Quantitative Finance

Prof. Raghu Nandan Sengupta

Department of Industrial and Management Engineering

Indian Institute of Technology, Kanpur

Module – 02

Lecture – 12

So, continue the discussion with it mention that CAPM model, which is capital model asset model, is basically a practical application of the single index model. So, in the single index model you considered; that r i or y which is dependent variable is dependent on x, which is the independent variable and your alpha, which was basically the hided which the line cuts the y excess plus and epsilon which was the white noise.

So, once you take the expected value, and when you basically tried to find the one to one correspondence with the CAPM model what you have is, on the left hand side rather then why you have basically r i minus r f which is the excess return of a particular stock over the risk free interest rate is equal to, basically the beta which is tan of the theta angle, which is in concept to the simple linear equation which we studying in class ten, is equal to y is equal to m x plus c. So, beta m r basically the slow and excess basically the excess return on the market with respect to the risk free interest rate and; obviously, the error term which is there, would both the single index model as well as in the capital model we consider that expected value is zero; hence and an average is zero.

And; obviously, will have a the white noise, the various would come into the picture, and we have seen in the concept was in thus market risk, and non market risk, diversifiable risk, non-diversifiable risk and so on and so forth. So, what CAPM is doing is there, basically triangle replace the concept of risk, from the domain of standard deviation on variance, for each and every stock, to one particular measure which is basically in the beta, which we already consider. Now if we consider that CAPM model, again i am repeating it, if in place of r i you have a certain portfolio r p with the actual stocks being exactly equal to the market in's the same proportions. Then the value of the beta will be one, if an only if the market and the portfolio which you have formulated basically go hand in hand, or they are in tandem; that means, they are just a replica of each other, but as you are not aware that how will basically buy some proportional's of that index, in actual case; hence you basically formulate of portfolio which mimics the market to the

maximum possible extent.

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Now continue with the CAPM concept, we will consider the security market line. The CAPM model can be expressed as a graphical form by regarding the formula as a linear relationship. Linear relationship between the market and the particular portfolio, or a market and the particular stock whatever we have, and that is considered as the excess returns. So, we are considering the case that we will invest in that particular stock or in that particular portfolio, in some positive amount, if i know if the return are greater than the r f.

But; obviously, the scenario will change if you consider short selling to be there, because in the case of short selling they would be in the negative proportions as we have seen in the last two days different type of problem formulation, or in the Markowitz problem formulation, whether the weights can be negative also, but; obviously, will follow the principle; that the some of the weight is equal to one. If you do not follow the link less equation, and some of the model the weight is equal to one, if you follow the link as model, but corresponding the fact the normalizations are done according. So, this security market line, this is a relationship termed as the security market line, as is shown and the next line.

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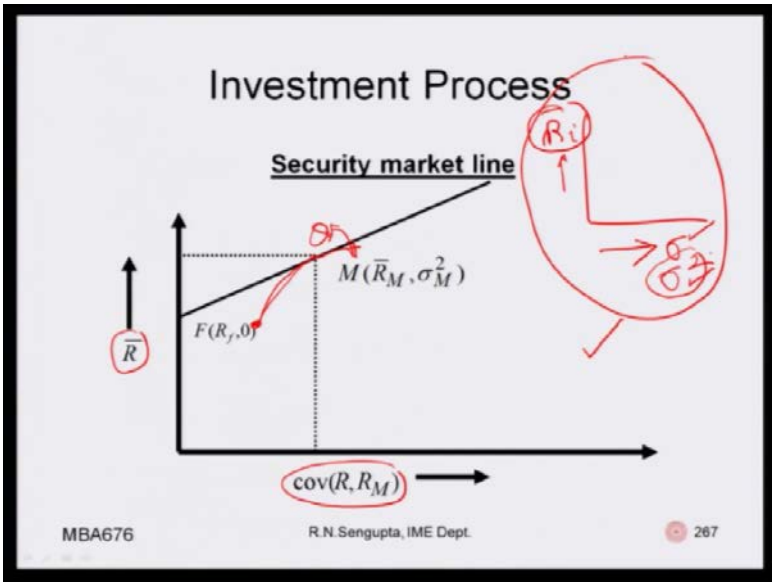

