Quantitative Finance

Prof. Raghu Nandan Sengupta

Department of Industrial and Management Engineering

Indian Institute of Technology, Kanpur

Module - 02

Lecture – 11

(Refer Slide Time: 00:11)

Continuing with the assumptions, we will also consider short selling, unlimited short selling is allowed, because solving the problem would also mean that, if you have particular stocks, which is giving me negative returns; which is r i minus r f by sigma or r i minus r f by beta, if it is negative so; obviously, it means that I can short sell that and utilize that amount, extra amount of money which you have and trying to buy particular other stocks which gives the positive returns. We will have unlimited risk less lending borrowing being there, which means I can go to the bank, I can borrow any amount of money, I can go to the bank and basically deposit any amount of money. Another thing which is important to note, is that the risk lending and borrowing values are same, but technically it is not true. In the sense, if I go to the bank and deposit or basically buy some dollars, or go to the bank and basically borrow some dollars.

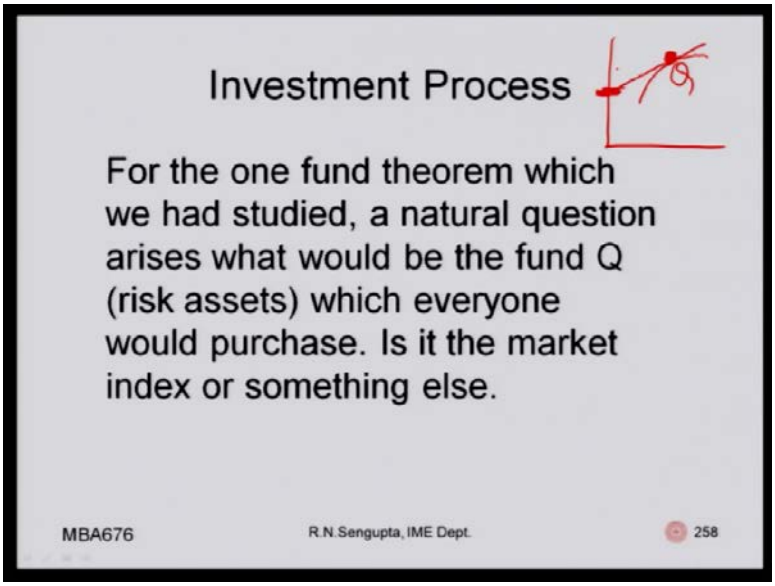
So; obviously, the rate at which I am buying and selling dollars is totally different, which means the same. If I go to the bank and get a loan or go to the bank and basically open a fixed deposit or occurring one, the interest are different, but in our case will consider

both interest rate are same; that means, r f r suffix f for the lending case, and r suffix f for the borrowing case both the values are same. Investors define relevant periods in exactly the same manner. So, I am trying to basically analyze a problem for one month period. So, the same or the different person, if he or she is trying to basically analyze a problem. Then as mentioned the input information which is available relate to the price, is same for both of us, and; obviously, will be analyzing our problem for the same time period. Obviously, it may mean that I want to hold a particular portfolio for a longer duration, but when I break up in time frames; the time frames would exactly match, such that any mismatch in the prices would not be reflected.

So, the ninth one would be investors identical expectation with respect to the necessary input from the portfolio. So; obviously, it may mean that my overall utility maybe different, but the actual input based on which I am trying to find my utility and the expected value would be the same. So, if I know if the prices are increasing or if there is an increasing in the demand, or increasing the supply or vice versa whatever it is. The exact information with the prices immediately available to all of us, all of the players they are in the market; such that depending on a utility and depending on nonsatiation, depending on the risk aversion property which any human being would have. He or she would able to analyze and get the best. So, called prices are best, so called portfolio would basically reflect, the overall characteristic of the market, with respect to the overall characteristic, with a particular human being as an investor has.

