Quantitative Finance

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Module – 02

Lecture - 09

So, welcome back to this class related to quantitative finance. So, as we were discussing that considering that short selling is there are not there, you have take a decision whether to include that particular asset in your portfolio yes or no depending on different criteria. Now, if you remember the criteria was that if the return of a particular stock is greater than equal to r f which is the risk free interested, you will include that if it is not; obviously, would not include that if short selling is not there. But if the short selling is there, then for those stocks where the return is not in access with r f, which is the risk free interested; obviously, it mean you will short sell those asset and try to utilize the money in trying to buy other assets.

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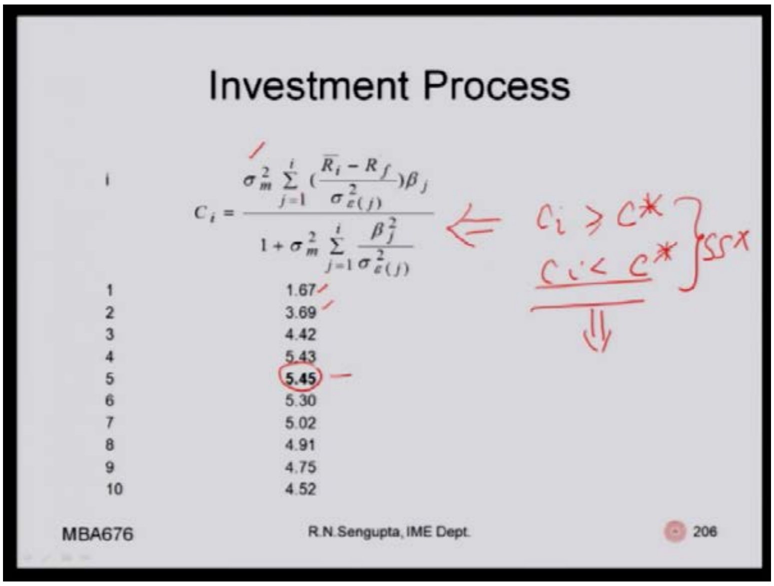

So, there is a certain C star cut off which we have already discussed. So, in case say for example, for each and every stock, you will find out the C of i value which is given by this formula which we have already explained. So, if this is equal to greater than equal to C star so; obviously, it will mean that you will include that stock; if it is less than equal to C star, you will obviously not include this stock, short selling is not allowed. And obviously, the vice versa would hold true where in this case the last case where C i is less than equal to C star you will consider them to be included, but in the short selling mode.

Now, the calculations I have in the, if you refer to the last class, the calculations are very simple. What you have is basically the value of sigma square m which is the market risk corresponding to whether you are trying to calculated for the Bombay stock exchange or New York stock exchange or NAC whatever, and this value R i minus R f is basically the excess return which you have over the risk free interested. This beta i j j is the corresponding risk which you have from the beta value for that particular stock, and the sigma square epsilon suffix epsilon in the white noise so that means, if you remember which we were discussed, there was one set of risk which was a non divertible and another set of risk which was divertible, the systematic risk and non systematic risk.

So, if you plot the values of write down for the values of C, c I, so if you include only one stock the C i value is 1.67; if you include second one, it is 3.69 and so on and so forth. So, these are tipping values C star after which as you keep including more stocks, the value start decreasing. So, this is basically C star value depending on the case. So, for the those particular stock, which you are include if it is greater than C star and for those stock you would not include if this no short selling if the values less than C star. Only one thing, I will try to recollect and also make a point to the students that when you find out C star, all the summation would be done or the C i value which you find out, all the summation would be done for the values of j is equal to one to i. So, if i is two, we will sum up all the values starting from one to two; if i is five, we will sum up all the values for 1, 2, 3, 4, 5 considering they are five different asset in your portfolio.

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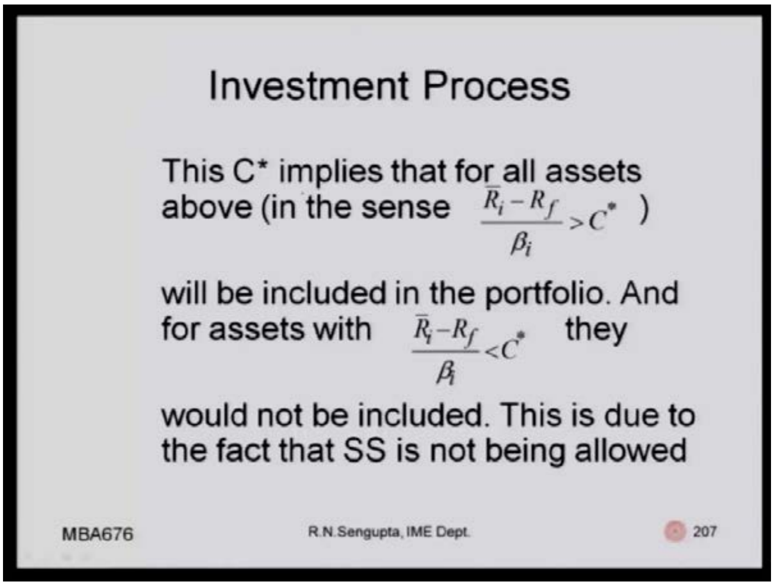

So, this C star which you have set is 5.45 is the maximum values implies that for all such assert above, you include them in your portfolio for values lower you do not include if short selling is not there.