So, what you have is again, the straight line with the tangent which is equal to the beta, and the y axis is touch by the line, the value of f which is r f which is the risk free interest rate, and it is tangent to the curve which you always should know, and I am sure you remember. If this is the value of market, so what you are doing is that, if have that value of q such that, is tangent to this. So, q and market exactly the same for the concept of one front theorem. And; obviously, if you extend it you have the minimum variance point which we are not going to consider, because we already discuss that earlier. And the security market line is this certain changes. So, if the in the efficient frontier what you consider was, you had the value of sigma or the value of sigma square; that means, this is the standard divisions, this is the variance, and along the y excess you basically had the r i's bar.

But now in this case the y excess remain the same as it is here, but the x excess rather than the variance is of the particular stock, we have basically the covariance existing between the particular stock and the market. So, even though we bring a different concept of risk between the market and the particular stock, or we consider the sigma square, but still the properties would basically be such, that they would portfolio same amount of the information, and same concept, if you consider that the single index model or the CAPM model and trying to basically compare that with the security market line. So, again the concepts are the same. The tangent would basically give you the same information. While the point m would give you the same information which is market and so on and so forth. As we can see in this curve, which is the security market line as

well as the, and in the graph which we have already considered the efficient frontier one which is the risk return profile which you are drawing.

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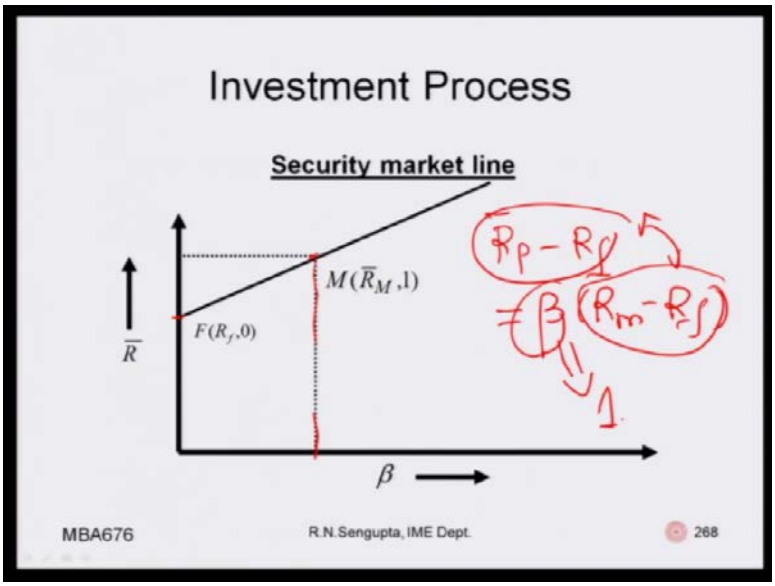

Now if we have a different version of the security market line. Like trying to basically visualize in a different way. So, rather than drawing sigma square in the x axis, or covariance of the particular stock with the market, or the covariance of the particular portfolio the market, let us basically try to depict another concept of risk which we already discuss the beta s. Rather than drawing the variances covariance's, let us draw the beta value along the x axis, while the y axis remains the return. If you know it again you will get the same short of graph, but remember one thing, here the beta values would be one exactly at the point, where the actual market value and the return of the portfolio exactly made; such that when you compare the x axis return of the portfolio to the risk free interest rate, and equated to the excess return of the market with the risk free interest rate, then the beta value would come out to be one, which means that the market on the portfolio are going in hand in hand.

So, if we have a look, this is again is risk free interest rate, this is the straight line, considering there are n number of such is risk as it plus n at plus one being the risk free interest rate. And the market value basically if you drape down of vertical line, then it basically will touch the value of beta, which will be one depending on exactly equal to the market as such, or the portfolio which you formulating which exactly that will come out of the market. It means you already have this. So, these two values are same, then the value of beta comes out to be one.

