**Quantitative Finance**

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**Module – 02**

**Lecture -10**

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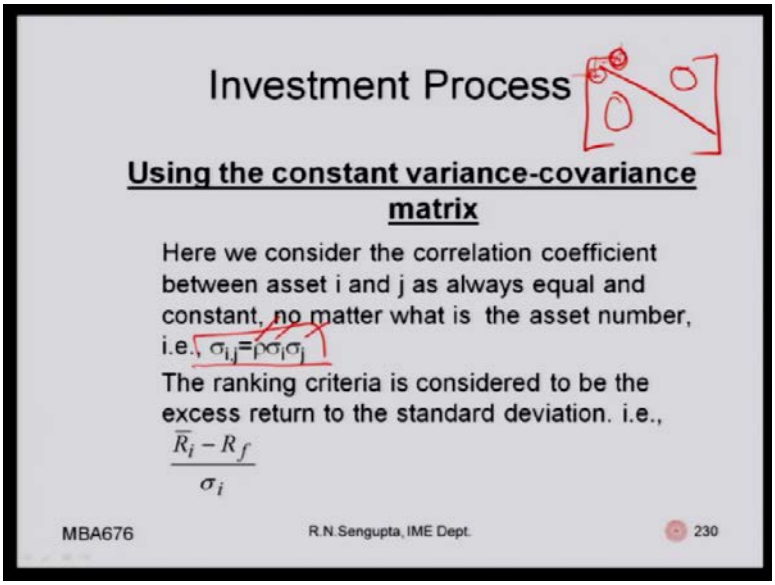


Now, here we consider the correlation coefficient between the next types of problem we are going to solve. Here, we consider the correlation coefficient between as the i and j always as equal; that means, rows i j which is the correlation coefficient existing is always fixed. So, if we expand the conveniences. So, covariance is a basically three terms; one is the correlation coefficient, one is the standard deviation on the i, one is the standard deviation in j. So, when you considering these three terms you will consider one as fixed. So, generally when you have the various co various matrix, what you have, is basically the principle diagonal is the variants of the first second third fourth in the nth one, depending on of the different matrix. And the off the diagonal element which we have, are basically the covariance variance values, covariance values of the ith jth of the first and second so on and so forth.

So, if we have in the first row in the second column, the covariant of the first to the second. In the second row, and the first column you will basically the covariance of the

second to the first, but considering the symmetricity; obviously, these values would be equal. Now what we are simplifying is that, you need to find out only the return which is arrivals. The standard deviation which is sigma i or sigma j, and the correlation coefficient you need not be bothered, because you will only consider a fixed value. So, now, you ranking would be done in the case where. Till now we are considering the excess return to the beta. The beta means, the beta which is of the risk for that particular stock with the market. So, now, will be basically considering the excess of the particular stock, with respect to its own standard deviation.

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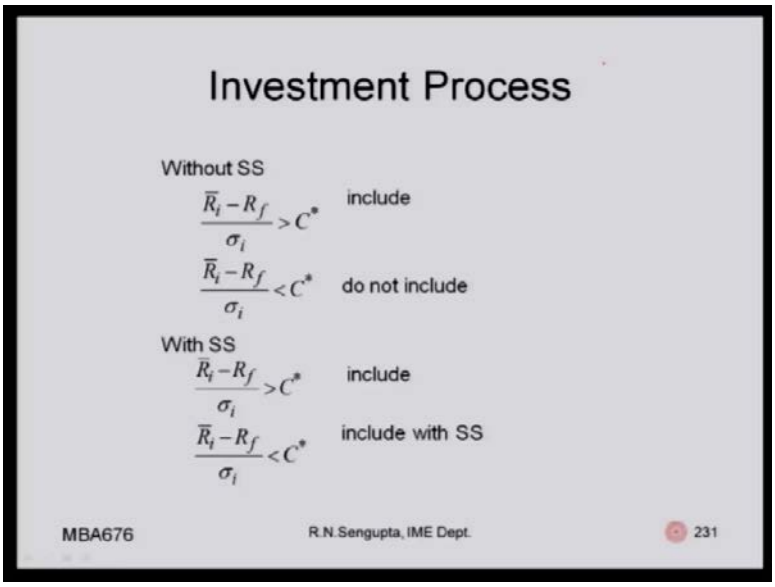


Again the simple formula; when there is no short selling; you will include those particular stocks in the S M in the portfolios. If access return to standard deviations; not beta. Standard deviation is greater than some c star. Again I am using the simple c star. And if the access return to sigma, is less than c star of obviously you will not include when there is no short selling. When you come to the sort selling case, again the same rule; access of return divided by standard deviations, if it is greater than c star, you include in a positive sense. excess of return divided by sigma, if it is less than c star, you include, but in a sort selling sense, but again in the sort selling sense, there will be two arms; non lankness definition, where the some of the weights is equal to one, and linkless definition with some of the merge of the weight is equal to one. So, again you will follow the same policy, and trying to differentiate the three cases; sort selling allowed, sort

selling not allowed, and sort selling allowed using the link less definition. When I use the word sort selling allowed; that means, I am using the non-link less definition.