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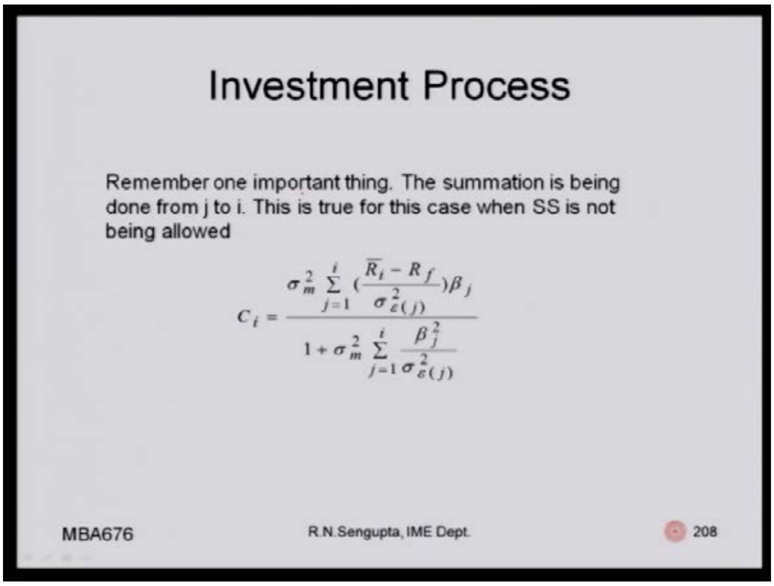

Remember one important thing as I mention the summation is being done from j is equal to one to i depending on how many I s are there.

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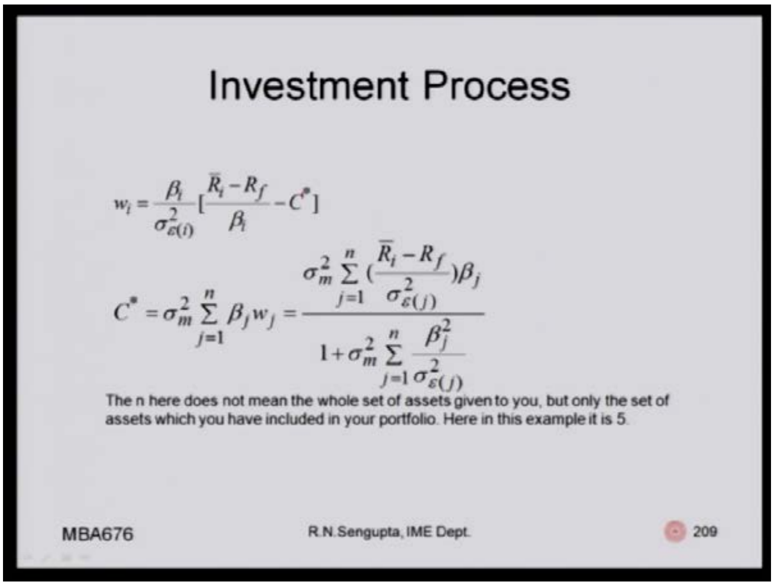

Now once you find out the value of C star and those values of stocks which you would be included; obviously, you have to find out value of weight, because weight is important. The reason being once you find out the weights, you have to find out they over all returns; that means, w i is the weight of the particular stock r i is the return. And similarly you have to find out the variance. So, where the first term is the weight for the i th one; second term is weight for the j th one; third one is covariance between i and j which has three terms. One is the correlation coefficient between i and j, the second one is the standard deviation of i, and third one is the standard deviation for the jth one. So, by formula without again any proof, we can find out that the value of w i is given by this, while C star value is that n now this n which have writing is not the whole set is then value f n which will basically make that particular portfolio, so called optimum considering the all the values of C i are greater than equal to C star value.

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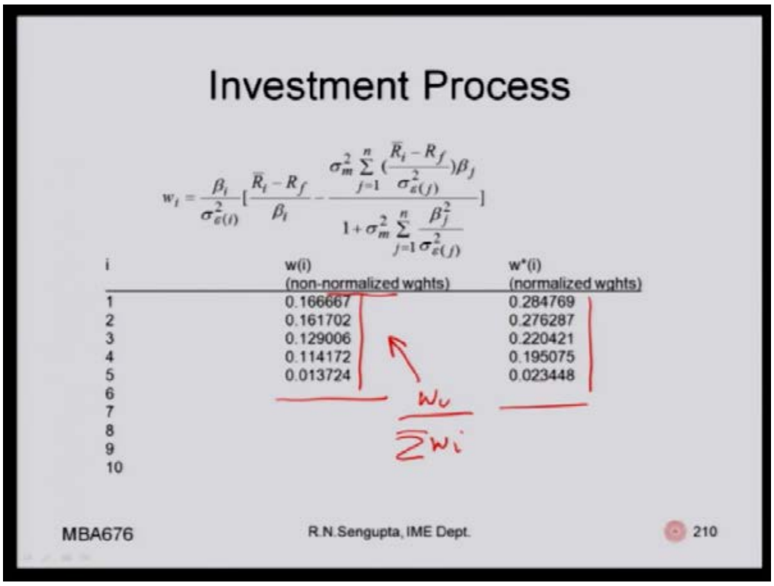

So, for the problem when there is no short selling, we find out C 1, C 2, C 3, C 4 and so on and so forth. And then once you find out the weights, the weights are given here; remember they are non normalized weights, you should add them they would never add up to one. So, once we find out the normalized weights using any normalized techniques which we have already discussed, a simple one where which would be w i divided by the sum of w i's. So, once you have that this w i would be the non normalized one. Once you find them you basically have the normalized weights; if you add up them they should basically add up to one which means the whole amount of money which you have to invest is being invested in the first, second, third, fourth, fifth stock in propositions of 28.4, 27.6, 22.0, 19.5 and 2.3 percentages is respectively.