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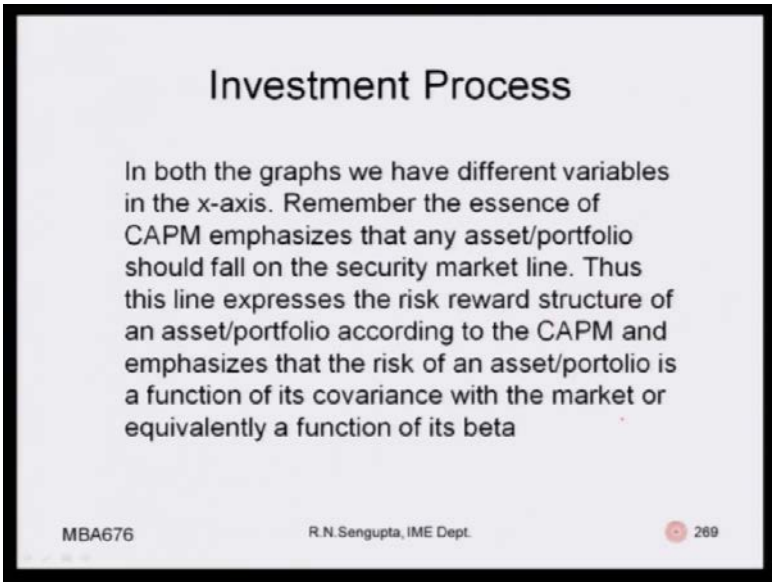

In both the graphs; that means, security market line, considering covariance's of have been product along the x axis, and the security market line will consider beta is being plotted the x axis. We have different variable in the x axis, as mentioned here again which I told in few minutes back. Remember that the essence of CAPM emphasizes that any asset or any portfolio, should fall on the security market line, does this line expresses the risk reward structure, of a particular asset are a set of asset is the portfolio according to the concept of CAPM, and consider the CAPM is basically true. And it also emphasizes the risk of the asset of the portfolio is a function of the covariance's of the market, or equivalent you function of beta.

So, what we are trying to do is that, if you remember few slides back we discuss the capable model is trying to basically replace the concept risk, using beta in place of standard deviation, is exactly what is being said again, that which our you look at that the concept, trying to basically analyze the concept of returns with respect to expected value of return. And trying to basically analyze the risk will respect to sigma or covariance or beta all of them give you the same concept, where the risk return profile is the way, how you try to analyze any particular stock or a portfolio with respect to the market. So, closer they are, they would exactly mimic each other. Further they are; obviously, the beta values or the standard deviations would be far removed from each other. Far removed means the market risk as well as the variance or the risk of the portfolio of the stock which you are formulating.

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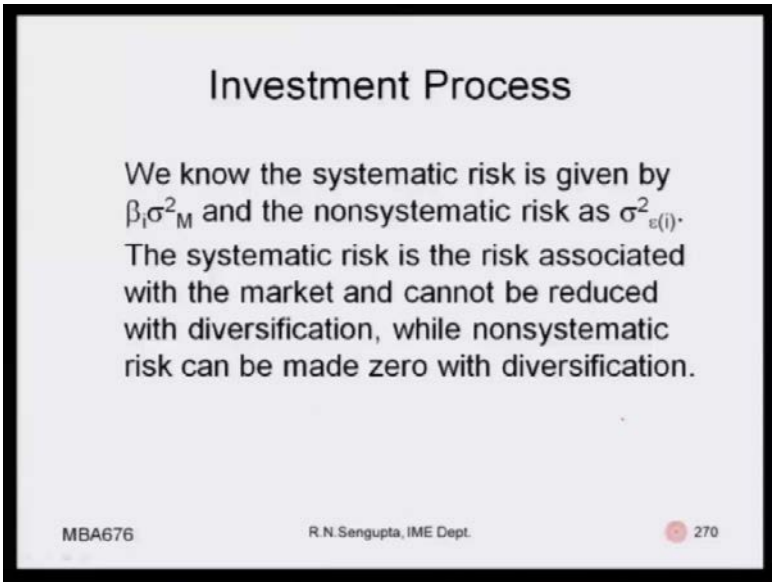

We know that the systematic risk which we have already discussed, is given by beta i into sigma square m, where beta i is the beta value of that particular portfolio of that particulars stock, and sigma square m is the market variance. And the non systematic risk is given by the variance of the white noise. The systematic risk the risk associated with the market and cannot be reduce, because; obviously, sigma square m cannot be zero and either can beta be zero, because beta zero means basically you have a certain stock which is not varying with respect to the market is basically in a way, a fixed value stock or a deterministic value, which is not possible. And while the market is cannot be reduced with diversification, but the non systematic risk can be made is zero with diversification, because if you remember we have to take weights are that particular stock in that portfolio. So, if we have n number of such stocks, and if you take weight of each of them as one by n, then technically increase n to infinity can be made in such a way that the overall and non systematic risk which is coming from the white noise can be made zero in the theoretical sets.

