**Quantitative Finance**

**Prof. Raghu Nandan Sengupta**

**Department of Industrial and Management Engineering**

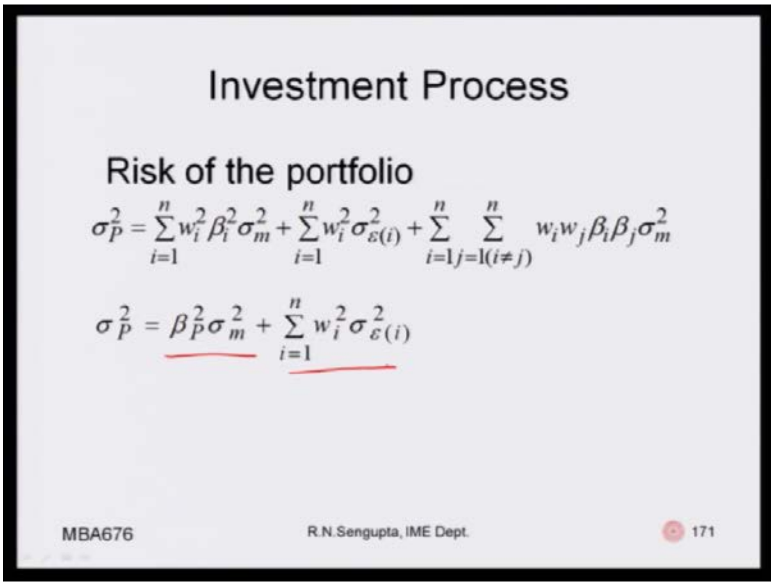
**Indian Institute of Technology, Kanpur**

**Module – 02**

**Lecture - 08**

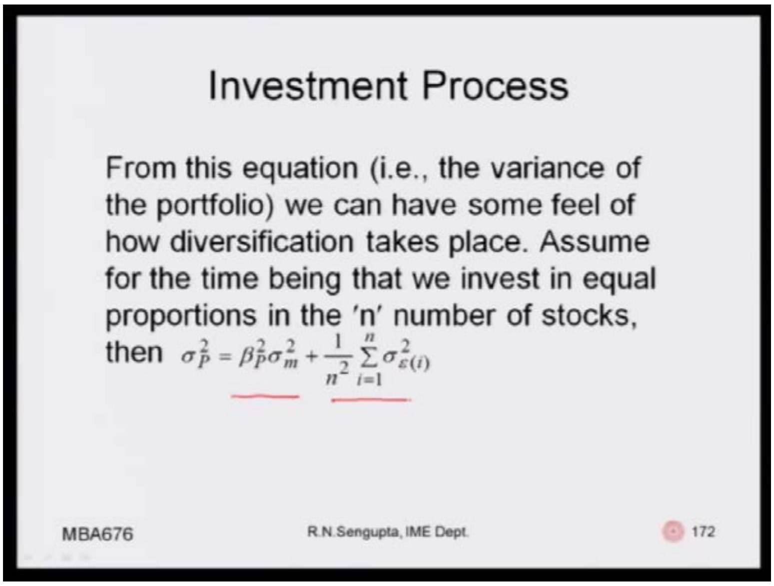
Now, the risk of the portfolio as we discussed is basically consist of two terms.

(Refer Slide Time: 00:09)



