce your disk, that would also consider later on. And certainly equivalent form would also be considered where it will give you a certain value based on which, you can find out, what is the value of a particular proxy.

If you consider are you remember you the utility concept, what we consider the expected value of the utility given some value. So, what you do is the repress that probabilistic or the ((Refer Time: 03:18)) or the lottery with the deterministic base and the value of the deterministic based on your utility function would basically be consider a certainty equivalent based on the fact the lottery, which you are trying to formulate for the certainty equivalent has a probability of one.

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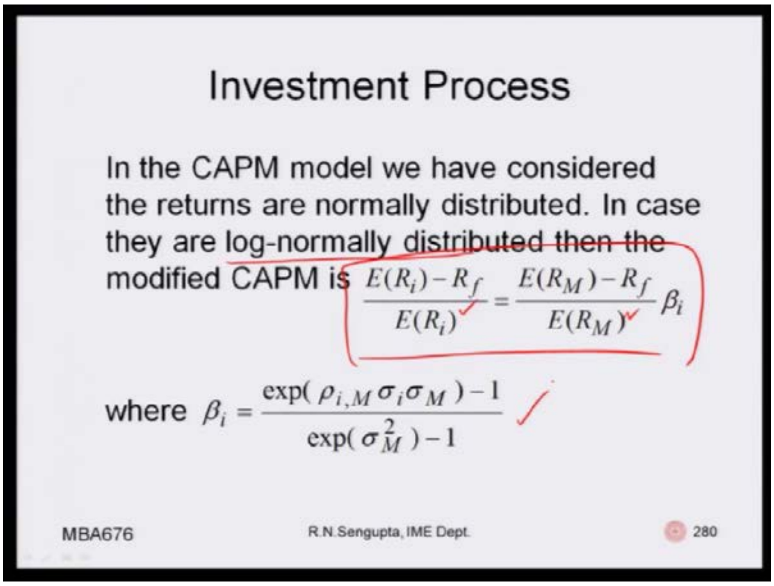

In the CAPM model we have considered the retunes are normally distributed, which means the utilities based on which, you trying to find out the returns or quadratic in nature in case they are log normally distributed, then the modified CAPM model is given. So, till now, what we are consider for the CAPM model if you remember the same index model we mention time and again the distributions of R i's R M's are R are x whatever it is you trying to denote as the set of random variables, which are dependent or independent on which, ever it is they had normal distribution.

And also if we recollect I did mention normal distributions as some properties as that convict combination on normal distribution would always result in a normal distribution, but incase if the a log normally distributed, then you gave the CAPM model as given. So, again it as the same replication, but in this case you can find out; that is basically the access return being given; such that you can find out access return with respect to it is own particularly term, which is E R i and E R M.

And, where beta very find out initially is basically given by this formula considering the log normal case. You would not be doing any problem exactly with the beta, what you found out, but will consider later on a or a betas can be found out for the simple normal distribution.

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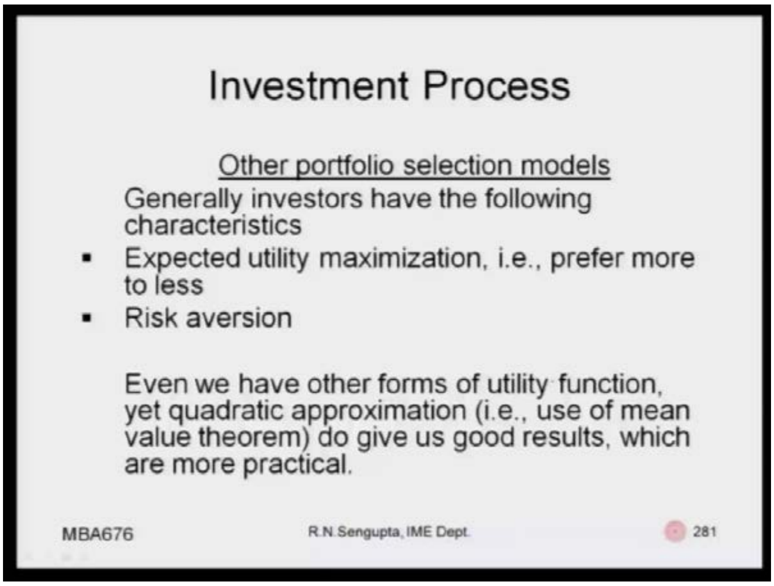

Now, will consider other portfolio selection models, so generally invertors have the falling characteristics whenever investing till now, we are consider concept of non sensation have a more I give you more you want and you are risk hours. So, now, will consider the expect, a utility maximization is to; that means, you will prepare more to less, which is also equal to the concept of non satiation.

But, now we are considering the concept of utility functions, utility functions would be consider and you will find out the expected value accordingly. And also funs consider the concept of only risk aversion later on will combine both expected value along with this risk aversion excepted value along with standard deviation expected value along with beta whichever it is depending on how the rick return profile need to be looked out and, what is the actual the concept of risk based on which, a particular person you trying to analyze is her portfolio.

Even if we are other forms you utility function yet quadratic approximation gives as better practical results, then in the case of when non quadratic utility functions are used. But, the point of better results has to be taken with the pinch salt in the sense, that when you are trying to analyze the Newton's laws of motion we always consider, then to be true in the sense that no friction is consider.

But, still we get very good results is exactly the same thing will consider normality be true when even though it is not true will still will see the results, which will obtain consider the normally will to be true for the retunes; that is considering the utility function to be quadratic still will get very good results. In order to basically be able to understand the practical situation, what is happening in the market later on will try to at the ((Refer Time: 06:43)) course will consider the concept of extreme value distribution will try to analyze that actually the portfolio use actually the indicates actually the options features for whatever there when combine together.

When you find try to find out the retuned they are the scud ether to the left or to the right depending on what you are trying to analyze, where is a positive return or a loss; such that those expected distributed and the if it is are extreme value distribution we try to utilize would be use in such a way they would give as much better results in the practical sense without losing the favor that have normality did give as a good picture about reality even though that was not true.

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People have other criteria also as I mentioned in the last night. So, what are the concepts people can basic analyze for the geometric mean returns they can analyze with this safety first criteria, they can analyze the problem can stochastic dominance, they can analyze in terms of characteristics of the return distributions are also.