We will also consider investment investors basically make the decision of selling a particular stock, based on only two criteria. According to Markowitz, the first movement with the expected value, and the second movement which is basically the variance, are the two criteria only based on which, any human being will make a decision to buy or sell a particular stock. And the last one would be basically the all assets are a marketable, in the sense, if they are particular assets which I want to sell, I have the full liberty to sell it to the market such that I can basically optimize the portfolio accordingly. On the other hand if I am trying to basically short sell a particular asset, and try to buy a another asset; that means, I am trying to short sell reliance and I am trying to buy Tata motors, then Tata motor is available for me in order to basically make my portfolio accordingly.

(Refer Slide Time: 03:34)

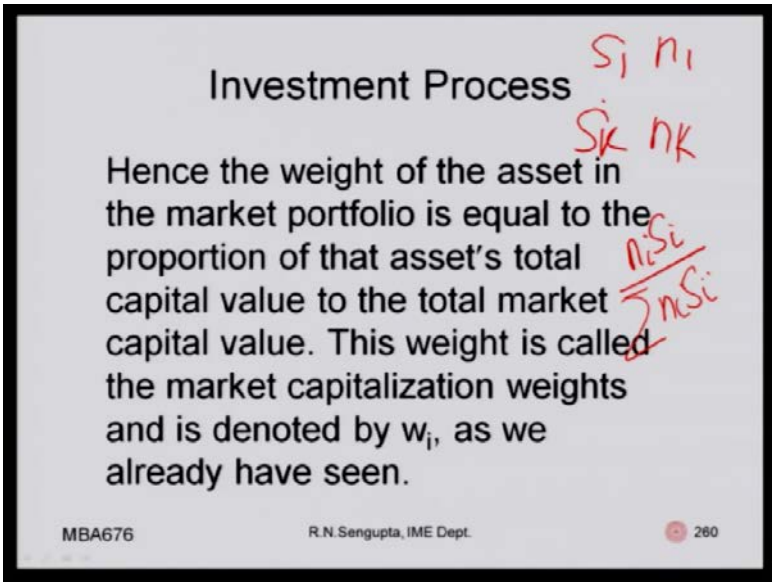


Now for the one fund theorem if you remember. What it was? This is the efficient frontier, considering n number of risk assets are there; this is the risk free interest rate. And if you remember this risk interest rate has only one value, not depending on borrowing and lending. So, if I draw the tangent, this is the value of q. So, this value of q is basically concept of one fund theorem; such that if I have this particular risk free interest rate and the Q portfolio, which has inside it n number of such risky assets in different proportions. Then combining them; that means, n such risk asset plus the nth one risk free interest rate. I can basically formulate by portfolio accordingly, and basically we at a certain level of risk return frame work, which basically it is my demand accordingly.

(Refer Slide Time: 04:21)

So, this is the fund in the market index. It is the fund that contains share of every stock in the market. So, what technically we are trying to implies that that q value, is a certain portfolio which is conglomeration all the assets in the market, with mimics the market to the maximum possible extend or which is in the market. So, then a question would be what is that; obviously, it is the market index, which is the BSE or the NSE in such a way, that whatever the price movements is happening for the BSE and NSE, it basically replicates the market. It gives as all the information the market at one go, depending on the demand and supply of each and every stock which is there. So, obviously, BSE or NSE does not have all the stocks in it, but price movement of those BSE 30 are NSE 50 such in a way, that whatever price movement is happening for other stocks, is immediately been reflected in the BSE NSE prices. So, the price and demand and supply of the BSE and NSE gives us the exact picture of how the market is doing.