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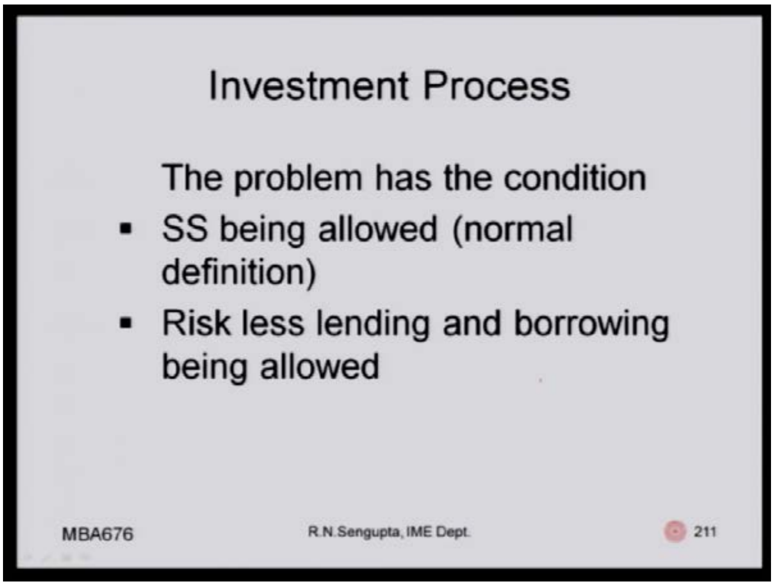

The problem has the condition that short selling is being allowed. In the second case we are consider risk less lending and borrowing is being allowed; that means, you can borrow from the bank, utilize that money to buy different type of assets. And obviously, if there are different type of stocks whether returns are less than r f; obviously, you include them in a short selling mode; that means, try to borrow them and sell them and utilize that money to buy different stocks for which the returns are more than the risk free interested which you are planning to invest.

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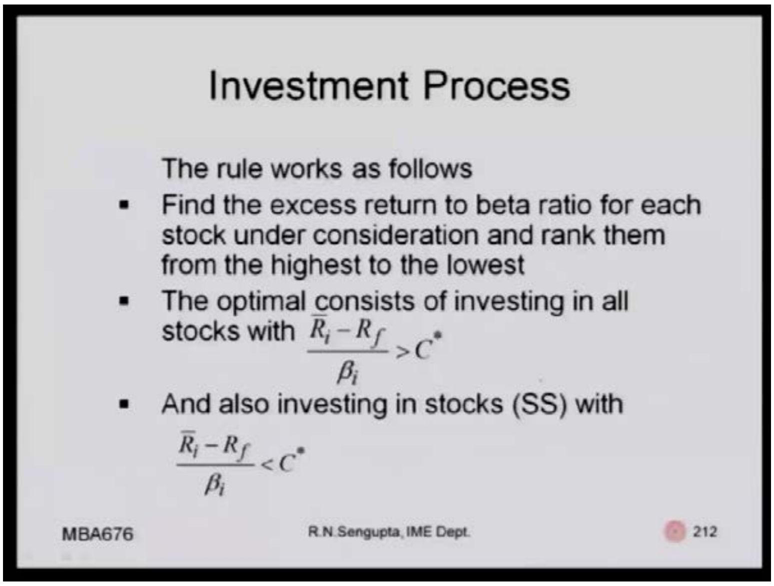

The rule works as follows. Find the excess return to beta ratio for each and every stock as we did for the last set of problem under consideration and rank them from the highest to the lowest. The optimum consist of inverting in the all the stocks where the values are the greater than equal to C star and also investing in stocks in a short selling mode for which the values of R i minus R f by beta is less than equal to C star.

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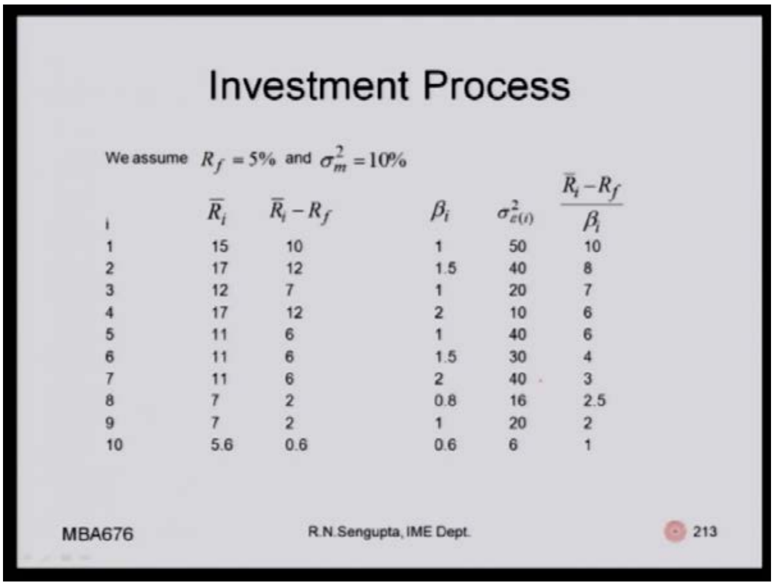

Investment Process

We assume $R_f = 5\%$ and $\sigma_m^2 = 10\%$

i	$\bar{R}_i$	$\bar{R}_i - R_f$	β_i	$\sigma^2_{\varepsilon(i)}$	$\frac{\bar{R}_i - R_f}{\beta_i}$
1	15	10	1	50	10
2	17	12	1.5	40	8
3	12	7	1	20	7
4	17	12	2	10	6
5	11	6	1	40	6
6	11	6	1.5	30	4
7	11	6	2	40	3
8	7	2	0.8	16	2.5
9	7	2	1	20	2
10	5.6	0.6	0.6	6	1

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So, again let us consider the problem, there are ten stocks R I s are given R i minus R f is also given in the third column, and the beta values are given, sigma square epsilon is the white noise. And finally, you have basically rank them according R i minus R f divided by beta which is basically the excess return over the risk free interested divided by beta.