So, rather than or for the last point rather than only considering the expected value and the variants you can also consider the excepted value and the say for example, the third moment or the second moment and the third moment or the third moment and the forth moment whereas the third moment and the forth moment are basically ((Refer Time: 08:05)) corresponding to particular distribution.

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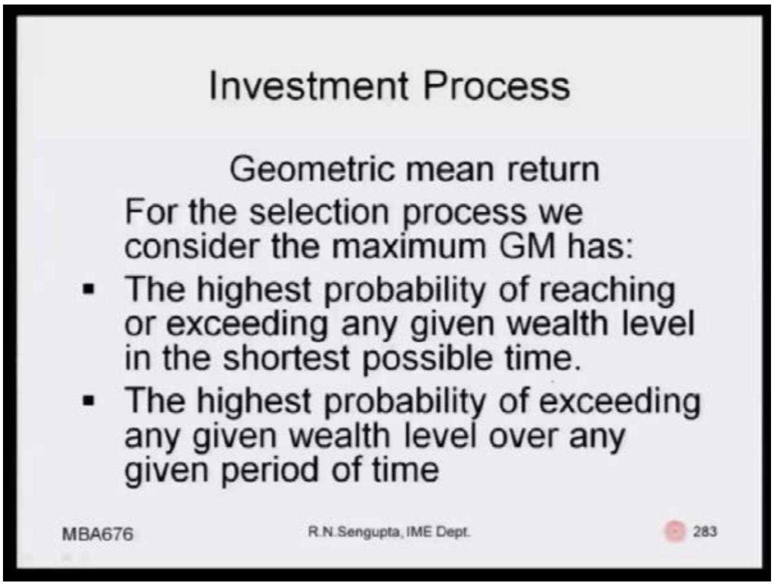

For the geometric mean returns for the selection process we consider the maximum GM; that means, geomantic mean. So, the highest probability of reaching or exceeding any given wealth level in the shortest possible time, that will be consider and the highest probability of exceeding any given wealth level over any given period of time. So, if these both concept of true, then will find out the geometric mean gives as mush better result than the arithmetic mean, arithmetic means the expected value or any other criteria based on which, we are trying to and an as the particular portfolio.

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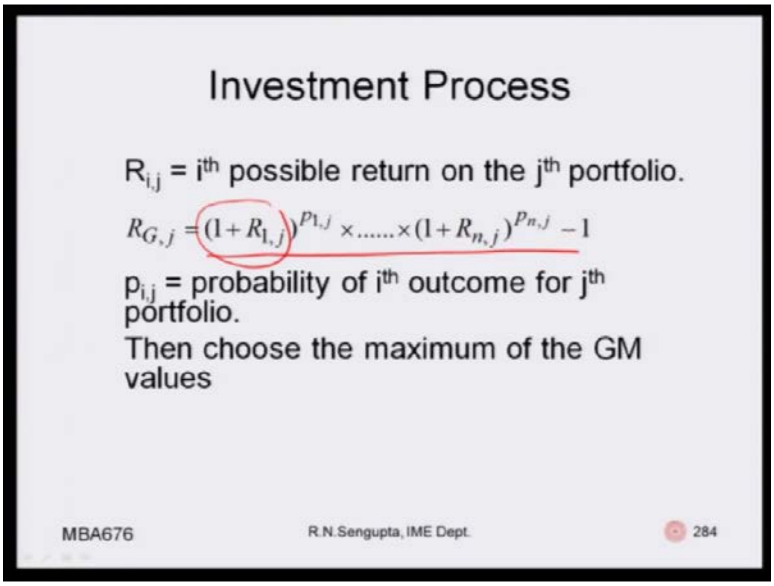

Now, in the geometric mean concept consider this very simply you have the retunes, so this capital R or small r is in martially you can consider the returns as it is. So, they are not two suffix in the suffix, which you have is i and j, which i is the possible return of the j th portfolio. So, what you have is basically the portfolios are given as well as the outcome as the portfolio, which you have which can basically we respect to time. So, rather than i you can have t th time frame also, which is basically a counter.

So, what you do is that you try to find out that, how many some such times that actual outcome as been true. So, if you consider the probabilities are given; that is p 1 j, p 2 j so on and so forth. Till p and j conserving the fact the sum the probabilities are always one then the geometric mean is given GM is given by this formula, where you basically find out the returns 1 plus R means basically you had some value of a and that value as a is increasing by the term a equal to 1 plus R in the simple concept. And then, there are being raced of the power of the probabilities they are.

So, this exactly the same concept as we used a geomantic mean minus 1, because you trying to find out you know the value of the access of the portfolio. So, p i is the probability of i th come for the j th portfolio. Then, once you find out the values of the GM you basically chose the maximum geomantic means based on which, you take your decision.

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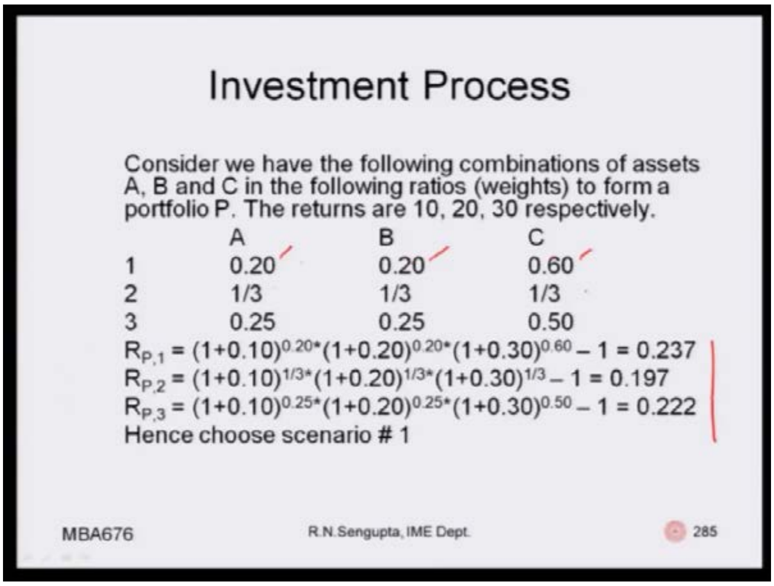

Consider we have the following combination of assets A, B, C in the following ratios or the weights. So, ratios are basically w I, returns are 10, 20, 30 and for A, B and C the first second third column gives you the situation. So, if you find out p R p 1 which is for the first instant, R p 2 for the second instant, which are the rows and R p 3 for the third instant; obviously, you will have the returns given as 10, 20, 30, which we are considering as fixed; it can change depending on the situation.