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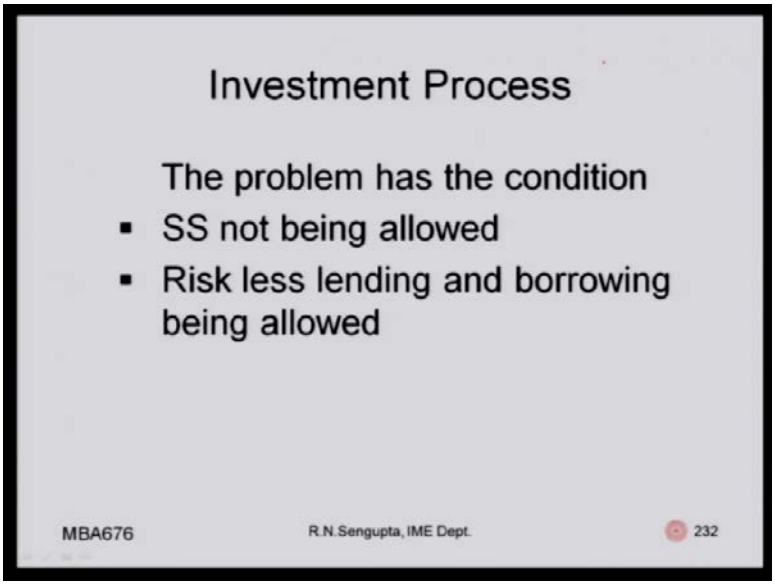


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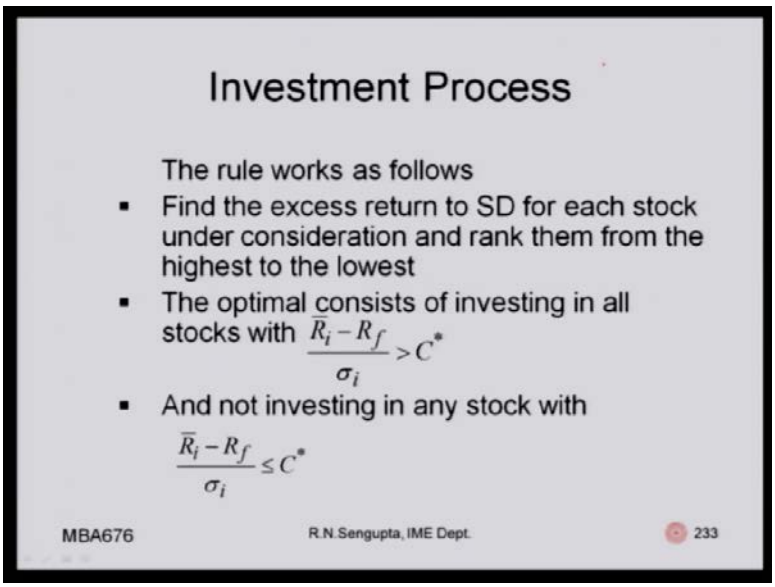

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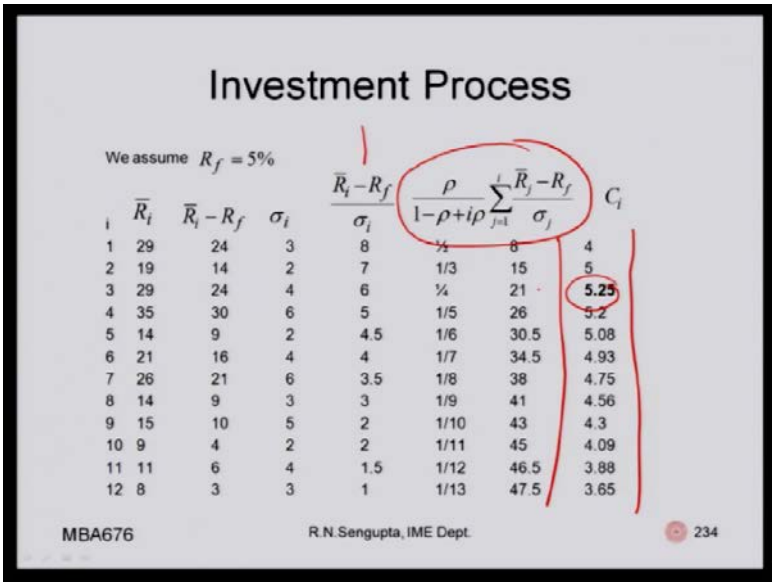


So, problem has the condition sort selling not allowed, risk less lending and borrowing being allowed. So, the rule focus as follows; access return, greater than c star, where the denominator is sigma include, access return less than c star, do not include. Again I am taking a different example, with 12 stocks. The first column is the stock numbers, second one is return, and the third one is the access return only, not divided by any standard deviations. Third one is the standard deviations, fourth one is basically the access return to standard deviations, so what I have done in this slide is basically. In the other one I have broken down into two different slides, and basically done the calculations.

So, I am trying to basically compact that and try to give to the results. Only remember the formula which we are using is different. I am not interested and trying to discuss. The proof of the formula, they can be found in internal group or any good book like investment science by Luin Berger which we have already discussed. So, I will just go through the simple calculations, without going to depth of the proofs and the theorems. So, again in the fifth column, you will basically have the access return to standard deviations, then again you find out the formulas. So, once you find out c i. Again you find for the case when this no sort selling, there is one particular value of c star which is the maximum value after which again c i value starts to basically decrease.

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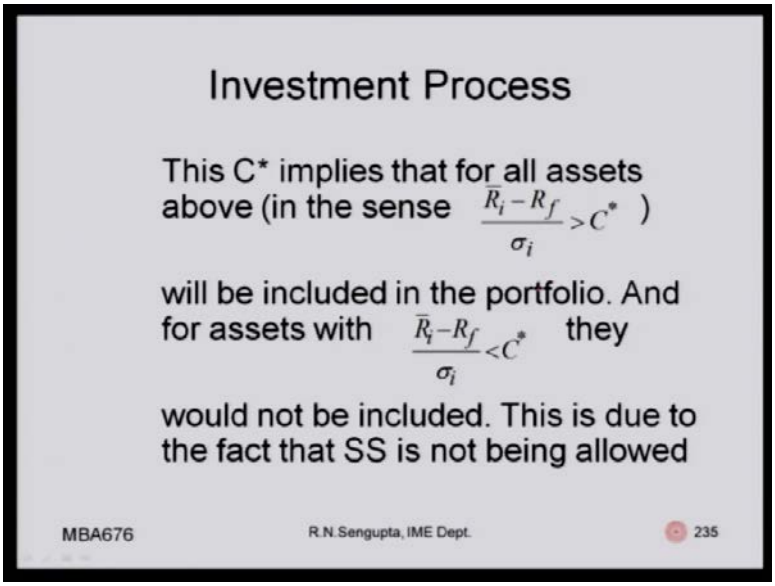