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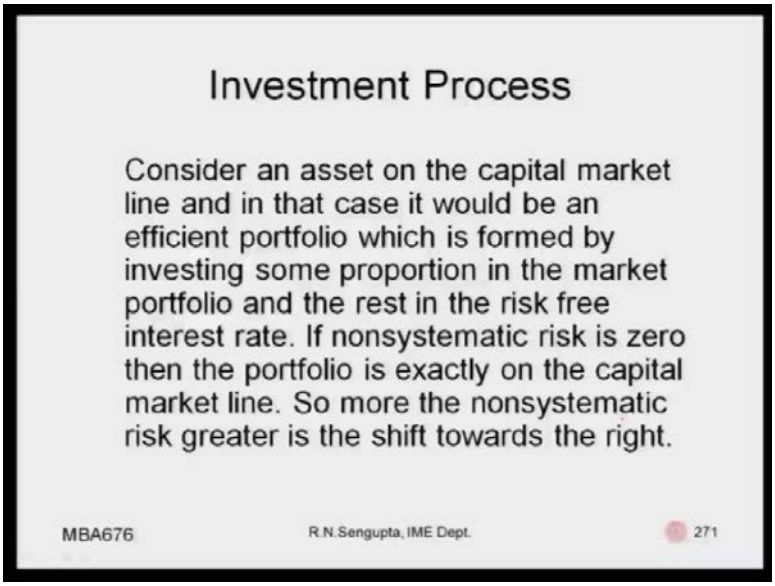

Now, consider an asset on the capital market line, and in that case it would be efficient portfolio which is form by investing some proportions in the market portfolio, and the rest in the risk free interested. So, if you remember rather than going back to the diagram is because we have been conceptually trying to analyze that diagram long time back; starting from say for example, the Markowitz model and so on and so forth, when we draw the risk return frame work. So, if you draw the efficient frontier in them in the efficient frontier problems when we considered. Their point q and point f are the q extremes. So, if you add short selling; obviously, could extend the straight line beyond q depending on whether risk less lending and borrowing was allowed from the bank. Now if you consider the same thing here also, again it gives you the same concept.

If you have the market, If you have the risk return frame work being their where the risk free interest rate is also there. Then you can basically follow depending in what you risk return profile is invest some proportional of you r money in the so called market or in q or m q whatever you denote. And some proportion you can basically invest in the bank; that means, either go to the bank invest, or take out money from the bank such that short selling is allowed and utilize that money to invest in q in some proportion or in m some proportion. What are the proportions that you can find out variously if you solve those problems as we have already discussed. Now this efficient frontier is formed by investing some proportions in the market portfolio in the rest in the risk free interest rate in non systematic risk is zero, then the portfolio is exactly on the capital market lines. So, more the non systematic risk is, greater is the shift that particular portfolio with respect to the

straight line which is the market line which you already discussed.

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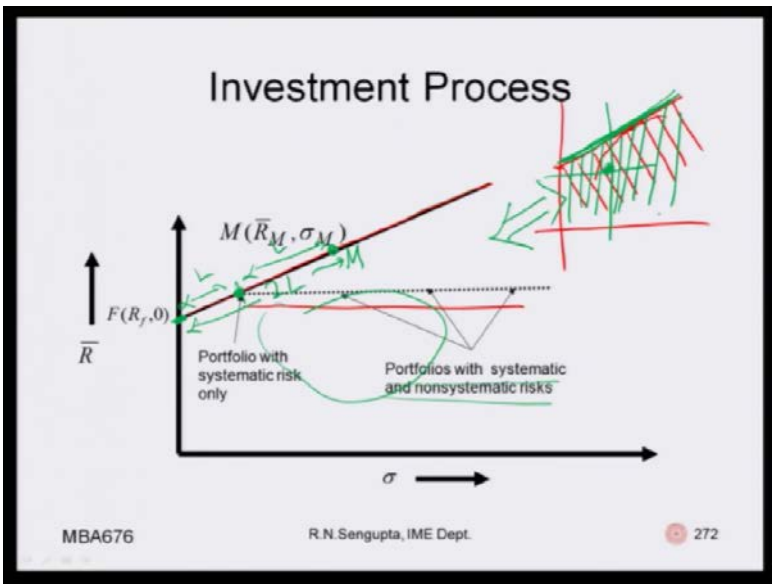