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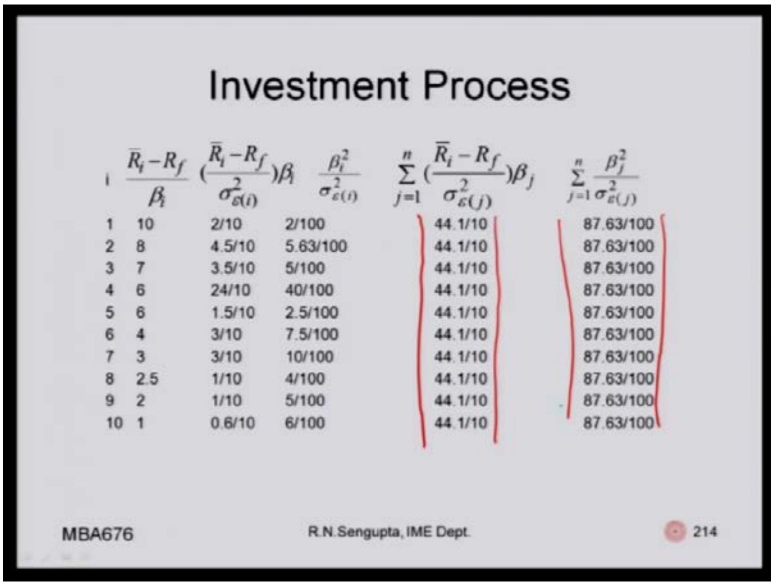

Investment Process

i	$\frac{\bar{R}_i - R_f}{\beta_i}$	$(\frac{\bar{R}_i - R_f}{\sigma^2_{\varepsilon(i)}})\beta_i$	$\frac{\beta_i^2}{\sigma^2_{\varepsilon(i)}}$	$\sum_{j=1}^{n}(\frac{\bar{R}_i - R_f}{\sigma^2_{\varepsilon(j)}})\beta_j$	$\sum_{j=1}^{n}\frac{\beta_j^2}{\sigma^2_{\varepsilon(j)}}$
1	10	2/10	2/100	44.1/10	87.63/100
2	8	4.5/10	5.63/100	44.1/10	87.63/100
3	7	3.5/10	5/100	44.1/10	87.63/100
4	6	24/10	40/100	44.1/10	87.63/100
5	6	1.5/10	2.5/100	44.1/10	87.63/100
6	4	3/10	7.5/100	44.1/10	87.63/100
7	3	3/10	10/100	44.1/10	87.63/100
8	2.5	1/10	4/100	44.1/10	87.63/100
9	2	1/10	5/100	44.1/10	87.63/100
10	1	0.6/10	6/100	44.1/10	87.63/100

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So, once you have that then again you follow the same procedure, find out the ratios of the excess return divided by the white noise multiplied by beta i for each and every stock find out the ratios of beta one i square divided by white noise which is the variance and then again stars summing in up. So but now remember one very interesting thing, if you see all the values are equal, now you may be thinking why this reason is very simple because if it is short selling whether the stock has excess return or less than excess return you are going to include that in the portfolio. So, if it is excess return is positive, they will be doing in a positive sense; if excess return is negative, they will be doing in a selling sense, but not for the other problem.

In the last problem, we say that you would basically goes step by step and include those stocks only where it was positive returns for negative returns where r i minus r f divided by beta those values and considering the values of C i which you found out C i was less than C star then you definitely not include. So, your summation was being done from j is equal to one to n where n is that number of stock which you are trying to include not the whole set, but here now you will include the whole set because all of the stocks have to be included either in the positive or the negative sense. So, once you find out the values of excess returns multiplied by beta divided by the variance of the white noise, so the values come out to be constant because n is fixed here which is about 44.1 divided by 10, and the values of beta square j divided by the variance of the white noise comes out to be 87.63 divided by 100.

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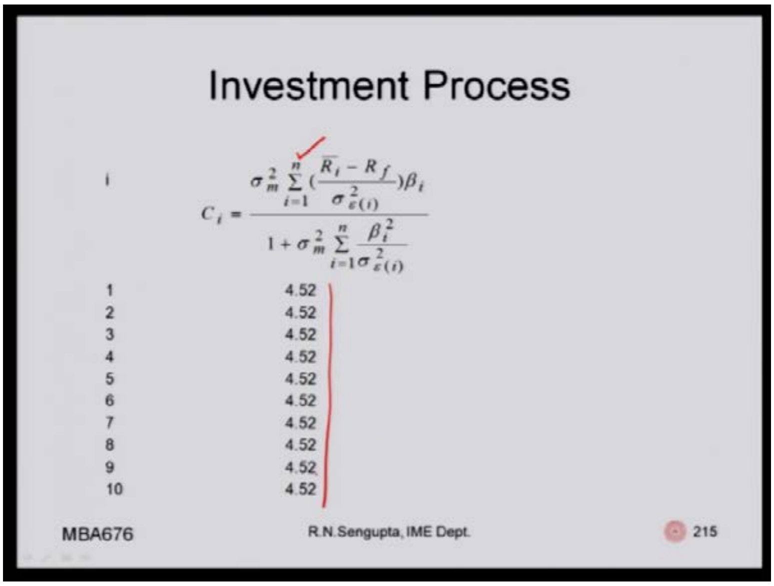

So, once you find out C I, again remember this summation is for the whole set, this point I am repeating time and again do not get confused by the end which we did for the case where short selling was not there and in this case where n and we have to find out for the whole set where short selling is there. So, this is conceptually two different concepts. So, once you find out the values, these are the values which you have for all the particular stock going what at a time. So, the values is basically 4.52.