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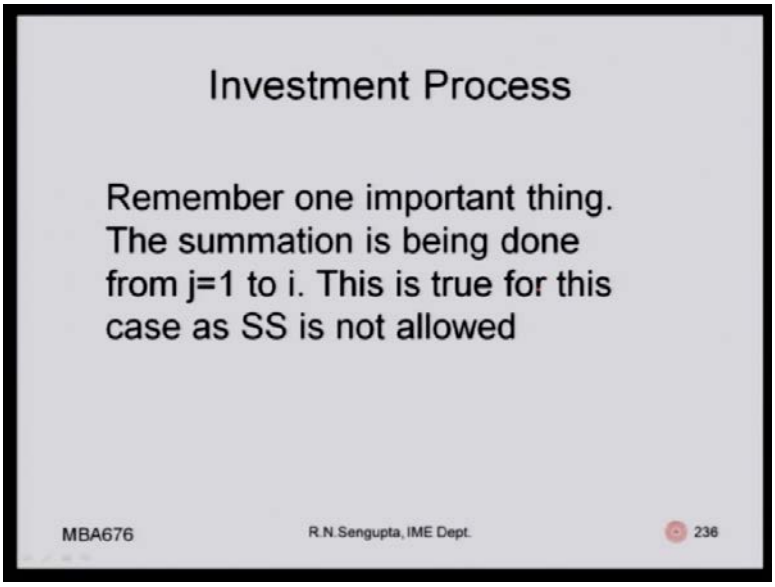

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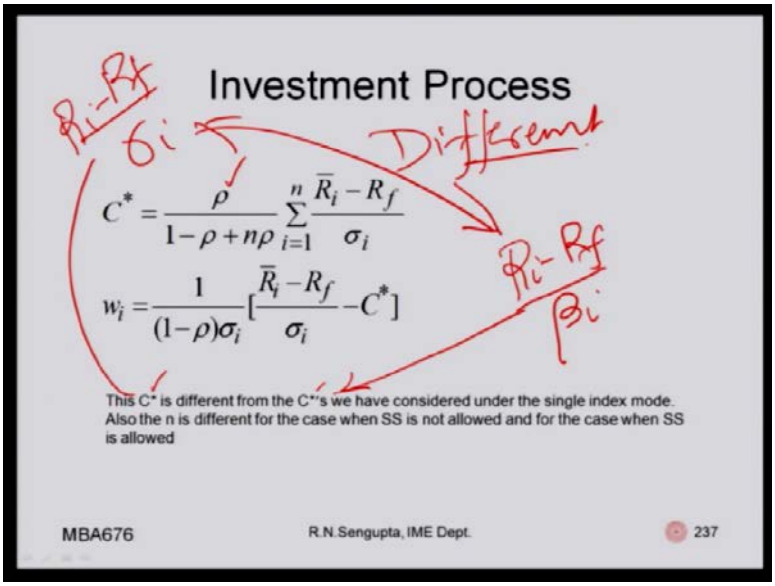


So, if you come to the these values remember another things, whenever, again the repetition, when we are trying to find out the summation, it is basically j is 1 to n or j is going to 1 to i in i basically means that at a particular asset till which you are trying to find out the value of c i, c star is given by this formula, where row is basically the constant correlation coefficient which we have already considered. And this weights are given by this formulas; this c star value which we are trying to find out with different from c star, which have considered originally, where your actual formula was this, where you trying to rank it according to beta i. So, this c star which we are considering, and this c star which you are considering for the case when is r i minus r f by sigma, these two are different; remember that.

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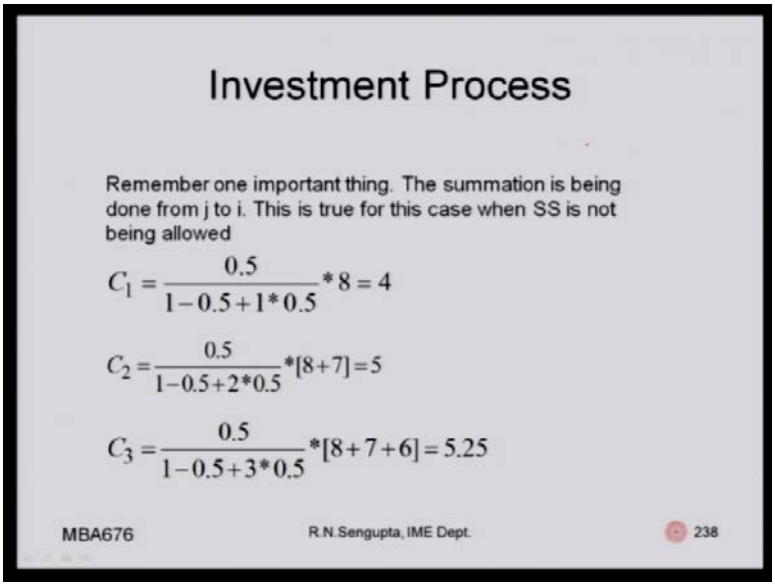