So, in the first case it will be to the power of 0.2, B would be 0.2 and C would be 0.6; such that if you see the probability sum up to 1. For the third case, the powers of one third, one third, one third again the sum has to be one. So, we find out this geometric mean, then the value comes out to be highest for the first case; hence we use the scenario 1, which will give the highest return, considering geometric mean is your main criteria based on which you take a decision to invest in the portfolio.

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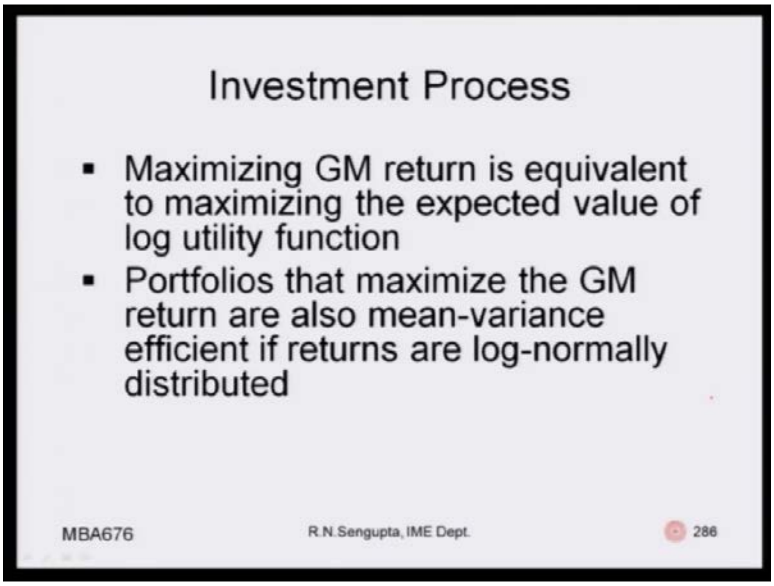

Maximizing geometric mean is equivalent to maximizing the expected value of the log utility functions. So, now, going back, we said utility function mean quadratic you had normal distribution, now we consider that you are trying to basically have the log utility function to be true, then trying to find out the geometric mean and trying to maximize would give you the same answer. So, this is basically in a very simple term, this one to one correspondence between quadratic utility to normal returns and log utility to the concept of geometric mean to be true, based on which you can find out the excess return.

Portfolios that maximize the geometric mean returns are also the mean variance efficient if returns are normally distributed. So, if you consider that in one case, you are considering the risk return frame work and trying to take the maximum return and the minimum risk and on the other case, if you have basically a log normally distributed returns and then you find out basically the geometric mean and basically maximize the geometric mean and take that portfolio.

So, the answer which you get in the second case where you take the log normal return and the answer you get in the case when you consider the utility function to be quadratic, they would be almost a one to one similarly between them. So, whichever concept you follow you will; obviously, get the same answer in trying to rank the portfolios depending on different criteria's.

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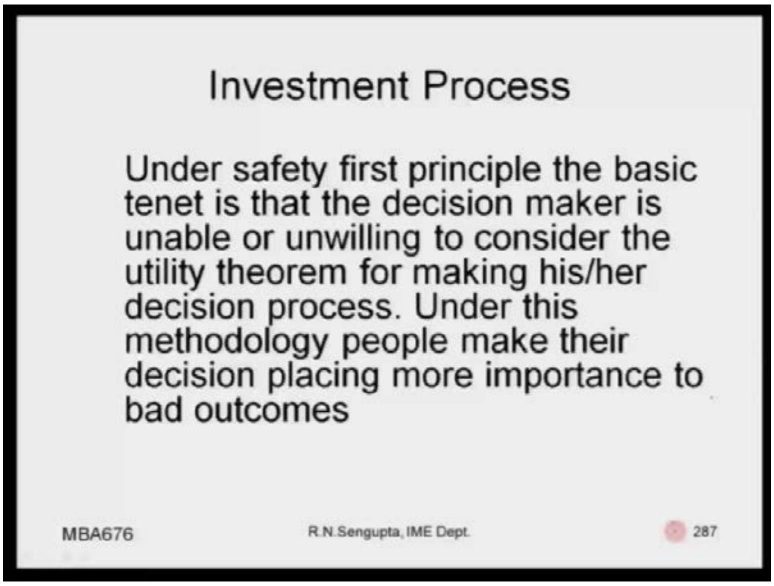

Now, the second one will consider after geometric mean is the safety first principle, the basic tenet is that the decision maker is unable or unwilling to consider the utility theorem for making his or her decision process; that means the person is not sure about his or her utility function or you are not, particularly not sure about the utility function of that person and you will take the concept of safety first principle in order to basically analyze the outcome based on which you are going to take the decision.

Under this methodology, people make their decision placing more importance to bad outcome; that means, words it is you will significant more bit, it is like this. If I get good results I am happy, but if I get very bad results, my over all tension level is very high; that means, I would basically lay more stress on negative returns rather than being happy on the positive return.

So; obviously, I will try to analyze the situation, will mean it case I would definitely not want any negative returns, but if positive returns come I would be happy, but not that mean related in the sense as I am very sad as in the case when the negative returns are possible.

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So, the safety first principle consist three different criteria's, sub criteria's. One is basically you minimize the probability of the portfolio return being less than equal to some stipulated returns which is R L, first case. Another case is basically, you maximize the level of criteria based on which you are trying to invest which is R L, another third case is basically very maximize the R P, so the return of the portfolio.

So, you should remember something that the value of R L and the value of R P, which we are trying to analyze, are not the same. So, R P may be the actual value of the portfolio which you have, but R L we mean the value of the portfolio which you want that portfolio to reach. So, say for example, you want your overall portfolio for the number of stocks which you are trying to analyze. To be say for example, 20 person, but your portfolio value can be 15 percent.

So, in whatever case you are trying to analyze, it may be either you want to basically increase the return on the portfolio or increase the overall threshold value R L over which you would like the portfolio value to. So, these are two different situations where you are trying to analyze, the same situation in two different cases or trying to look at it at two different angles.