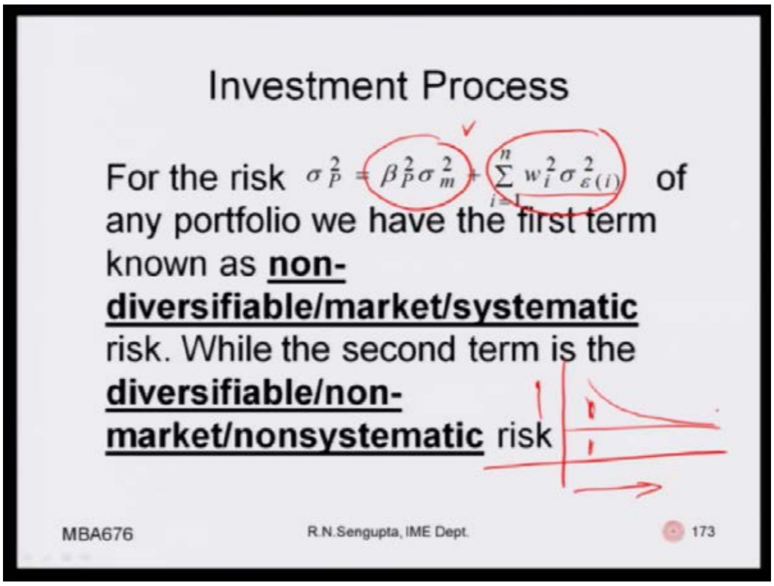

One is beta square p, which is the beta of the portfolio multiplied by sigma square of the market; another is basically the white noise.

(Refer Slide Time: 00:25)




From this equation, that is, the variance of the portfolio you can have some feel of how diversification can be done. Now, assume for the time being that we invest in equal proportions n number of stocks; n or k if you remember, because I am using ((Refer Time: 00:39)) If there are n number stocks and if we expend the equation, what you have is basically the risk coming from the market and the risk coming from the white noise, so on and so forth.

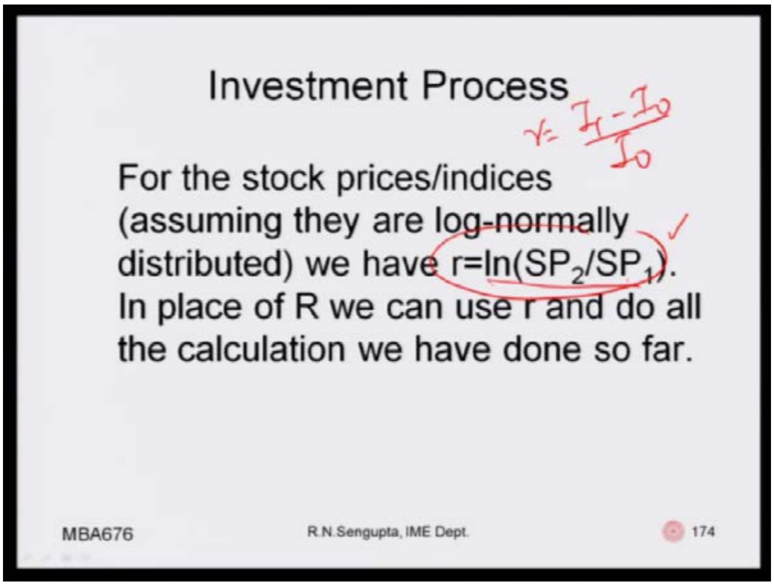
(Refer Slide Time: 00:49)




Now, for the risk, for the portfolio, which you are forming we will have basically the first term, which is known as the non-diversifiable and the market risk; because if you see, you cannot diversify that. There would be some beta p whole square and there would definitely be some sigma square m. And, in the second term, if we see is basically being multiplied by the squares of the weight multiplied by the white noise. So, that is known as the diversifiable risk, because we are able to proportionally put some proportions of that particular – in that particular portfolio p. And, the stocks are put in huge numbers such that each of them weights are n.

So, what we have is w i's are equal to w 1, w 2, w 3, w 4 – all of them are equal to 1 by n. If n can be increased, then we see that, technically, the overall risk may be decreased almost to 0. So, if we see, in a very simple terms, the number of stocks, which is there and the overall portfolio, which you have; so, what actually can be – there would be a fixed line such that it basically consist of the market risk, which cannot be decreased and basically the white noise. So, in the long run, the overall risk can be minimized to the maximum possible extent such that overall risk, which remains as the market; while the white noise, which is the non-market risk of the non-systematic risk can be made 0 theoretically.