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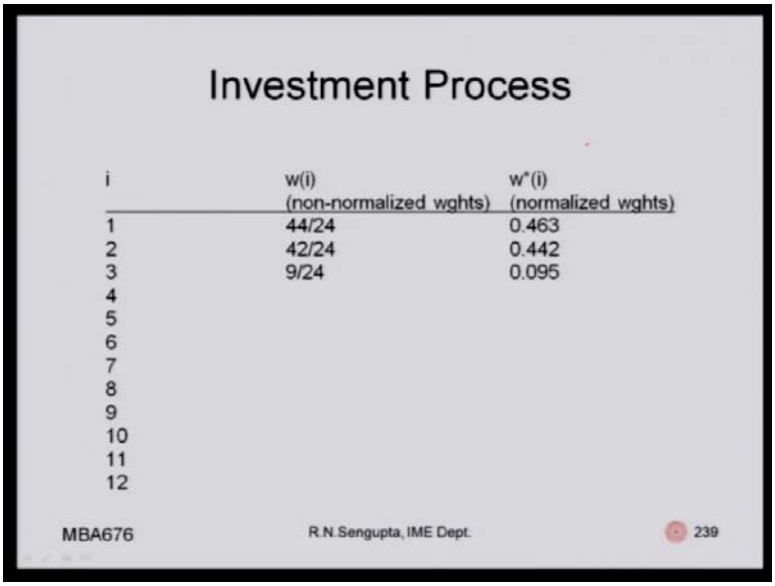


So, the values of c i you find out four five so on and so forth. 4. 5.25 which is the third value in the last column, which we just saw in the overall calculations. Using the formula if you find out non normalized weights, they come out to be 44 by 24 42 by 24 9 by 24. So, if we add up, it does not does not come out to be one. If we normalize them, it is in the concept. Normalization as we are doing. If you see the weights come out to be almost equal to one. so; that means, you will invest 46 percent in the first, 44 in the second, and about nine percent in the third, and the rest would not be included, because there access return divided by c sigma is negative, with respect to the c star value which you found

out, and obviously the case that sort selling is not there. Extend the problem four to the case where you have sort selling allowed link less definition not being allowed.

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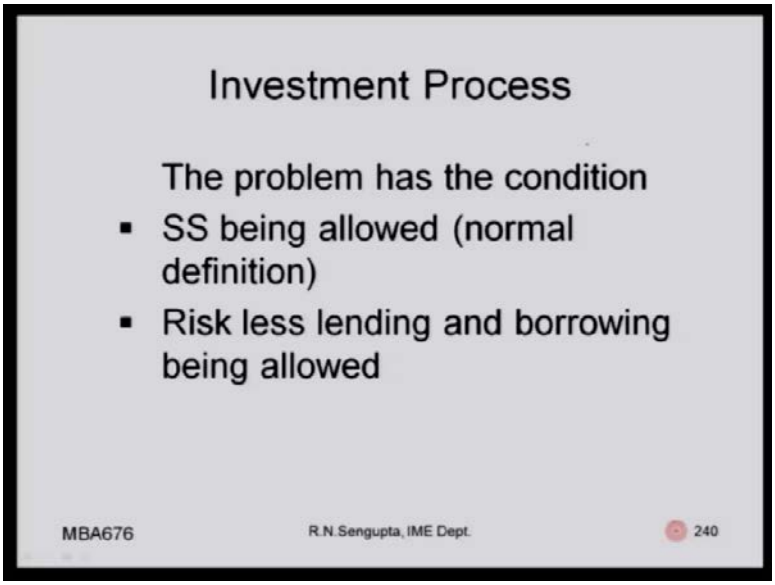


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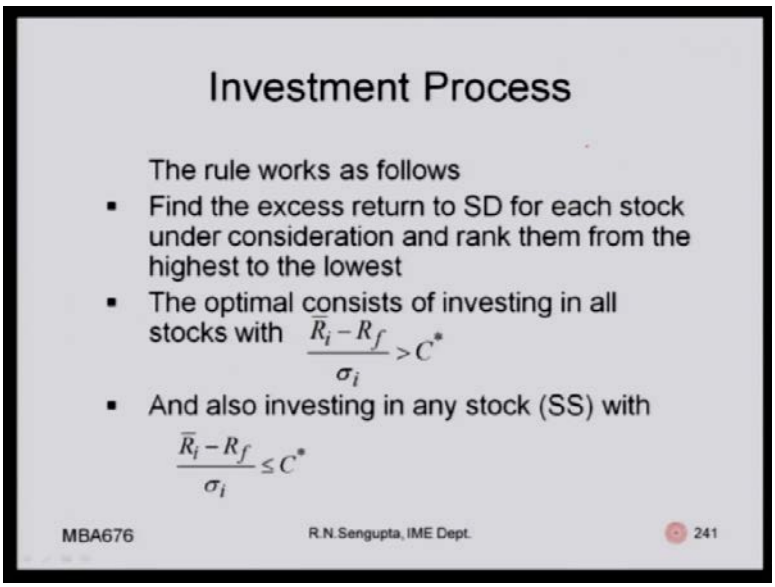

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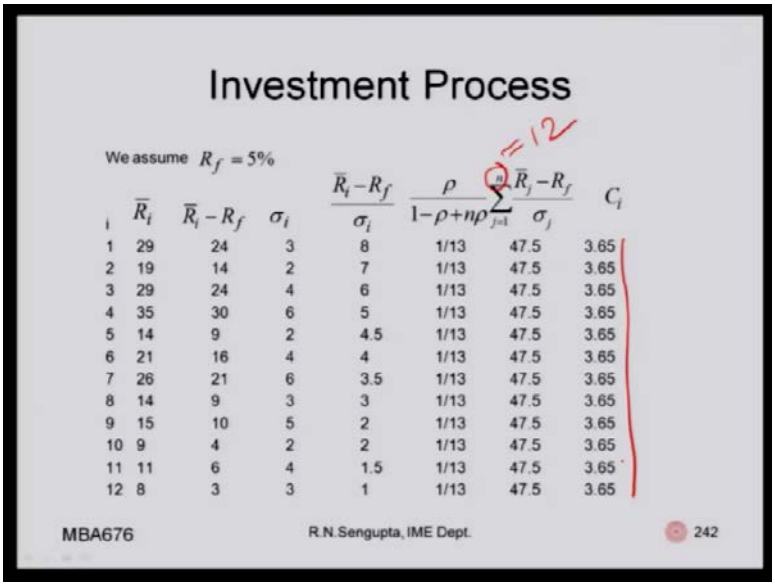