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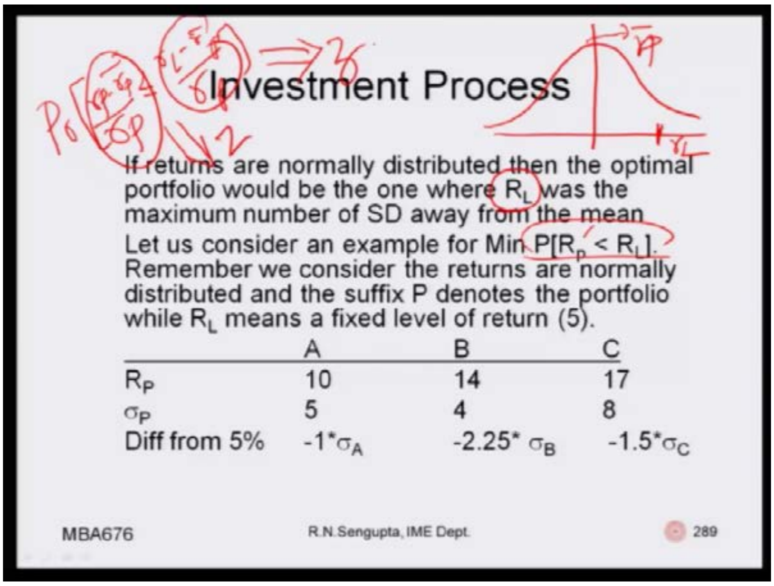

So, if returns a normally distributed than the optimum portfolio would be the one, where R L was the maximum number of standard deviations away from the mean. So, let us consider an example of the first case which you are trying to minimize R P less than equal to R L. Remember, we consider the returns are normally distributed with the suffix P demoting the portfolio, while R L means the fixed value of the return.

So, what you are trying to do is like this, you have returns, which are normally distributed. Remember, it is normal and you have the average value of the portfolio. So, this average value of the portfolio is given by R P bar. Again, small r capital r does not matter here. So, what you have done is that you have fixed some value of R L, this is the R L which you have done and this random variable R P is such that you are trying to basically find out how far it is from the R L value in terms of standard deviation.

So, if we expand this formula using random variable to be normal, what you are doing is this; probability of r p minus r p bar by standard deviation is less than equal to r n minus r p bar by standard deviation of p, which means you are trying to convert those values in to a standard deviate or a standard normal deviate case, where this is the random variable z and this is the realized value of z depending on what is your level of significant or the number of standard deviation far away it is from the actual case.

So, consider this R P is 10, 14 and 7 and standard deviations or the portfolios are given at 5, 4 and 8, then the difference with from 5 percents are given by one standard deviation 2.25 and 1.5, which means that how far the dispersion is. If the overall dispersion is 2, so; that means, you are going plus and minus 1 to the right and the left of the mean value. If it is basically 4, you are going plus and minus 2 value on to the right of the left of average value R P.

Similarly, if you plus minus 6 will basically have plus 3 and minus 3, depending on which side you are going. Now, the level of the difference which is 2 sigma, 4 sigma, 6 sigma would basically give us that concept; such that you can find out what is the overall dispersion. So, if it is 2 sigma, overall dispersion is average. I am giving you very average value, it is 67 percent; if it is basically 4 sigma, it is about 97 percent and if it is 6 sigma, it is about 99 percent.

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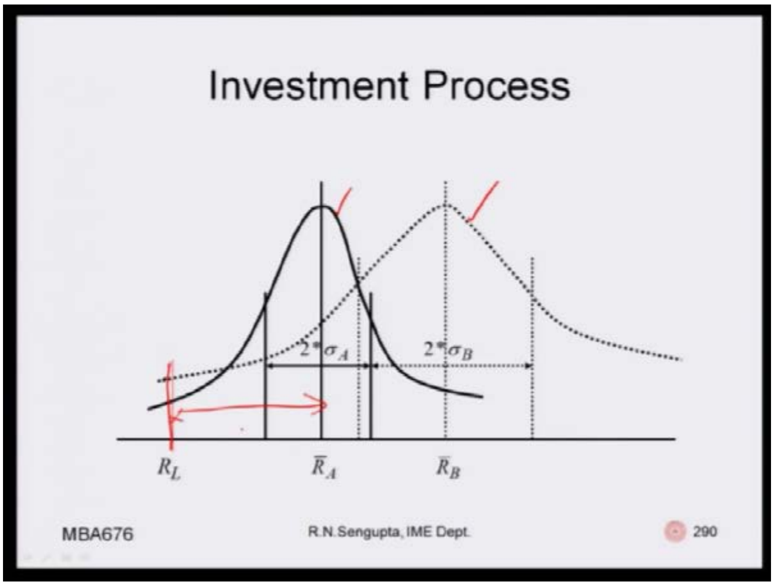

So, it means that if we have this graph and consider the R L value is given. So, and your actual distribution is given by this R by this, whichever it is I will consider 1 at a time. So, in the first case you have with a bold normal distribution returns are there, which is the bell shape curve. So, this is basically removed from R L, this average value by this distance, but what you are trying to do is that you are trying to basically minimize or maximize the differences depending on which side of the graph it is.

So, if you go more on to the right, then the corresponding problem to the differences; obviously, will increase which you want. If it is on to left, you would try to basically bring it is R P value as close as possible to R L; such that it minimize. So, in this case depending on the standard deviation and the expected value, standard deviation would basically given by the dispersion on the left and the right hand side.

So, what you are trying to do is that, you are trying to basically either find out the ratios in such a way that the value of R P with respect to R L is either maximize or minimize, depending on which side of the curve it is; such that you will try to basically gain the maximum amount of probability; such that the returns are increased to the maximum possible extent; such that you are more concern about negative returns and less concern or less happier about basically positive returns.