So, consider this is the market line. So, along and you have basic along the y axis r bar returns and along the y axis we have sigma. Again we have now sigma, it could be beta, it could be covariance of the particular portfolio, or the particular asset with the market. So, that does not matter conceptually. So, what you have is that, if you see the dotted lines a shown here. So, any particular stock, which is their exactly on the line, is exactly line with the market, and it has got all the information show the market, but any point which is on the right, is definitely not efficient. Now you may be asking and trying to visualized how is it. So, let us go back to the risk return frame work which you already done. So, this was a risk return frame work, when n number of risk assets are there, then you had basically risk free interest rate, continued the tangent. Now any set of points in here were feasible, but they were not efficient in the sense only the efficient line, would be in this case, let me use some other color.

So, any efficient line would be the straight line, and any set of points which is a contended to be efficient, or is a contender or a set or the feasible set, would be this set of points. Now if you are below somewhere here; it means what. If you consider a risk return profile, your overall risk or overall return is such that you can at an some portfolio some asset, which will definitely give you some higher returns for the same value of the risk, or if you go to the left you can basically achieve a low level of risk with respect to same portfolio, for the same level of return. Now if draw a singly between the concept which we have consider there, and if you see that diagram here, is exactly the same

thing. This point is any portfolio, which is being formed in some proportions of the market m and the risk we interested. So, it is if it is in between; the length is exactly equal to l and l here, and consider the total length is 2 l. It means that you are investing 50-50 percent of your amount of money in the market, or in some proportion in the asset inside the market such that is exactly 50 percent of the total amount of money. And you are investing rest of the fifty percent of money in the bank.

Now if you again, if you go extended at year and it is basically short selling from the bank. Now, the set of points which are here. All the sets points which are below the straight line, they are according to the concept which we already done. They are some feasible set, but they are not optimal. So, more away from they are in the straight line, or below they are from the curve, or below they are from straight line, in whichever diagram you draw, whether you are trying to basically draw in along the x axis, the covariance of the of the particular stock with market, or the beta value or the standard deviations whatever it is. Further it is from the straight line more inefficient it is. So, it means that, if you follow the straight line dotted near here. It means the portfolio is you systematic risk and non systematic risk being there, but the systematic risk is such that they would be in efficiency in that trying to find out the actual price of the portfolio based on which you are trying to basically mimic that particular portfolio of that particular stock with respect to the market.

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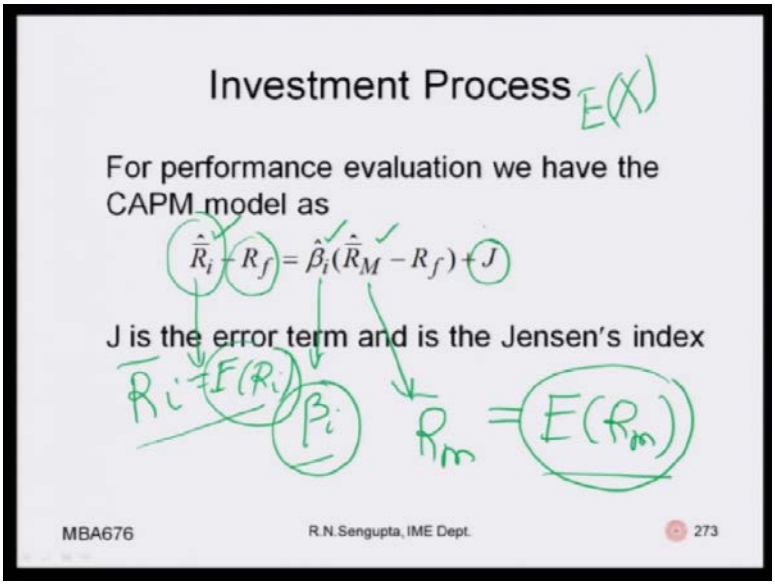