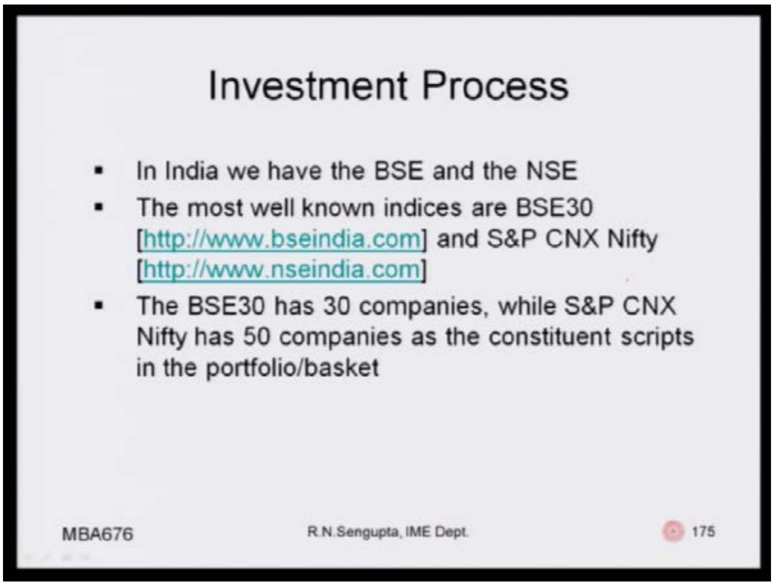
(Refer Slide Time: 02:13)




Now, for all stock prices, if you remember that… Going back to few slides back, we consider the returns R is equal to the difference of investment, which you have which is i 1, which is the investment, which we are getting as of today – minus i 0, which is the investment, which we started with – divided by i 0. Now, generally, we consider that, if concept of continuous compounding is there; what is continuous compounding? We will be discussing that within few days later on. If continuous compounding is there and if you consider the overall stock prices to be normally distributed; we will see the returns can be found out using the concept of ln; ln means this logarithmic with the Naperian log – e log. That is given by the stock prices for day 2 divided by day 1.

So, what we will consider is not the opening stock price; the closing stock price of the adjusted closing stock price. So, if we have Monday, Tuesday, Wednesday, Thursday, so on and so forth; so, we will take basically the ln of Tuesday's closing price divided by the Monday's closing price to give the returns between Monday and Tuesday. If we have basically in Wednesday's closing price divided by Tuesday's closing price; we will basically have the returns between Tuesday and Wednesday, so on and so forth. So, the odd values, which you find use this formula; is exactly the r values we have been talking about is basically this. And, we were interested to find out what were the returns and so on and so forth. And, we will see that, the returns, which we consider in the simple sense r related to this. And, we will use the expected value of r using this formula and the variance of r using this formula and try to analyze that whether they are normal or not.

(Refer Slide Time: 03:57)




So, in India, we have basically BSE and NSE. The most well known indices are BSE 30. And, the BSE 30 as 30 companies; while NSE has basically 50 companies depending on how the structure is formulated.

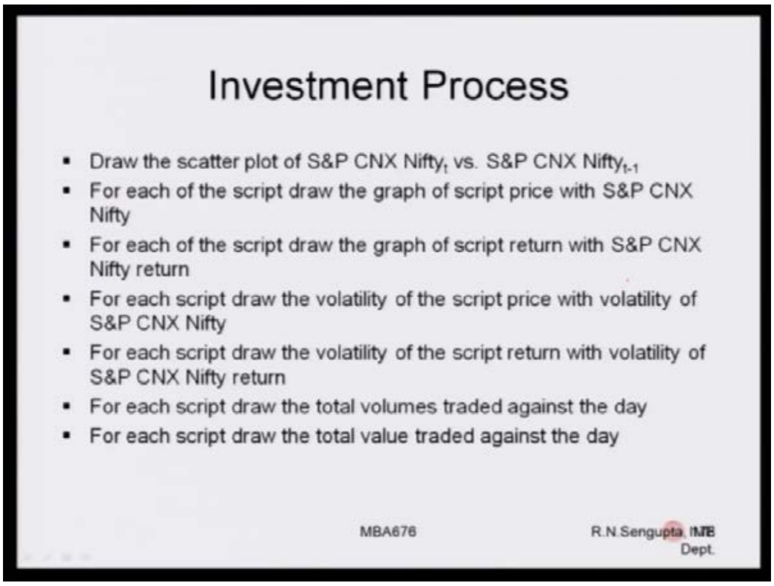
(Refer Slide Time: 04:10)


So, as on 2009, this is just a snap shot. We have this company starting from ACC in the BSE 30 – ACC to Wipro.