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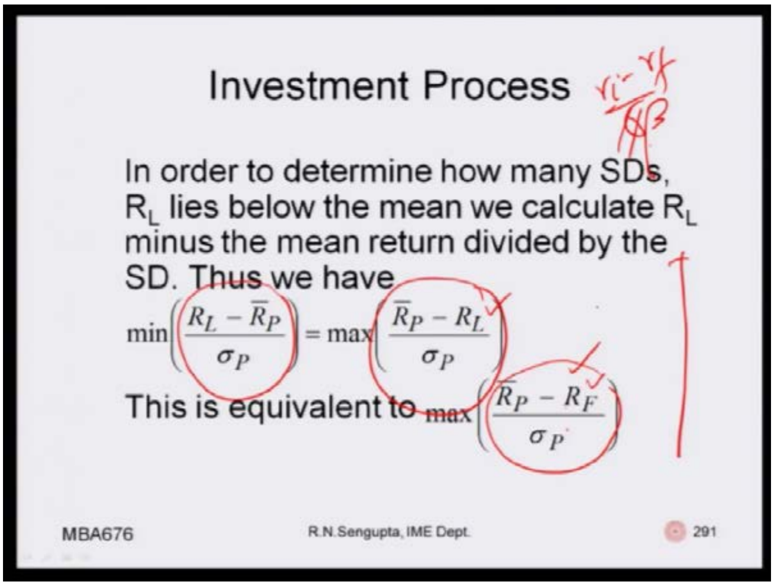

In order to determine how many standard deviations R L lies below the mean we calculate R L minus the mean and then, once you find out; you basically find out what is the ratio. So, if you consider this value, this R L minus R P divided by standard deviation or we change the sign, it becomes R P minus R L divided by standard deviation and this is equivalent to exactly this. So, what you are trying to do, you fix of some R L value.

So, now, you will ask what is the R L value, let us consider it is the risk interested. It can be risk interested from delta value also, that does not matter. So, what you are trying to do, you are trying to basically analyze the portfolio, rank them according to the principle of excess of that particular portfolio with respect to the risk interested, divide by the standard deviation with risk and rank them from the positive ((Refer Time: 19:42)) to the highest to the lowest.

If you go back to the principle which you have already done, it was ether R I minus R F by standard deviation or standard deviation being replaced by beta. So, whichever you look, now this, our different perspective of this ranking SD return to risk interested divided by sigma is basically a concept of safety first principle, where you will consider that if it is normal distribution, it is easier for you to analyze the problem in a simple sense, where you use the standard normal deviate and the tables and the charts to find out these values.

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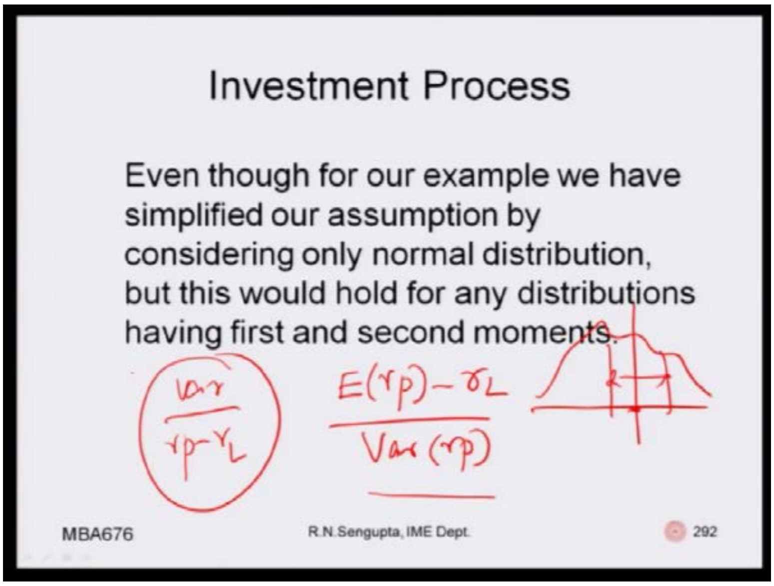

Even though for an example, we have simplified assumption by considering only normal distribution, but this would hold through for any distributions, depending on that we have the first and the second moment, because the first moment is the expected value, second moment is the variances. What we are trying to do is, find out the expected value minus some R L divided by the variance of R P which is the second moment and rank them accordingly.

So, if you have rather than normal distribution if you had a graph of this, so this is the mean value and this for the standard deviation and the variance, we will basically find out the ranking based on the same fact that what is the excess return divided by the risk and rank them from the highest to the lowest. Another think can be that rather than trying to rank them from the highest to the lowest, it can be done the other way around also, but the question is that how would you do that.

In that case, rather than take the excess return to the over the risky interested and divide by the value of standard deviation, what you will do is that, will rank the ratio of the variance with the respective value of excess return. So, now, we are trying to basically put the risk on the numerator and the return on the denominator and then, rank them from the minimum to the maximum and take the minimum value.

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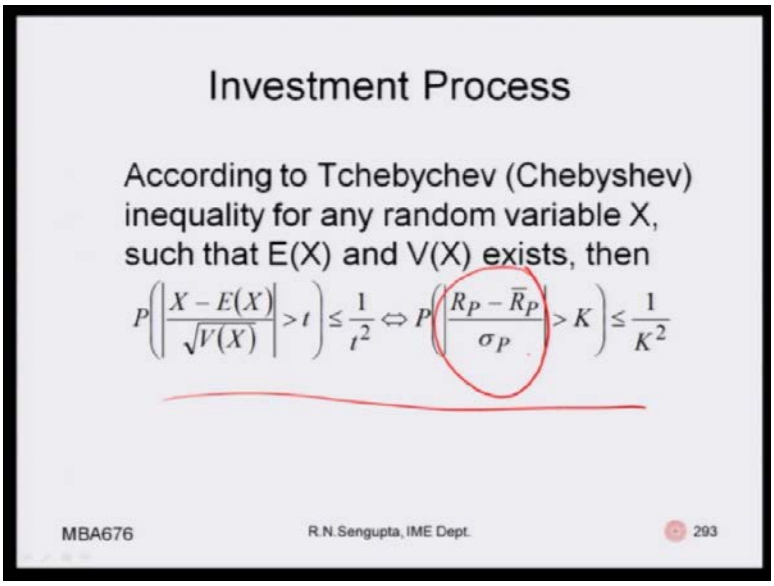

So, now, if you can recollect, in statistics we have the Chebyshev inequality. The Chebyshev equality basically means that given the two moments the expected value and the variances, then the difference of the e random variable with respect to it is expected value divide by the standard deviation. If it is greater than some t value, whatever the t is, then the probability is given by this formula.

So, if we look at this it is exactly the same thing, what you are trying to find out is, you are trying to find out the excess value of the particular stock random variable which is the expected value divided by the standard deviation and that value is greater than some

K value, depending on what your safety value is. So, if your safety value is very high or very low, then the probabilities have to be adjusted accordingly.