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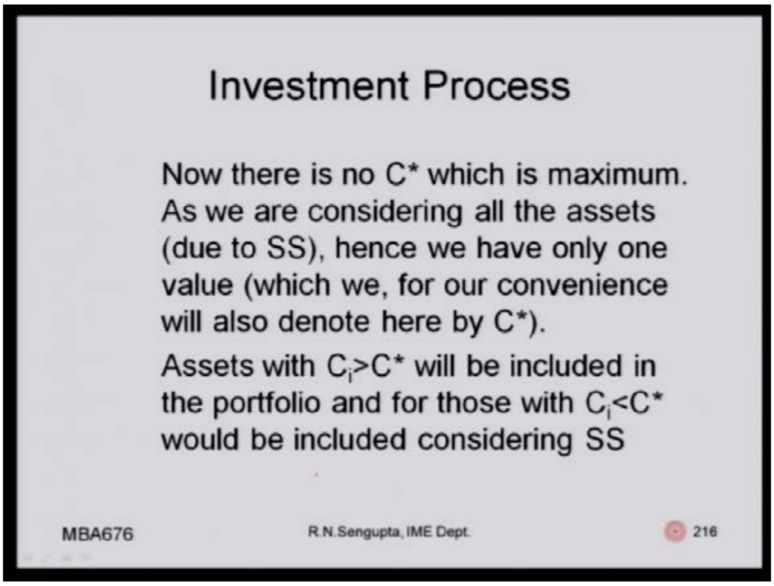

Now there is no C star which is maximum because all of constant as we are considering all the assets due to short selling as well as a not short of selling to be there, hence we only have one value which for our convenient we denoted is as C star. For the time being considers that is C star which is the value which we just followed is 4.52. Assets with values of C i greater than is value would be include in the positive sense, asset with less than C star would be included in the negative sense.

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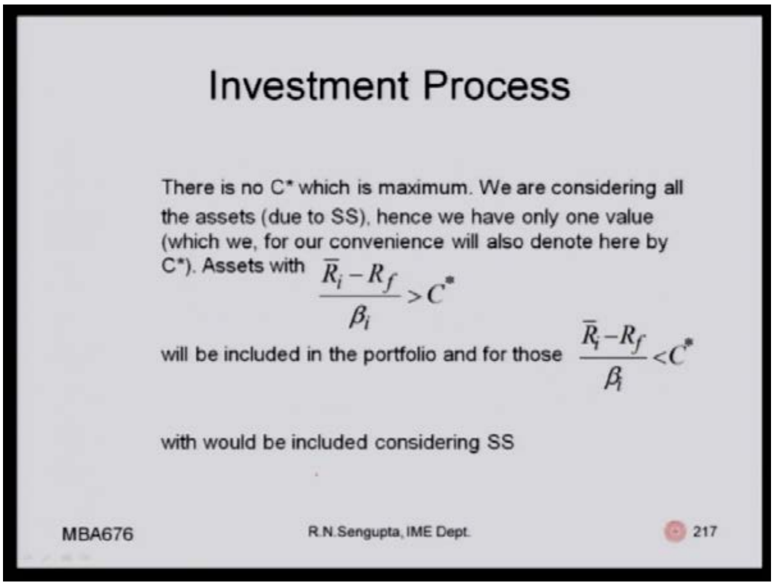

So, there is no C star which is maximum we are considering all the assets due to concept of SS, which is the short selling, hence we only have one value which for our convenience we will denote by C star. Assets with values of R i minus R f divided by beta i greater than C star include in the positive sense, less than equal to C star would be included in negative sense

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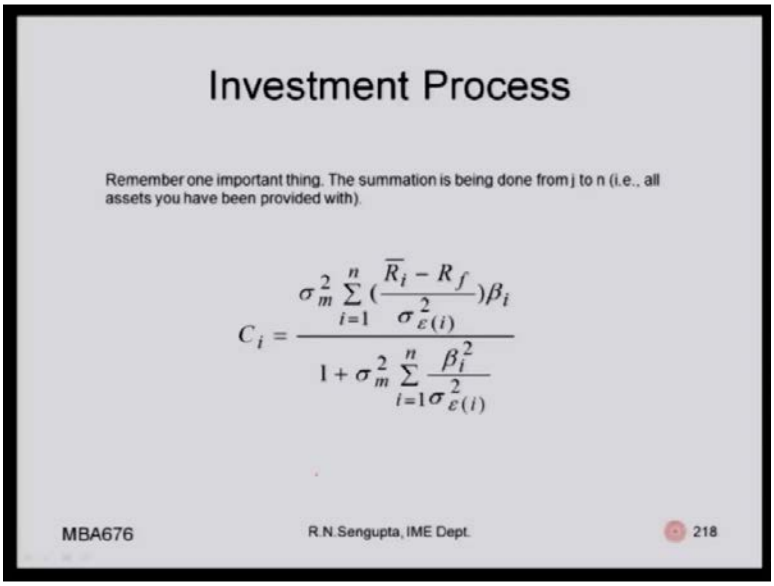

Remember one thing, the summation has to be done for all the particular stocks which are there in the box, where n is the whole population of assets which we have in front of you to formulate the portfolio.