(Refer Slide Time: 05:21)

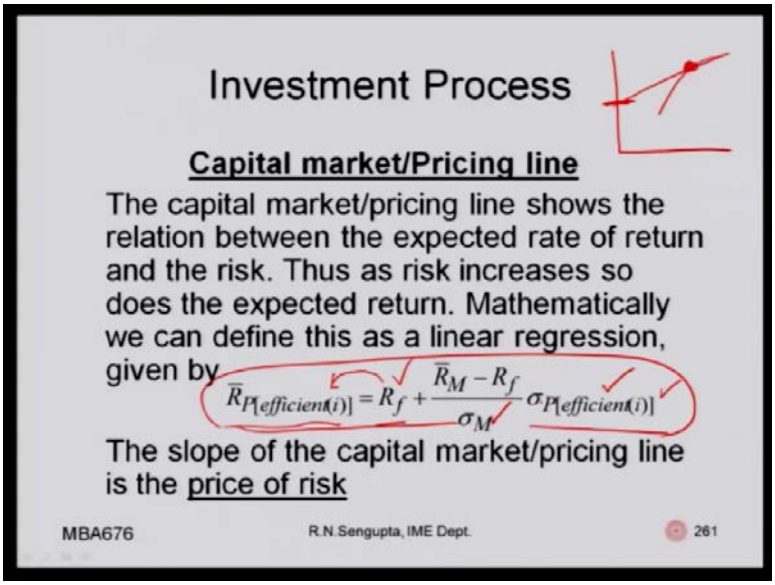


So, now, second question would be, how do we find out the weights. So, if you remember is in the concept of the optimization we found out the wi's, whether short selling is there or not their; such that it gives us the good picture about how all the portfolio should be formulated. So, our main question is that, how do we find out the weights. So, the weights of the asset in the market if portfolio is the proportion of the assets total capital value. So, now, I want to pause here for one minute. So, if you remember that I mention that if I have hundred rupees in my pocket. So, total amount of I am investing in stock one if it is 10 rupees. So, it will be 10 by 100 this is a proportions. If I am investing twenty rupees in the second stock, the overall ratio would be twenty by hundred. So; obviously, w 1 would be 0.1 w 2 would be 0.2 so on and so forth. Now it is basically stock, what is more important to note, is that how do you find out the ratios.

So, consider that, say for example, you have stocks s 1, 2 say for example, s k, k is the number, and the corresponding number of such assets you are going to buy for the first stock is n 1 till n k. So, now the question would be, what is the total capital net worth of my total investment. If I invested, n 1 number of stock in the first one, n 2 in the second one till n k in the kth one, corresponding to the price which is s 1 s 2 till s k. So, actually the weights which we have found out, will be very simply given by this. So, n i into s i is the total amount which i investing for the i s asset divided by the summation of n i into s i. It means i multiply each and every stock number, number I have bought in that particular stock multiply by the price of the stock is the total amount of investment which you have, and that goes in the denominator, and the numerator is basically the

proportions of that particular asset which I am going to invest in that particular stock. So, coming back to the continuation or what the slide says, it says, it is equal to the proportions of the assets total capital value to the total of market value. The weight is called the market capitalization weights, and it is denoted by w i which we have been discussing. So, as we have already seen in the portfolio optimization problem using a Markowitz problem.

(Refer Slide Time: 07:38)



Now, the capital market pricing line, is very important for us to find out how optimum or some optimal the pricing of particular stocks. So, it means the capital market or the pricing line shows, the relationship between the expected rate of return and the risk. So; obviously, we have the risk return framework that is true, but we want to find out that risk return framework, or the risk of the particular stock with this risk return, is how good or bad with respect to the market. So, obviously, you cannot beat the market. What you can do is that, you can basically formulate a portfolio in such a way, that the overall weights in that particular portfolio is such, that it mimics the market to the maximum possible extend. And the movement of the particular price of the portfolio which you formulated, moves in tandem of the market. Obviously, the market will increase and decrease. So, what you want to do is that, you want to invest in such a proportions all the assets which is their in a portfolio; such that the price movements exactly goes as hand in hand with the market. So; obviously, it means, the market is increasing the portfolio value which you have would also increase.