So, if the probability file is very high it will give you a picture that whether you are really conservative and the probability value very low; obviously, it mean that you are not the correct conservative; that means, your safety level based on which you will invest would change from person to person, depending on his or her utility perceptions or the risk return values.

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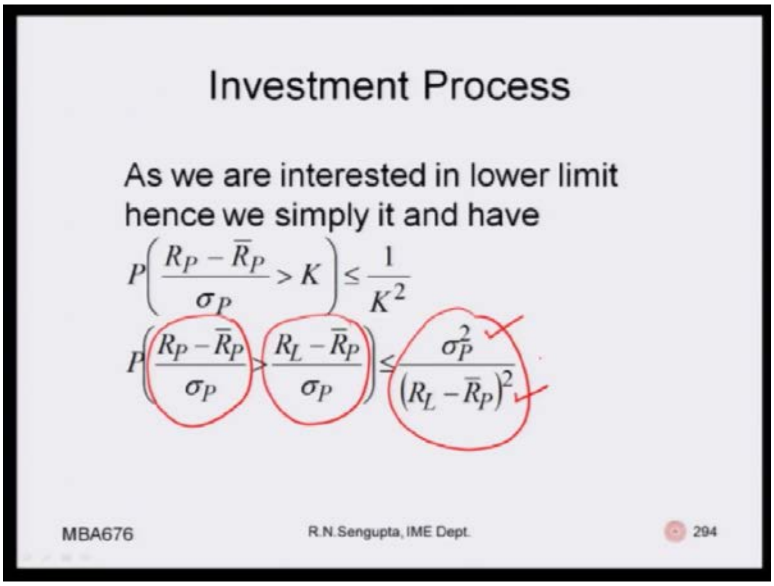

As we are interested in to in lower limit; hence we simplify it; after simplification if you see that on the left hand side you have basically the excess value over the return value, average value. On the left hand side is basically the excess of the risky interested or the R L value over the R P value and that should basically be give us in a way or concept of variances of these two cases.

So, if you find out, the numerator is the variance of that particular portfolio and in the denominator is basically the variance or the differences between the deviations or the minus values between the return, which you want that particular portfolio minimum to have, which is R L or R f minus the average values squared. That, why you are doing the square one, because you want to find out the ratios in the variance.

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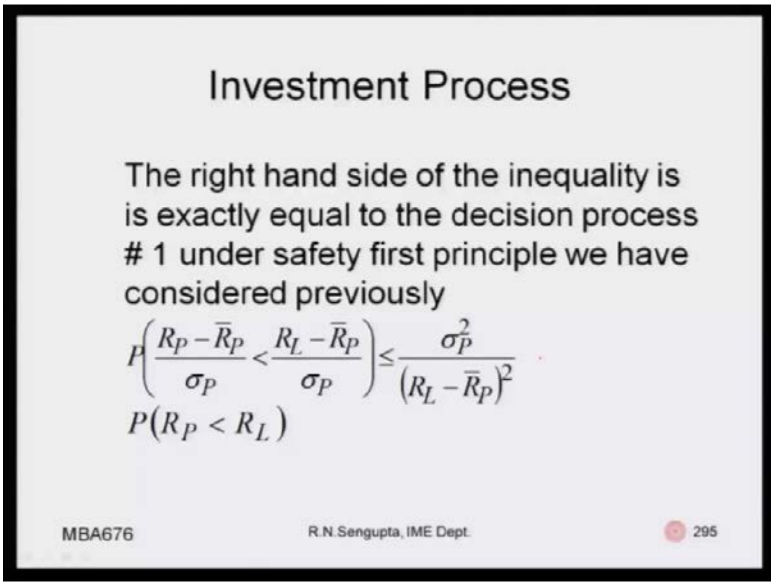

The right hand side of the inequalities is exactly equal to the decision process number 1 under safety first principles. So, once you simplify you get the probability that the R P value is less than R L or greater than R L depending on whether you want to, you are more concern about risk in the negative side or so called risk on the positive side, which is positive to.

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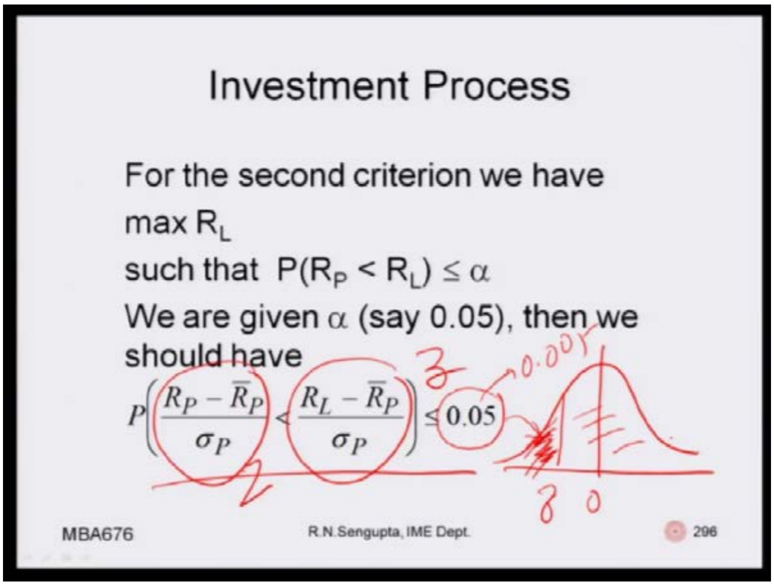

For the second criteria which we consider maximization of R L, we consider the probability to be in such that they would be a constant, which will given by probability R

P less than equal to R L is alpha. So, higher or lower alpha would basically with the conference level alpha s.05 or 0.10 or 0.15, then which basically should have considered normal distribution as this. So, what you have is basically this would be the small z value as I have already discussed, this would be the capital z value which is the standard deviate.

So, if we have the distribution this is the value of zero, because the expected value of the z value is 1 and what you want to find out is that, how the deviation is based on which you take the decision. If this is small z, you want the return of that particular portfolio, excess return in the sense R P minus R P bar, not in the excess return of the sense of the R f, but excess return divide by standard deviation. In this case, out of this 10 case only 5 number of them should be on the left and the 95 of the cases should be on the right.