For performance evaluation we have the CAPM model as given. Now if you look at this equation very carefully; the left hand side is again x axis return on the particular portfolio

of that particular stock with respect to the risk free interest rate. Only fact is that r f is fixed, so hence is value theoretical value saying is constant, but rather than r i bar we have now basically HAT. HAT means basically estimated value of the actual population, which means that if you add the whole population all the number which you have a HAT for a particular stock or a particular portfolio. The expected value we would have found out, the expected value found out which you already discussed, is basically given as the actual average value of the overall population, but now if it is not available to you, what you do is the, you pick up a small junk from the population which is known as the sample, and then take the average of the sample which is known as a sample size. So, will consider the sample size to the best mimicking characteristic of that population; such that the difference between the population mean, and the sample mean is as low as possible, and it basically has some characteristics in the satirical concept. So, you will consider the HAT to be there.

So, what we have is the excess return of the sample average over the risk free interest rate. And on the right side again you see we have the same thing. We have beta. Now it is a beta had because you do not have the overall population you take a sub sample of that sample of that, and try to find out the best proxy or the best value of that beta; such that is given by beta HAT. And again inside the bracket you have the excess market return over the risk free interest rate. Again the market return is given by the same sample average corresponding to the market only. So, if you had the actual population, these beta would have been that actual beta i. This r m would have been the actual r m, or in the actual sense it is this; expected value. And this one would basically be again this or the expected value. Now if the left hand side has e of r i and the right hand side has beta i and e of r m, and basically if the exactly match, then we will say the beta is equal to one. That means, the market is exactly equal to the stock are the portfolio vice versa.

Now even in case if it is not, still the beta value would be there, but in the long run if that is certain difference between the expected value which you are trying to find out along with the sample average, or the expected value or the actual value beta with respect to the sample beta i which you find out, or this is a difference between the expected value the market along with the sample average. Obviously, there would be some difference. That differences we will basically denote in the financial terms in the CAPM model as j are the Jensen's index. So, higher the value of j is, more is that the difference between the average values from the population in the sample, which means that sample, is not able

to mimic or port the exact value from the population. Less the value of Jensen index is, we will to basically assume and consider that the sample average is able to mimic the actual population average with the maximum possible extent.

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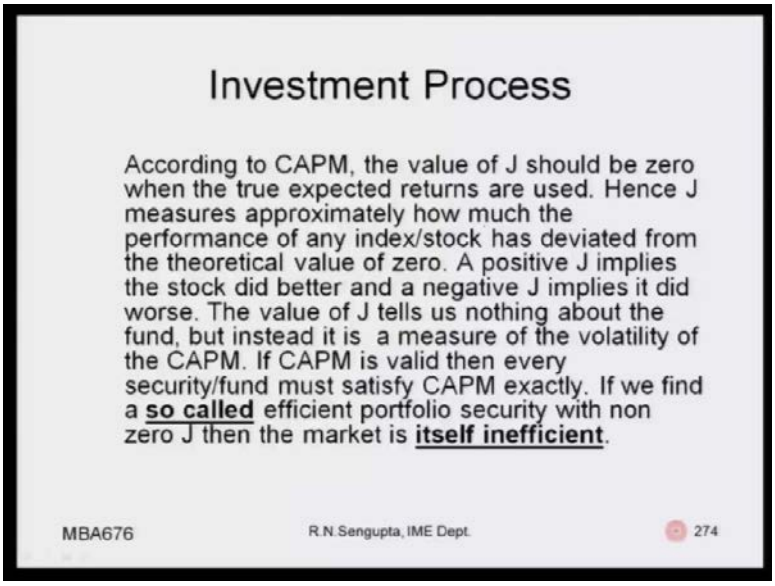

According the CAPM model the value of j should technically is zero, which is right, because if the population is average or exactly the same then; obviously, the equation which you have for the population, where expected value have being used, and not that HAT values, and the HAT values equation which you have. So, they would not be any difference which means the j value is zero. Hence j measures an approximately how much the performance of any index or a stock or a portfolio, has divided from the theoretical value of zero. A positive j implies the stock did better, and a negative j implies did worse, but the question would be on the longer one what is the value of j. So, j has both positive value is both negative value. So, if you add up all the values in the long run for given samples, theoretically exactly be zero. So, you can consider in some sense j is a short of error of the white noise. So, on the long run if you add up all the values j, or if you add up all the values of white noise, the expected value comes out to be zero.