Again the same concept, risk less lending borrowing there, sort selling being allowed. Again with repetition r i minus r i divided by sigma is greater than c star include, r i minus r f divided by sigma less then c star include, but in a negative sense. Again, the same formula; first column is the stock number, second is the returns, third is the access return, forth is the standard deviation, fifth is actual ranking which you are doing by dividing the ratio of the access return divided by the standard deviation. So, again we use the same formulas, but with the important note that the n which you are trying to find out in this problem, is for the whole set of asset which are in front of you. So n would be basically twelve for your case. just an example which is resolved, considering sort selling was not there, will consider that end would be that particular asset which we are trying to find out the c i values. So, the values come out to be of c i comes to constant which is 3.65.

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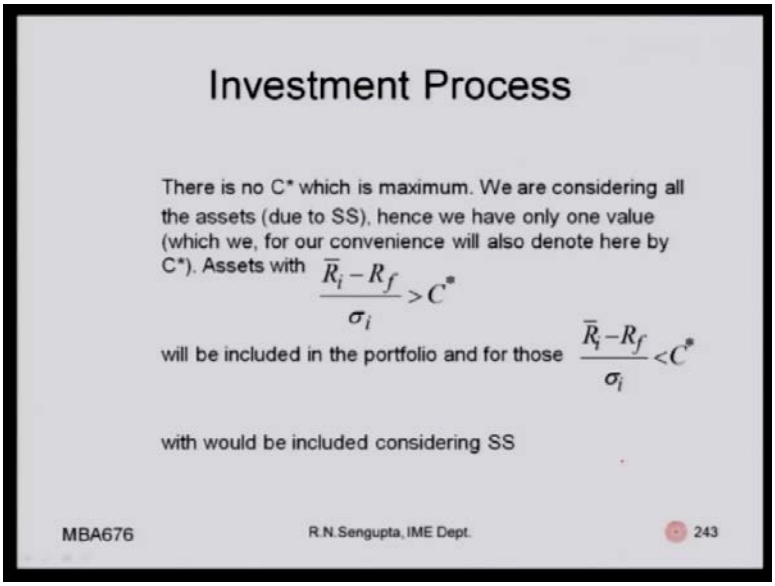


So, we will include those assets in positive sense, where it is greater than this three point the value which you are find out for c star, so called c star and in a negative sense for other stocks. Remember one another thing; the summation is always for n which is 12.

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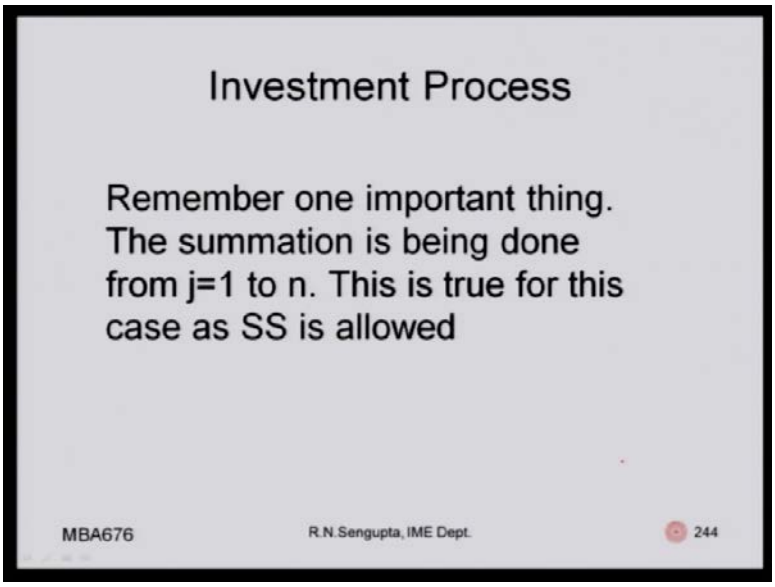

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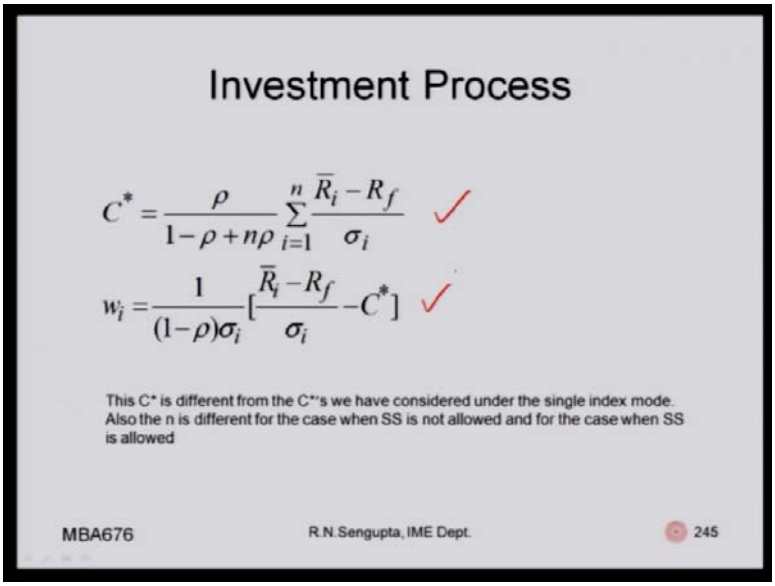