So, this 5 percent or point zero number which is coming is basically giving you how many number times it will be on the left. So, if this value say for example, 0.005, it means out of the thousands such runs which you do, rather thousands such scenarios which generate in the portfolio, 5 numbers of them would like on this side and the rest 1000 minus 5 would basically lie on this side; such that you will always happy that out of the 1000 such instances only 5 are not favoring you, the rest are favoring your outcome.

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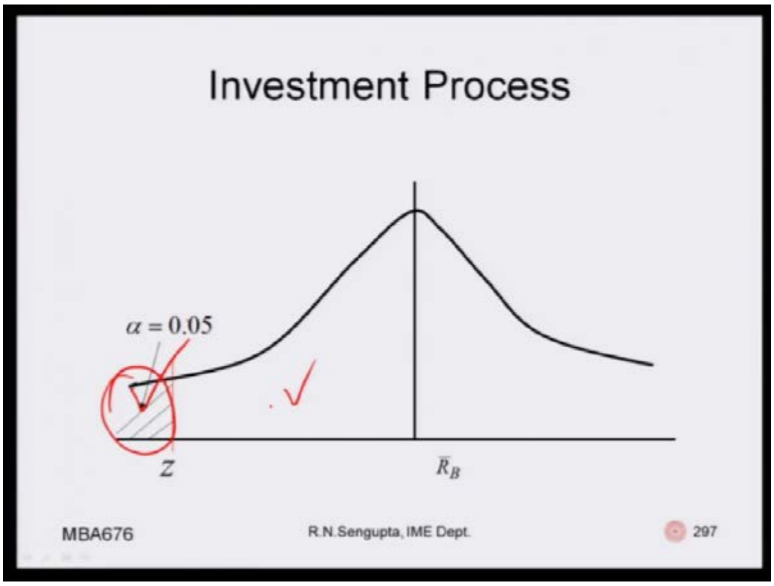

So, this is the scenario which have drawn, consider it is a normal distribution of point 0.05 in the left and any deviation on the left is definitely not wanted, you want to minimize that. Any deviation on to the right, right means this portion is good for it. So, you are trying to minimize the number of chances or occurrence on to the left.

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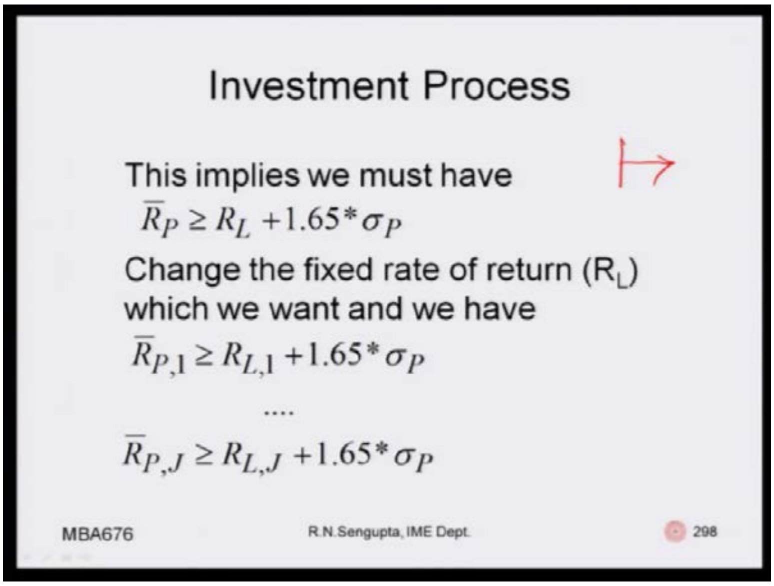

Now, this implies that depending on the value of the standard deviation, so what you want to do is that you want to have R P bar greater than equal to R L, which is the fixed level which you have put for yourself, it can be again R F and if this is 1.65 times into the standard deviation; that means, you have a value of R L and you want to basically move away from the R L on to the right 1.6 time more than the standard deviation. So, higher values are good for you, lower the value is definitely not good for you. Change the fixed rate of return; that means, change R L as R L 1, R L 2, like as we did R F 1,R F 2, R F 3, R F 4, so on and so forth. And once you do that you will basically have different scenarios based on which you can basically rank them.

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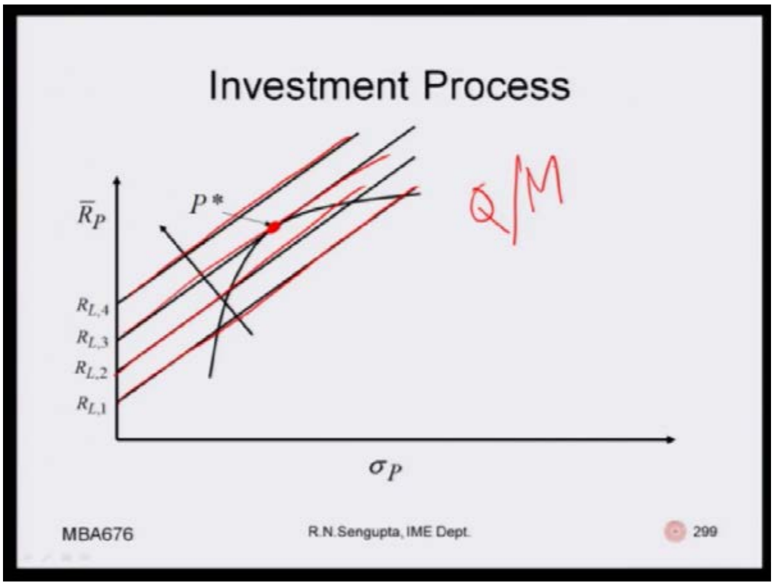

So, what you are doing is this. Again the state line is there and then you keep changing; that means, you keep changing R F 1, R F 2, R F 3. Should all of them are parallel? Why they are parallel? Because, none of the values are changing, so it is basically R L 1 plus 1.65 into standard deviation, R L 2 plus 1.65 into standard deviation. So, the values of standard deviation and that 1.65 value which you have put depending on the level of confidence is same. So, as you keep changing them they are all on parallel lines and the point when it touch, this is the efficient frontier theory exactly the value of Q which you have done already or the market value which you have done.

So, in a way the concept of safety first principle, the concept we are trying to basically use the CAPM model, the concept we are trying to use the utility function or the quadratic, the concept we are trying to increase the return and minimize the risk, all lead to the fact that they give you the same set of results or same type of results based on which ever output we are trying to basically find out.