If the market is decreasing it would mean that portfolio value which you have would also

decrease, but the decrease and increase should be to the maximum possible extend. Which means that the correlation coefficient existing between the so called portfolio which we have, which you formulated, and the market which is their, which you are trying to mimic is plus one. so; obviously, it mean that any one unit increase the market would have one unit increase in a portfolio, or one unit decrease in the market value, would have one unit decreasing the portfolio which you have. And risk increases shows that the expected value, which is true; that means, more I want; obviously, they would we more risk, or if there is no risk; obviously, I will invest in a particular portfolio which as more returns. Mathematically we are trying to find out that what is the so called market line equation which mimics the risk and return of that particular portfolio which you performing, or the particular asset which you have.

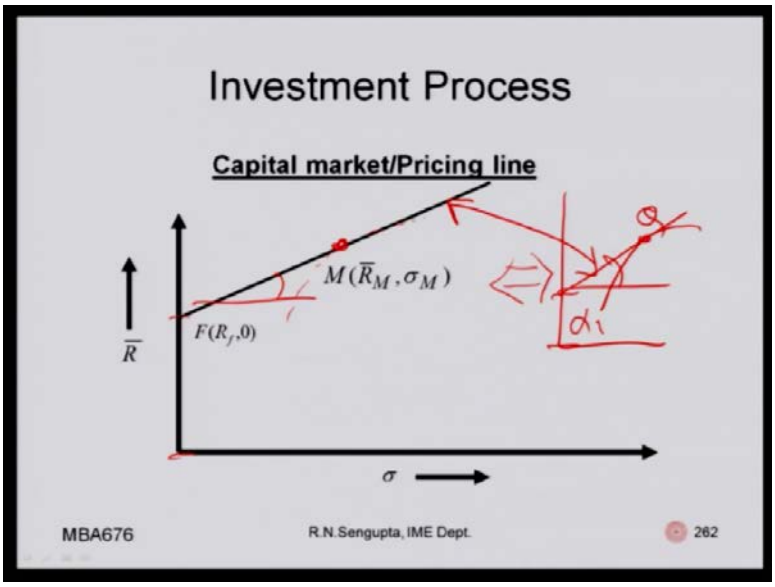
Now, if you basically see this equation, they simulate to the one of the single index model. In the single index model what you have is, r i s equal to alpha i plus beta i r m plus epsilon, and if you take the expected value; obviously, epsilon vanishes, because expected value of epsilon zero. In the similar way what you have is the, average return of the portfolio which is efficient, or particular stock which is efficient, is equal to r i. So, consider r f is in some way a proxy of alpha i, because if you remember the y axis where it cuts, it means that it does not have any risk, but it has return, but then if you go to the portfolio analysis which you have done, and try to find out the tangential portfolio. This value which is equal to alpha is also equal to the r f which means r f and alpha which you are talking about are generally the same.

And will consider that that single index model when converted to the CAPM model, and try to basically find a similar between the capital market and the pricing line, is exactly the same, where the equation is given in such a way; that the return of the portfolio which we have, or the return of the asset which we have which is efficient, would be given by two terms; one is the risk free interest rate. Or if you take it to the left hand side it will give you the excess return. So, the excess return of the portfolio and the asset, would be equal to the excess return of the market multiplied by the ratios of the market. Ratio means, ratio of the risk of the market, and the risk of the portfolio. So, what you have. This is the risk of the portfolio which you are forming.

This is the risk of the market which you have, risk means standard deviation. So, the ratio of this, is exactly equal to the excess return which you have for that particular stock of the portfolio divided by the excess return of the market. So, obviously, if r p is r m,