The value of z else is nothing about the fund, but instant is a basically a measure of the volatility in the CAPM model. If CAPM model is valid, then every security fund portfolio whatever you formulate form that particular market, satisfies the CAPM model exactly, if you find a so called efficient portfolios security line with non zero j, then the market itself in inefficient. So, in case the market efficient so; obviously, j value can be zero and positive or negative. And if the market and the portfolio mimic each other

exactly, or the portfolio which you found falling is following the CAPM model exactly, then the j value would be zero. But if there is an inefficiency which means the two things; either the market is inefficient, which is possible, or else the portfolio of the prices which you following up does not follow the exact efficient CAPM model to the maximum possible extend. So, how do you basically find it out.

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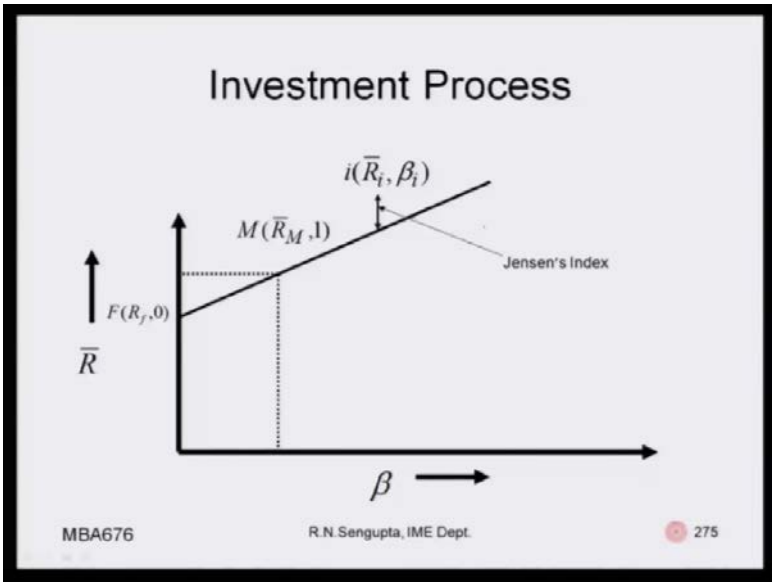

So, consider again the same market capital market line or the security market line whatever you say, but only remember in each and every slide, be careful about what is the x value, what is being drawn the x axis. Here again we see rather than sigma square rather than covariance. Again we are trying to depict the value and along the y axis it remains as r. Now if your actual market line is the straight line, then depending on the value of j positive and negative your actual portfolio which you are formulating, which is not the market, if the portfolio which you form p, or the actual stock which you formulating can be ether above or below, but on the general sense, if you add up all the values, and try to find out the that differences in the long run, average value it comes out to be zero.

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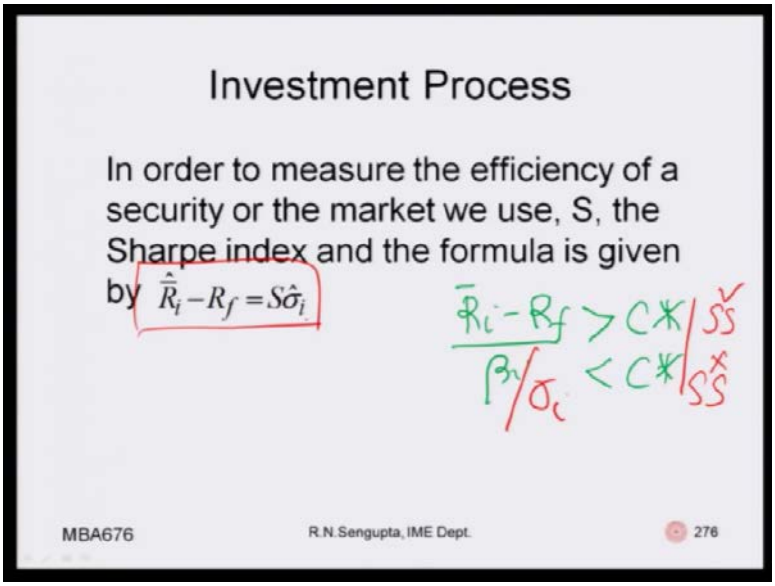