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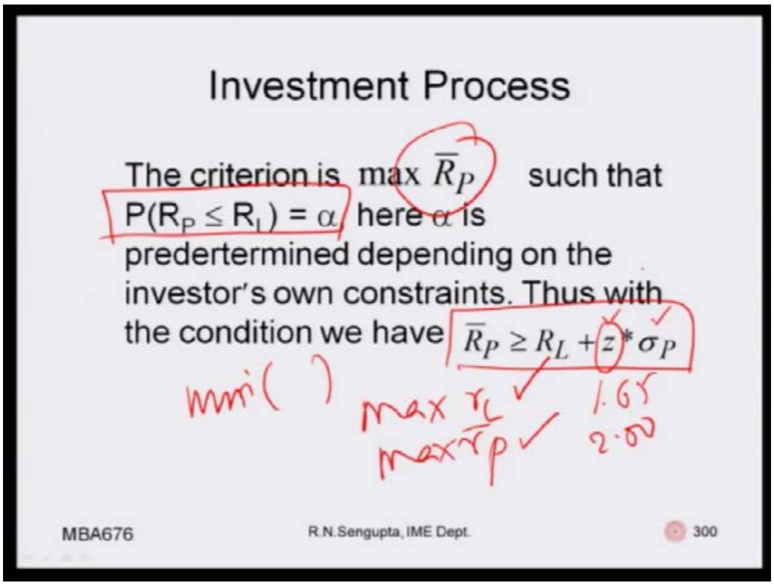

Now, the third criterion is basically maximizing R P. So, you basically maximize R P, again set to the same conditions, where you try to find out the probability R P is less than equal to R L, some alpha value so; obviously, alpha value in the long run should be made as ((Refer Time: 28:12)) possible. Here alpha is the predetermined value depending on the investors own constraints. Thus with the condition we can easily find out that R P should be greater than equal to R L plus z in to sigma p.

So, this is the value of z which you portfolio yourself, so 1.65 or 2.0 whatever it is depending on that you basically try to find out. So, in the first problem we had basically the safety first you want to minimize, in the second case you want to basically maximize R L, third case as it is you want to basically maximize R P. But, one interesting fact is that both for maximizing R L and maximize the R P, you always use the same constraint, which means that you are trying to basically use this equation; in such a way the return on the portfolio R P star is greater than suffix value R L or R F whatever it is by a certain distance; that distance which is basically K or z in to sigma p would be detected by two things. One is the standard deviation of the portfolio which you have and the level of confidents which you have at the level of the risk perception you have about this problem.

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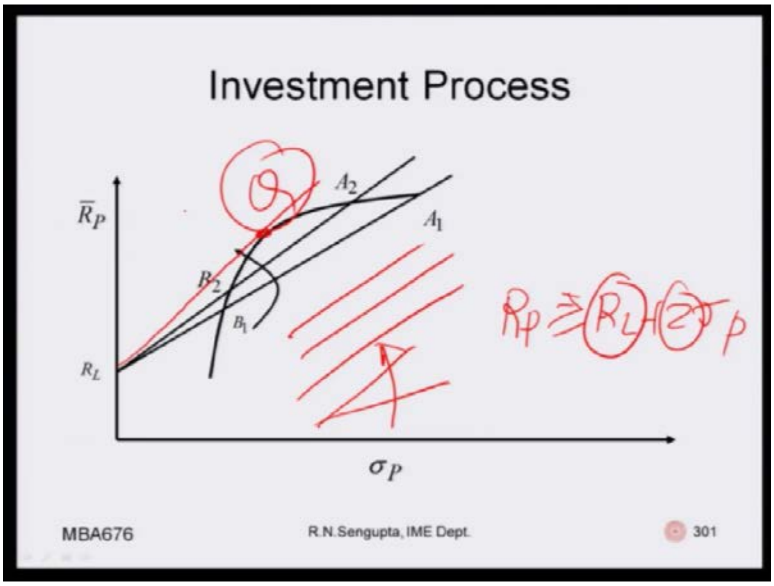

So, again in this problem, now rather than changing R L you change the value of K or sigma which is basically so called slop of that line. If you keep changing, it starts moving and at some points of time again it basically staging, again this value is Q. So, the value of Q which you obtained by using this equation either changing this, such that they are parallel or changing this such that line moves anti clockwise gives you the same point of Q; such that you can find out other Q of the cases.

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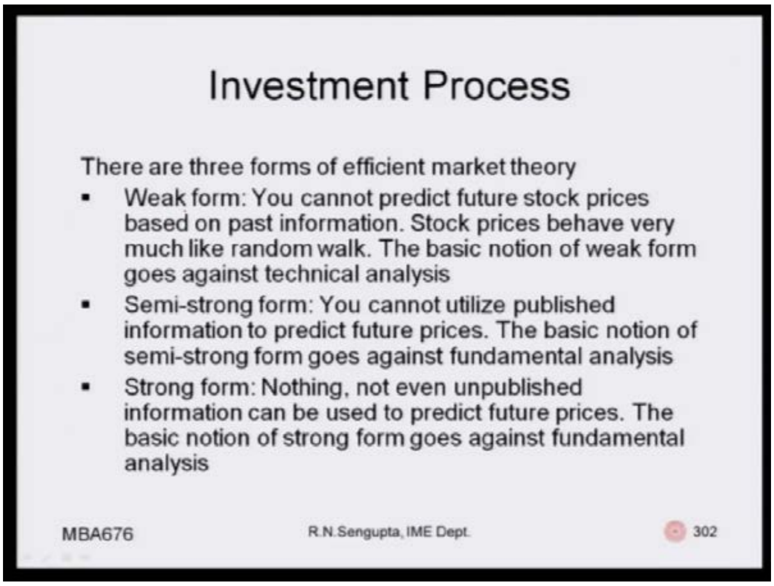

So, now, there are three forms of efficient market theory, one is the week form, one is the semi strong form and the strong form. In the week form, you cannot predict the future stock prices based on the past information; that means, stock prices behave very much like the random walks. Semi strong forms you cannot utilize published information to predict future prices that is the basic notion of semi strong form goes against the fundamental analysis and the strong form means, nothing not even the unpublished or published information can be utilized to predict the stock market.

So, in every case which you are trying to analyze, we will consider the prices to be such that whatever information is there is easy level would all of us and the market mimics the actual ((Refer Time: 30:42)) to the maximum possible instant.