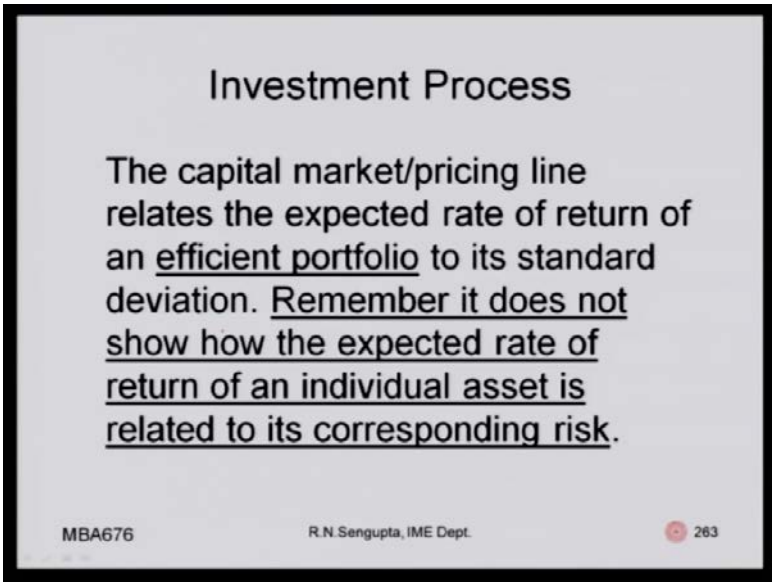
then the ratio is r m minus r p divided by r m minus r p minus r f divided by r m minus r f would be one. On the right hand side also sigma p will be equal to sigma n; hence the ratio also comes to be one. The slope of the capital market line, is basically price of the risk. So, if you remember the, in the single index model we consider tan theta which was beta. So, basically this is to the concept of the price of the risk which we have to formulate in your problem, in order to find out the best price of the particular stock, and whether it is according to the capital excess pricing line.

(Refer Slide Time: 12:27)



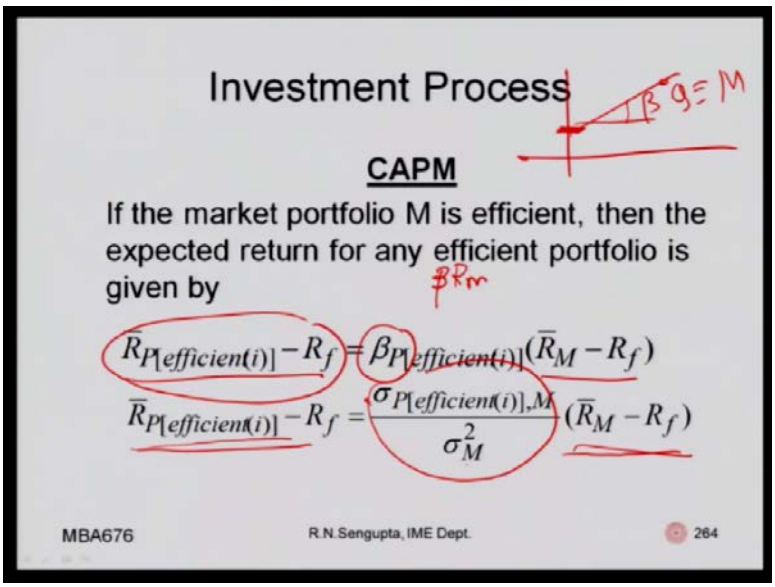
So, now, if you basically plot the capital asset pricing line and recollect. So, this is basically the, exactly the straight line. So, here we have the market, here we have the q, here we have basically alpha i's, here is basically have the r f. So, again this is in a similar way, to the exactly the single index model which you have already formulated. So, what we need to find out, is what is the tan of that triangle, which is this, or tan of angle which is this, and try to find out what is the price of the risk, based on which you are trying to formulate your so called portfolio of the asset.

(Refer Slide Time: 13:03)



The capital market pricing line relates the expected value of the return of the efficient portfolio to it is standard deviation. Remember it does not show how the expected value rate of the return of an individual asset, is related to the corresponding risk. So, what we are trying to do is, that trying to find out the so called efficient portfolio with respect to this standard deviation of the risk. And efficient portfolio would be compared with the market, because we consider the market to be most efficient.