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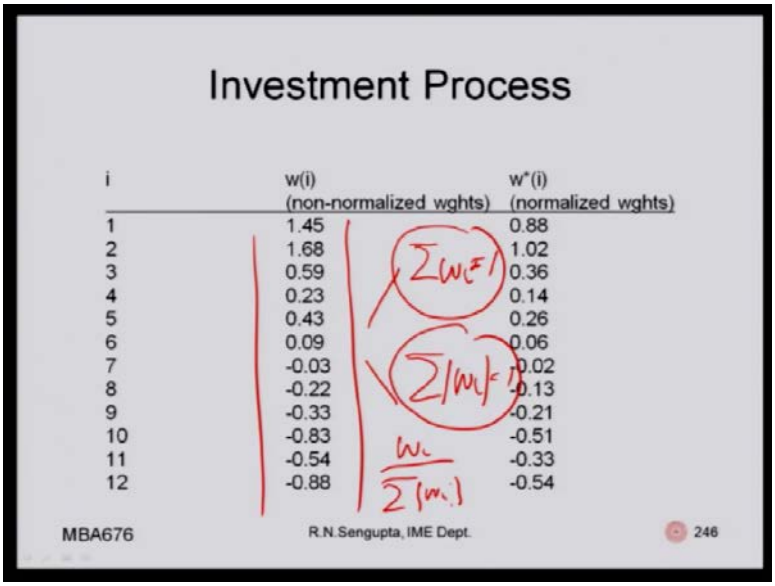


So, here is the formula for c star, and here is the formula for w i. Once use this, your non normalize weight come out to be this, which technically means that if we add up; obviously, there is one, but if you do the concept of normalization using the definition; obviously, you will get two different answers. In then non link less definition, this is true. link less definition this is true. So, whichever you use, be sure that depending on the formula which you are trying to used, your normalized would be done according to that, because if you are doing the normalization case. For this one, so; obviously, if you are following the interest definition in this case you are trying to be basically compensate

some collateral such that, if there is any risk, or non-payment, or non-returning of the stock for your part to the person from whom borrowed then he or she basically has a collateral in a order to overcome the overall risk interest definition consider, again the same thing greater or less. You have to include, was in the positive sense negative sense, but only again remember the division which you are doing for ranking is sigmas not betas. Again we use the same formulas, but find out the values again.

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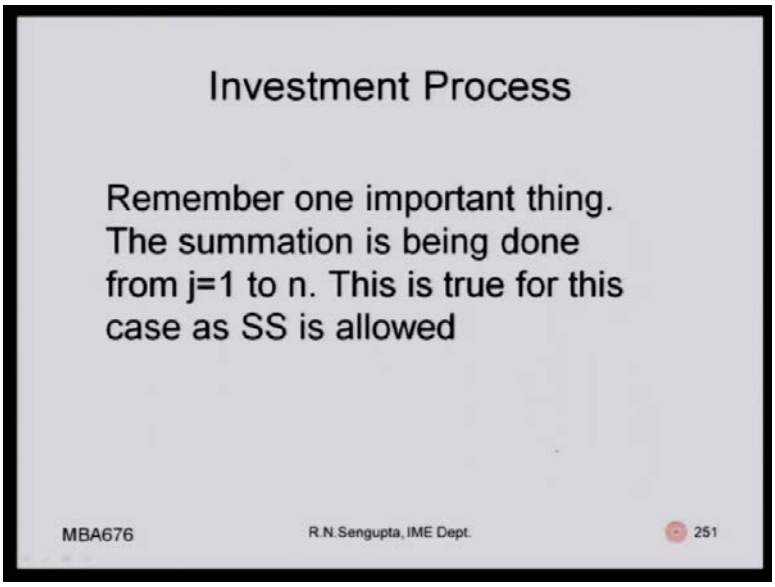


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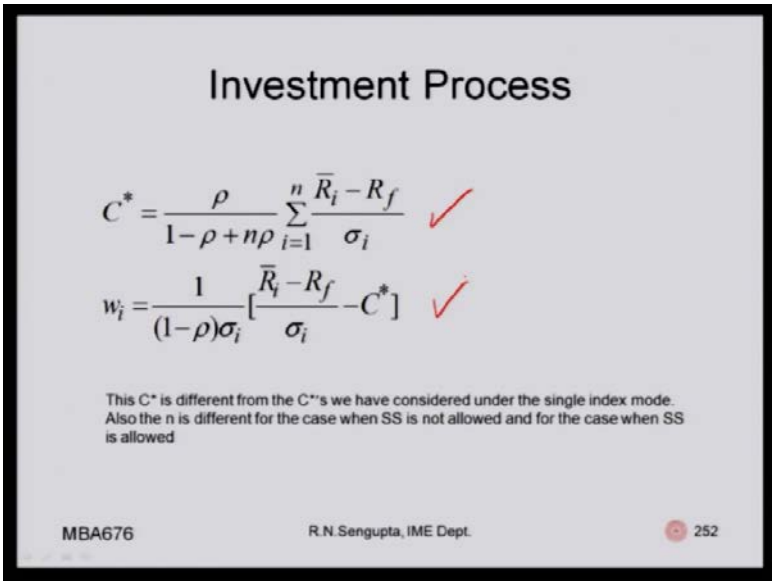