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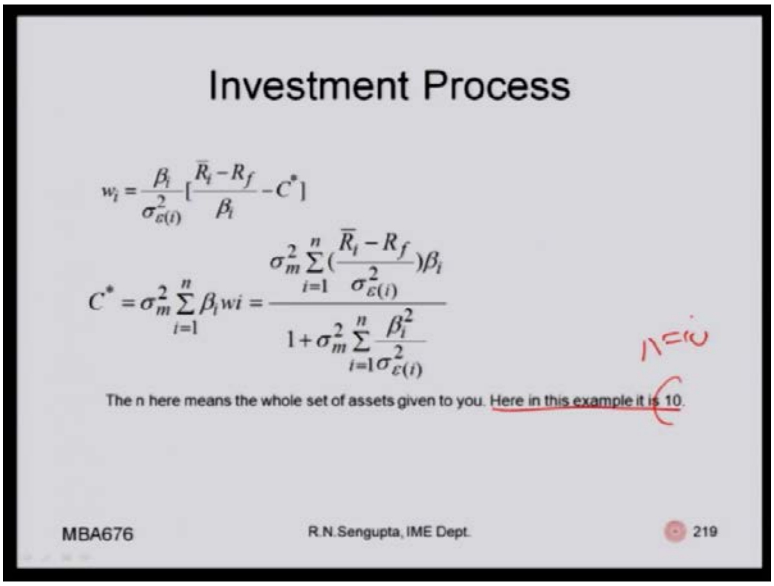

Again you use the same formula, find out w i which are the weights and basically C i star or C star is given by the same formula, but bit of slight change. So, change is this, n is always equal to 10.

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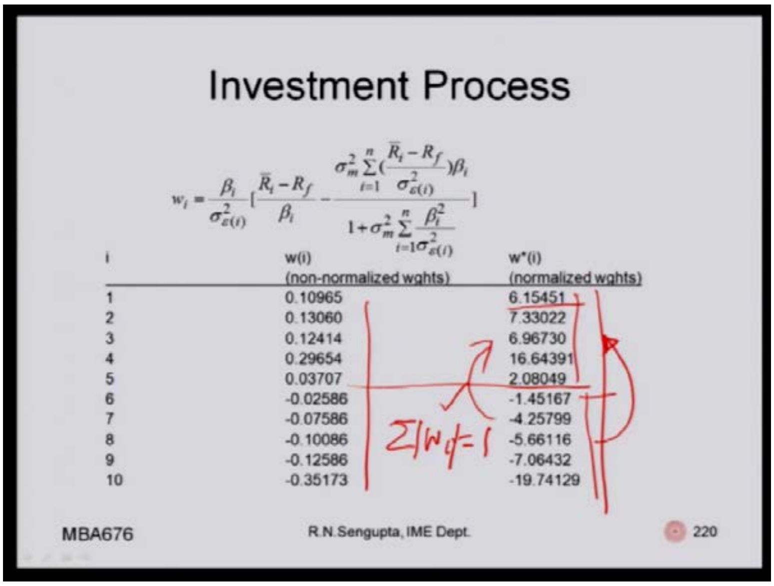

So, now with this, let us do the calculation. If you do the calculations, what you have is w i is which is the non normalized weights given in the second column. So, first talk would have about ten percent, while the last talk would basically have a percentage which is basically negative. So, what we are trying to find out is that whether the sum is some sort to be one; the answer is no, if you add up they would not come out to be one. So, what you do that again normalized that, but remember that if you use the lindens definition, this should always w two that the sum of the mod of the way should always come out to be one. In case if you do not use the mods, obviously, you will get a difference answer, but in that case practicality does not sound very good in the sense that when you doing short selling, they should be some collateral. And if you read the paper will internally means basically when you are doing the mod is basically means that whatever you do short selling, they should be compensatory collateral such that you are able to utilize that money in case you default.

Say for example, I am trying to basically go for a short selling and buy particular stock at loan a particular stock from a certain person, so obviously, they should be collateral because in case if a default the person can utilize that money in order to compensate the overall risk you are faces. So, considering the concept of intra definition non intra definition whatever is you can solve the problem. So, once you find out the normalized weights, see very interesting thing one is for all the till the fifth one, if you remember they are being done in a positive sense; that means, all this stocks would basically be done because their values of access return divided by betas r positive. And for the second set that is from the six to the tenth all the stock you would be done in the negative sense because the values of R i minus R f which is the excess returns which you have with respect to the beta is negative, so obviously, those would be done on a short selling sense.

And try to basically divest or utilize the money in those stocks for which there are basically positive returns excess positive returns. So, what actually it means that you would basically be utilizing about minus 1.45 in the total value sense or percentage wise it will be 144, 45. The second one will be 425, third one will be 566. So, these are the percentage wise excess amount which will be borrowing from this six to ten stocks, and trying to basically invest in the first five one. So obviously, it will mean that you will borrow some money from six to the ten and invest such a way that total amount of percentage investment in the first one would be about in two naught six hundred and fifteen percent.

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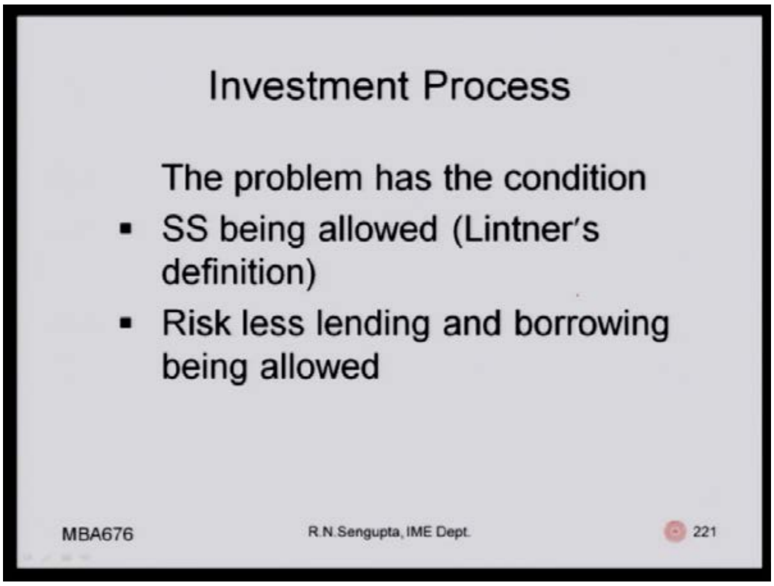

The problem has the condition that SS being allowed using this Lintner’s definition risk less lending and borrowing for risk free interested can also be done, but remember Lintner’s definition need not hold true for any application where we will consider the some of the weight is equal to one, but not the mod concept.