(Refer Slide Time: 13:33)



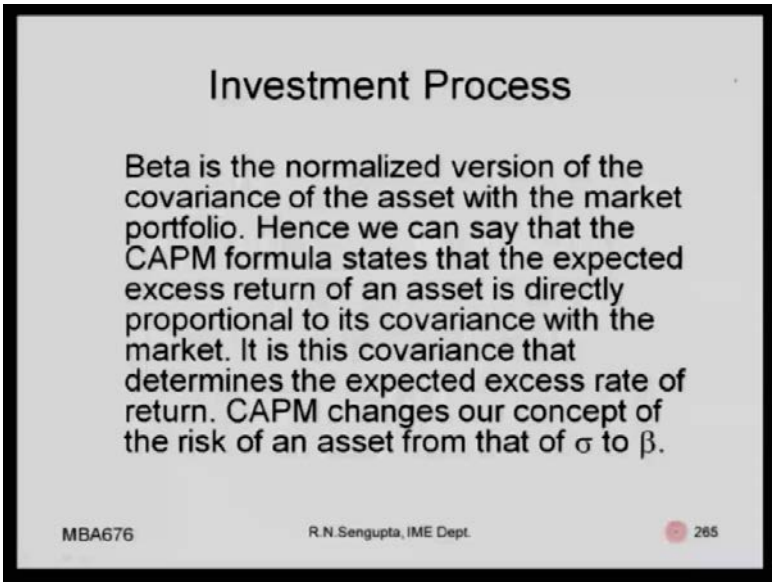
If the market portfolio m is efficient, consider is efficient. Then the expected value of any efficient portfolio, as we have already discussed would be exactly the same. So, if you come back to this formula. So, what you have, is the excess return, excess return. Now in

this term, it is basically standard. The ratio which is basically the standard of the covariance's extended between the so called portfolios in the market divided by the covariance of the market with itself, which is the risk. So, if you find out the ratios and if this is one; means that the covariance of the portfolio with respect to the market divided by the covariance of the market with itself would be one, which means that if we expand that term; the covariance terms basically three terms; one is the correlation coefficient, one is the standard deviations in the market, one is the standard deviation in the portfolio.

So, if the correlation coefficient is one, and the market on the portfolio is are exactly the same. Then this ratio which we have in front of you would be one, which means the excess return is the market as it has exactly equal to the excess return is the portfolio. So, any positive negative movement of market would be immediately mimic in the same proportions in the portfolio. Another formula would be exactly the same. Here what we have is the excess return of the portfolio, or any particular asset is equal to. If you remember and go back to this formula, initially you have basically beta in two basically r m, so exactly the same. This is basically the tangent, and this is the excess return of the particular market with respect to the port portfolio. So, if you have a particular straight line.

So, this is the excess return which you have. This is r f minus r p minus r f, this r f is given, which is the risk free interest rate. If you take it to the left hand side, you have basically the excess return, and you have the tangent. So, what you want to find out is that beta, and where the value of q is r m is, and whether the particular slope is greater or less, based on which will find out that the pricing of the stock has been done perfectly, and what is a relationship between the efficient portfolio and the risk of that particular portfolio. Nothing to do with the market so, you are trying to basically compare the efficient portfolio, if it is efficient with the market and find out, how good or bad it is.

(Refer Slide Time: 15:51)



So, beta is a normalized version of the covariance of the assets with the market portfolio; hence we can say that the CAPM formula, states that the expected excess return of an asset is directly proportional to it is covariance with the market. It is this covariance that determines the expected excess return of the rate of return. And what you are trying to do is that, for any particular stock you have a standard deviation. So, what CAPM actually does is that, converts the standard deviation which is the best measure of risk for different type of assets into only one corresponding symbol or use which is basically the beta. So, beta or a particular stock, or a particular portfolio, or a particular efficient inefficient portfolio whatever it is, gives us the overall scenario, what the risk of that particular asset is, or the portfolio is with respect to the market.

(Refer Slide Time: 16:44)

Now we will consider the security market line. So, the CAPM can be expressed in a graphical form, by regarding the formula as a linear relationship. Remember that, this very important, and; obviously, you can extended for the case when you have the non-linear relationship.