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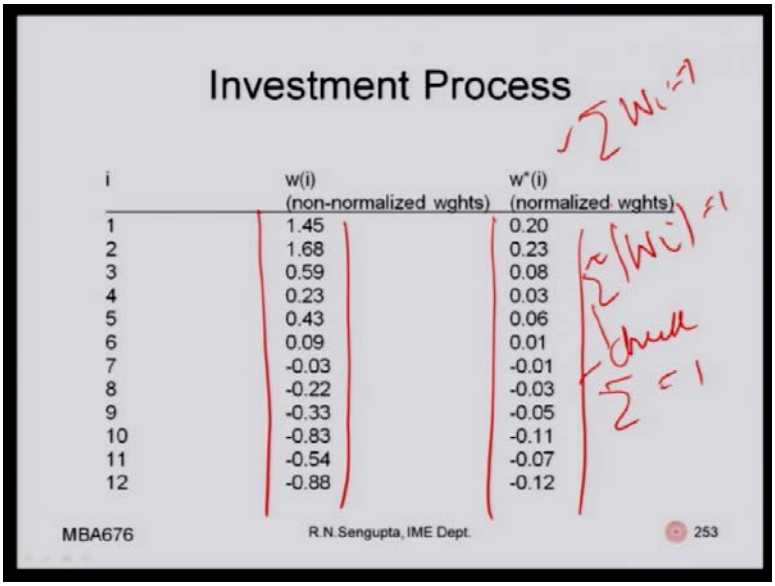


Sums are always from j is equal to 1 to 12, which were n is 12. The formula for c star remains the same. Formula for w i remain the same. When you do it you will find out the normalize weights again comes out to be, as for the formula what you are using. The normalized weights come out to be the inter definition. Check that summation should be one, but this summation which I am using for the link less case, this is the summation. For the non link less case this is the summation. This is very important to be noted in both the cases.

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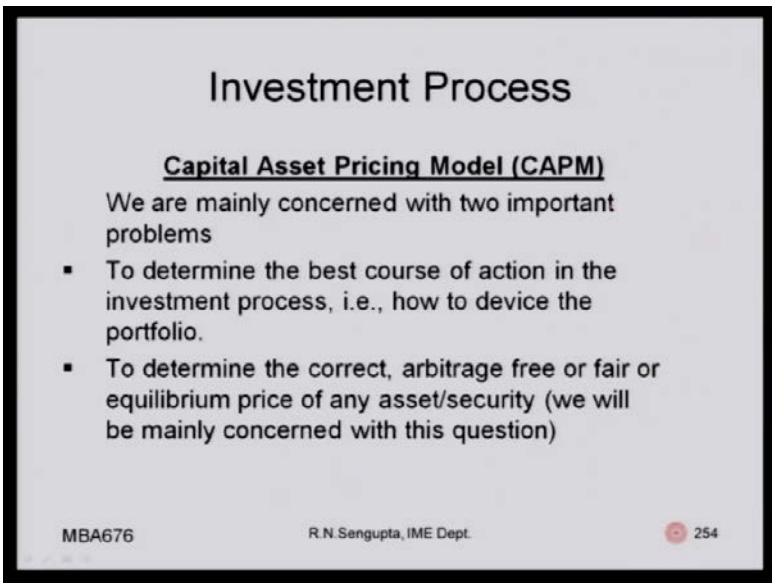

Now, the capital asset pricing which we have already dealt with that, but we will now going to the details for that. So, what we are going to consider is, that given a particular stock, how can you find out this overall change in the price, or find out a returns, given that market is changes. So, if you remember in few class back, we consider the single index model. So, in the single index model problem was exactly the same. We are not going to the details of that. We will skip these slides if they are there, but only going to the actual concept of the CAPM model is being utilized. So, when you considering the significant model we consider there was one dependent variable which is r i or which is say for example, y. And on the right hand side if all equation there is basically alpha, which is the, where the line straight lion cuts the y access, and there is basically beta i in to r m bar, basically bar we are considering the average value. So, beta is basically the tan of the angle, which is basically are slop. And if you are not taking the average values; obviously, they would be a quite noise which is epsilon.

So, if you take the expected value, then on the left hand side you have r i bar is equal to alpha i because this is the constant. Theoretically we are considering as a constant plus beta i into r m bar plus epsilon is not this so; obviously, you would not be there. And based on that you find out the best line and try to based find out how the prices are changing. So, this is the multi index model; obviously, in case of only one index now will basically have more than one index. So, those index is indices would be given, but say for example, i 1 i 2 i 3 i 4 whatever is there, any one to basically find out the relationship of the rate of the change of the prices of the particular stock with respect the each and every i's. So, obviously, in the multi index model, you have a relationship where the price have particular stock would be dependent on the different type of independent variables, but the CAPM model consider is basically the way how we try to analyze the problem from the single index model.

So, having said that I am sure that student have already gone through in the single next model would be in a much better position to appreciate how CAPM model are be utilized, not from the sense of theoretical knowledge how you derive it, but from the sense that how it can be utilized in the market sense. In order to understand, how the particular stack is doing with respect to the market, and how changes in the market would basically give us some information in the change of the particular stock. And how good or bad the particular stock is with respect to the market whether one should invest

or not invest. So, in the capital asset pricing model we are mainly concern with the important problems, but is to determine the best course of action investment processes, and that is how to devise the portfolio, and how to make a decision whether you may include not include that particular stock. And to determine in a very correct sense the arbitrage free or fair equilibrium based on which you can decide whether you can particular stock should be included and not included, and find out the rate of return for the particular stock.