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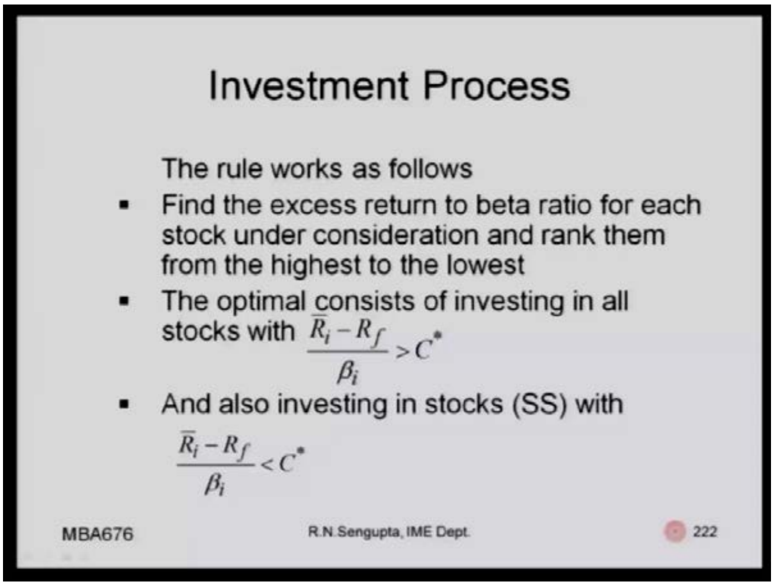

The rule works as follows; find the excess return to beta ratio for each stock under consideration rank them from the highest to the lowest. The optimum on investing in all the stocks is greater than C star include less than C star include short selling sense.

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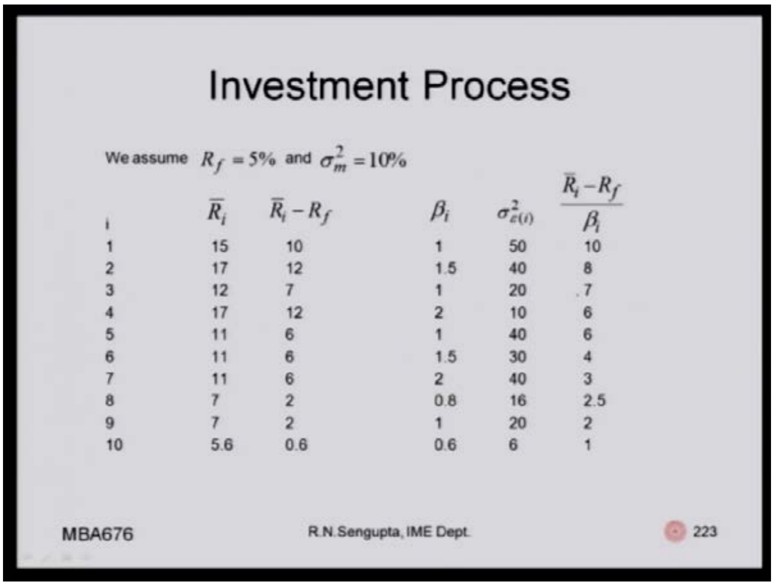

So, now we are doing the Lintner's one, again the same formulation have ten stocks their returns second column, excess returns to the risk interested in the third column, the betas are given, the white noise are variances are given and then you find out the excess returns which is R i minus R f divided by beta is basically ranking them from the highest to the lowest.

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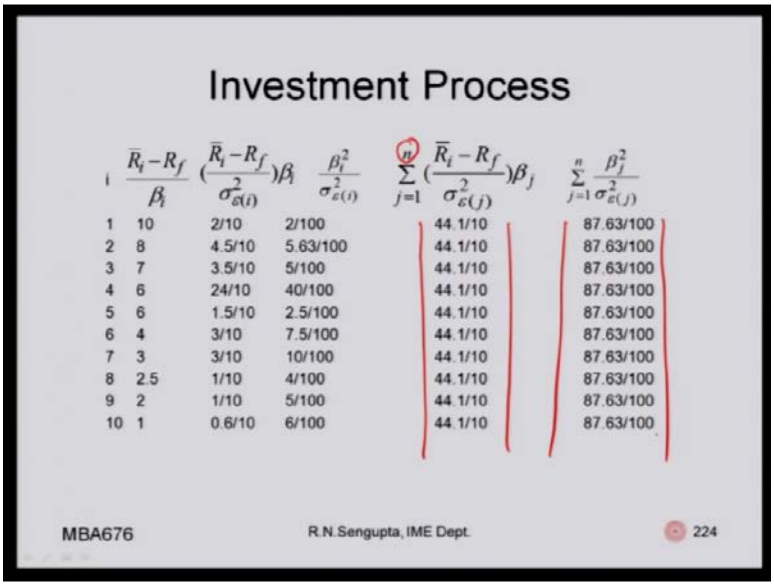

Again do the same calculations, but remaining the fact which I am repeating time and again, this n is always ten so that means, you are including all the stocks whether in the short selling sense or not in the sort selling sense that will come out very easily as we continue the doing the calculations.