Now, there is another index will also consider. In order to measure the efficiency of a security on the market, we use s which is basically the Sharpe index. So, Sharpe index basically gives you the ratio of the access return or particular portfolio or a stock over the risk free interest rate divided by standard deviations. Now we have already in Coordinate Corporation if you remember, when you were trying to basically rang the portfolios. According to the excess return of the particular portfolio or a stock with respect to the risk free interest rate in the numerator, and in the denominator we had either the beta or the standard deviation, and based on that we had that. The rule was if can recollect we had this r i minus r f; this is average value divided by beta, can be greater than equal to some c star, can be less than equal to some c star. Or else in place of beta, you can also replace by sigma. So, whichever concept you follow you can basically have both the case of short selling being there, short selling being not there. And if you remember we had discussed that in the last class. Now if you find out the ratios r i minus r i by sigma is exactly equal to the Sharpe ratio which we have consider. Sharpe ratio can be utilize as a proxy, in order to decide whether you will invest in a particular stock, or not invest in a particular stock in what proportions, depending on whether short selling is allowed or not allowed.

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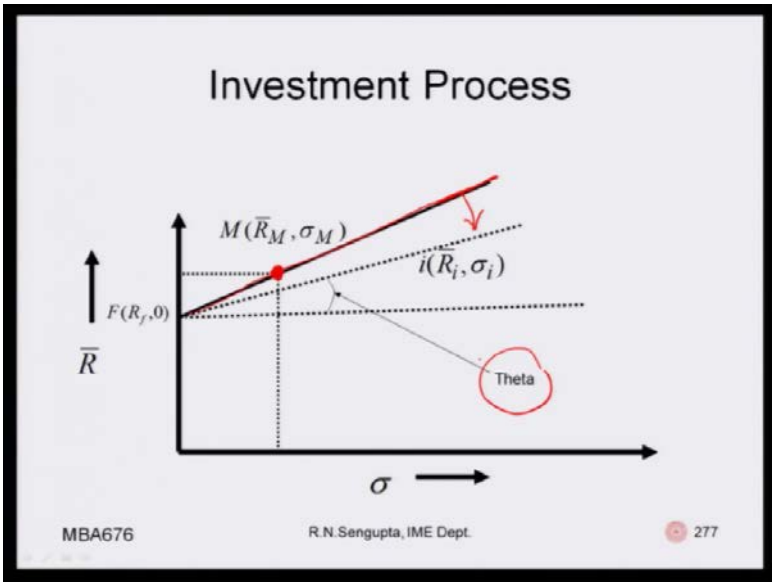

Now if you draw the market line, again see this a change in the x axis we are just replace that with sigma. So, whether you see, use sigma whether use beta, or whether use covariance the conceptualization of the of the actual result is same in all the cases, is basically trying to look at the same thing in different angles, or trying to basically put our different colored lens or colored goggles, and trying to see the same thing in different perspective. So, if you have the market line, and if the market is by itself inefficient. So, what you will do is that, basically you are actual efficient front portfolio q or the one font portfolio the market if this one, but if it is basically inefficient. Then; obviously, the market line which you have which is the bold one which is just I had drawn would basically fall; that means, it will go down in this direction, which means more inefficiency being there.

It will mean the market line would basically fall more towards the zero degrees; that means, which will the theta angle which you have here will start decreasing. So; obviously, it will mean that in order to basically fit a particular portfolio or an asset. You basically need to be aware of the two things, whether the market is efficient, or whether the stock price itself, based on which you are trying to draw the particular stock point, along the market line whether the pricing is inefficient or efficient. So, both of them can be inefficient, and in that case basically Sharpe index and the Jensen index are basically a proxy of the amount of in efficiency which is there, in a combined format for both the market as well as the pricing of the particular asset of the portfolio.