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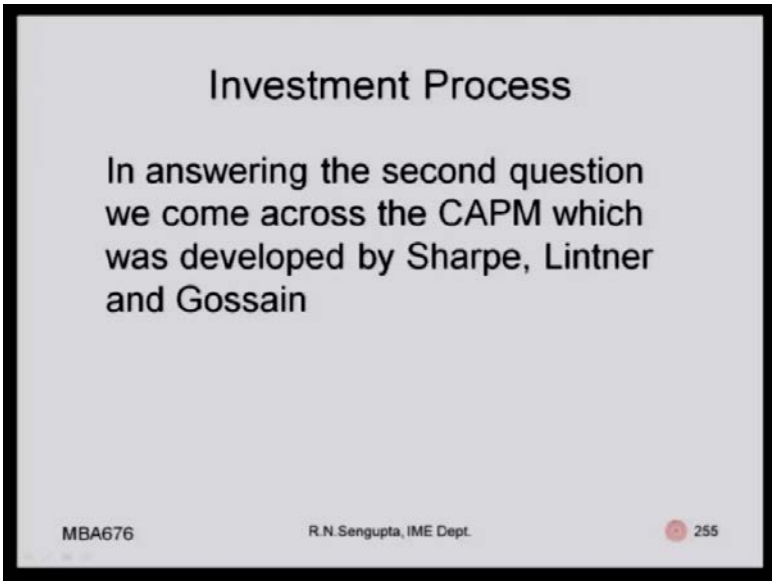


So, answering the second question, will come across the CAPM, which was developed by the Sharpe, Lintner and Gossain. So, basically I will just discuss their general trend or their general concept of CAPM, and basically going to the other concept which would definitely give us a good picture of how the prices can be found out for the portfolio point of view.

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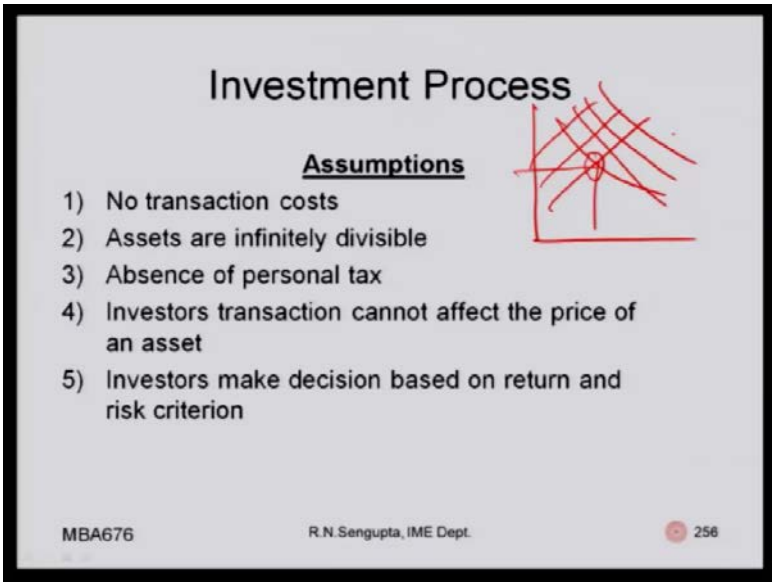


So, now, if you remember in the single index model, I only dealt with the assumption which are there from the theoretical point of view, from statistical point of you; one was expected value of the error is zero. Covariance between the errors is from time one to time two to time three so on so forth is zero. Covariance exiting between r m and the white noise is zero, and we consider also normality to redraw for the returns of the market as well as the return of the stocks. So, those are all from the statistical point of view. Now the question would come that. Well that was fine from many of us solving this single index model from a theoretical point of view. What if you try to utilize those the exact model in the CAPM one.

So, now, obviously, there would be some assumption. So, the assumptions are; will consider there is low transaction cost. And if you remember did mention in one of the classes, when you are trying to do the portfolio one when we are discussing the Markowitz model, we did discuss that there would not be any transaction cost, there would not be any dividence, there would not be any brokerage cost so on and so forth. So, those are on the very simple stick sense.

So; obviously, we will later on that using formula, using some solve examples, how they can be relax in order to get much more practical and much more solutions, which are totally removed from theoretical sense and more active and more closer to the to the actual market. We will consider assets are in finite devisable; that means, you can buy

and sell assets in one unit only, but there may be cases where assets you cannot buy in contains of one. You may be required to find buy and sell hundred of such assets at one go. So we will relax those assumptions. We will assume that there is no personal tax. So, reason being that if this personal tax; obviously, the buying and selling; like the case when you have the brokerage price, that will effect for over all selling. So, we would not consider that to be presenting in a problem. We will also consider investment buying and selling does not effort the equilibrium price. So, if you consider in the demand and supply case there was basically price and the quantities. So, this price would not change dramatically as the demand and supply curve changes.

So, it will basically instantaneous in the sense, any movement in the demand and supply could be immediately reflected in the price and the quantity of the particular stocks; such that the prices which we have in front you reflects the market to the maximum possible extent without any asymmetricity in the market. We will also consider investor make decision based on the risk return profile only. So, if you go back to our initial problem where we considered the utility function, where it was a person invests depending on the person invest depending on the case that he or she wants to reduce the rest to the maximum possible extent. We will consider invertors invest only for the case where they want to maximize the expected value the utility, and this utility is always on increasing trend. And they want to basically invest in particular stock or set of stocks to form the portfolio where the risk is minimum.

If you also recollect, I did mention time and again, and I will defiantly mention it timely later on that the utility function was quadratic. So, if it is quadratic the square terms means, that whenever I invest more amount of money w; obviously, may utility would be on increasing trend; that means, I will get much more net worth from my investment. So, this would always hold true. And also I will consider that I will always take a decision where at the end of the day. If I take different decisions, where both the returns are same, but in one case the risk is low; obviously, take the one where the risk is low. So, with these two assumptions, which is basically the property of nonsatiation, and the property of non being the risk averse, would also whole through for the CAPM model, apart from the first to fourth, which are very simple concept on the practical sense. So, if we consider the these assumptions; the five one, and also the statistical one, then it makes sense in order to basic implement, and combine this statistical assumption along with the

economic assumptions, and then try to basically formulate of one to one simply between the single index model and the CAPM model, so that wants you to understands the theoretical concept of single index models of the simple linear regression, which we call the statistical sense, then; obviously, it will make sense, but how you can utilize the same concept in the CAPM model.