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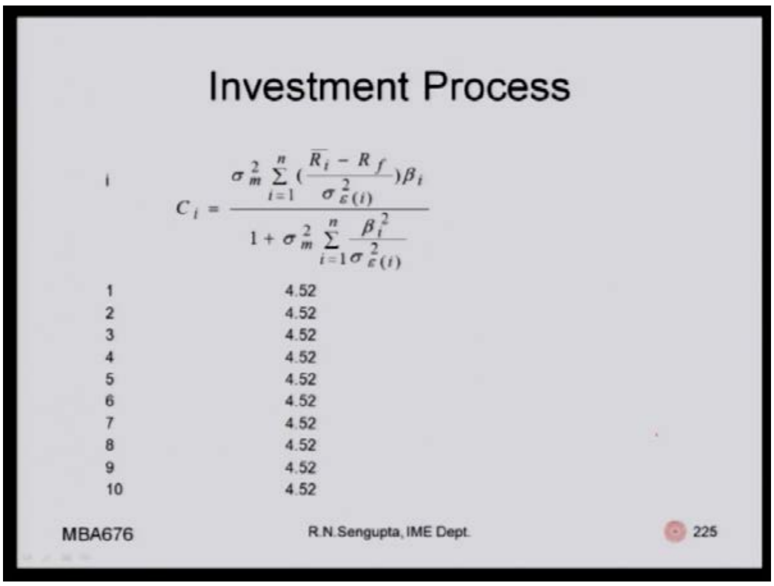

The value of so called C star, there is no maximum value of C, so called C star let us mention it as C star for our convenience comes out to be 4.52. So, again you will include not include depending on the values being greater or less

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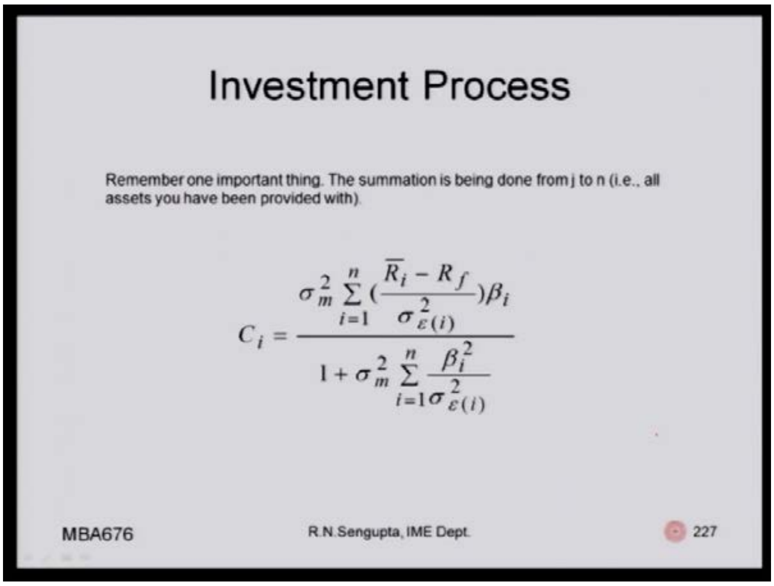

The C i values are calculated using n is equal to ten. Weights are found out using the same formula and C star value as you have noted is four 4.52.

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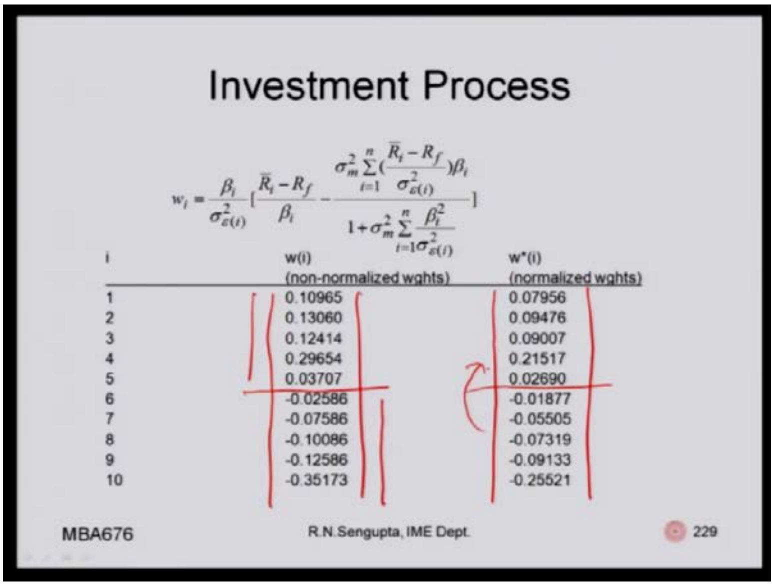

Once you do them, now you see a very interesting thing. These are the cases where you basically invest in a positive sense for this five number stocks and negative sense in the five number stocks. When you do the normalization on the weights considering the Lintner's definition, they come out to be quite practical in the sense, because in the other cases in the last problem where you consider they were percentage about 600, 500, 400 which practically is not right. So, if you watch this problem and solve it using the Lintner's definition with respect to the non Lintner's definition and usage of that formula in the just solve problem, this means that you will invest in a negative sense about one point eight percentage of you total money in the six stocks. While for the last one you will invests about minus twenty five point five percent in the tenth stock.

So, corresponding the amount of money which will basically divest in the first five stock would be given as in ratio sense it would be 0.07, 0.09, 0.09, 0.21 and 0.20. So, obviously, you can check double check, this is the sum of the mod of the weights come out to be one. Remember that when you finding of the weights for the Lintner's definition is some of the mod of the weights for all the tenth stock. For the non Lintner's definition is some of the non mods, you would not take the mods and find it out that comes out to be one.