(Refer Slide Time: 04:18)

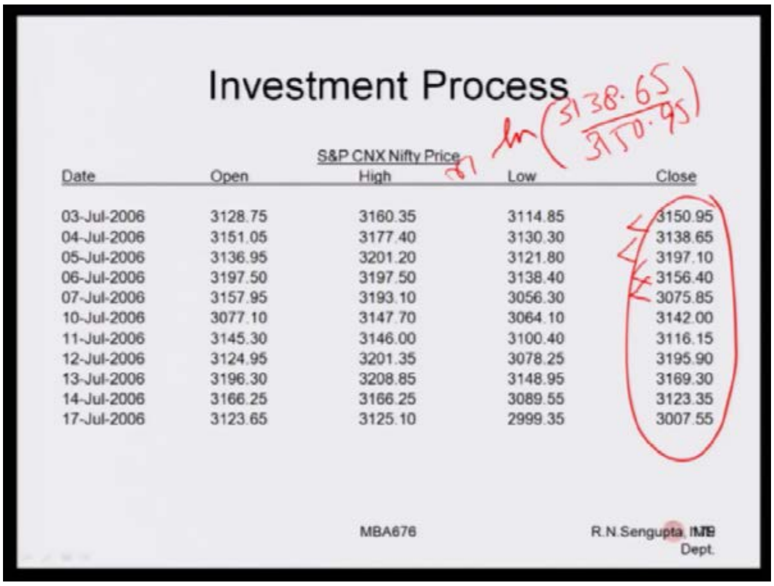

Similarly, for the S&P’s Nifty, we have stocks starting from ABB to Reliance communications.

(Refer Slide Time: 04:24)



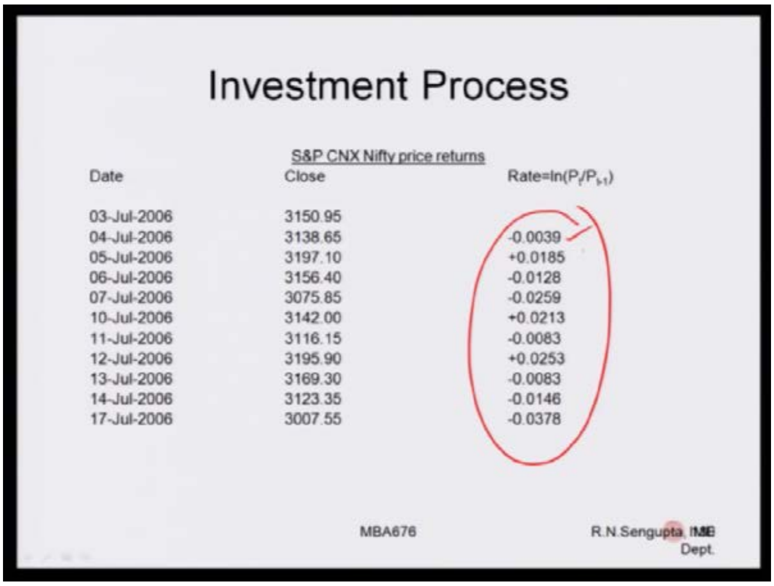

So, what can the students can do is that, later on, we will come to simple assignments. Draw the scatter plots of S&P's Nifty; or, for each of the stock, draw the graph of the scripts along with the S&P Nifty and so on and so forth. And, have a look at the – how they basically vary with the market.

(Refer Slide Time: 04:42)



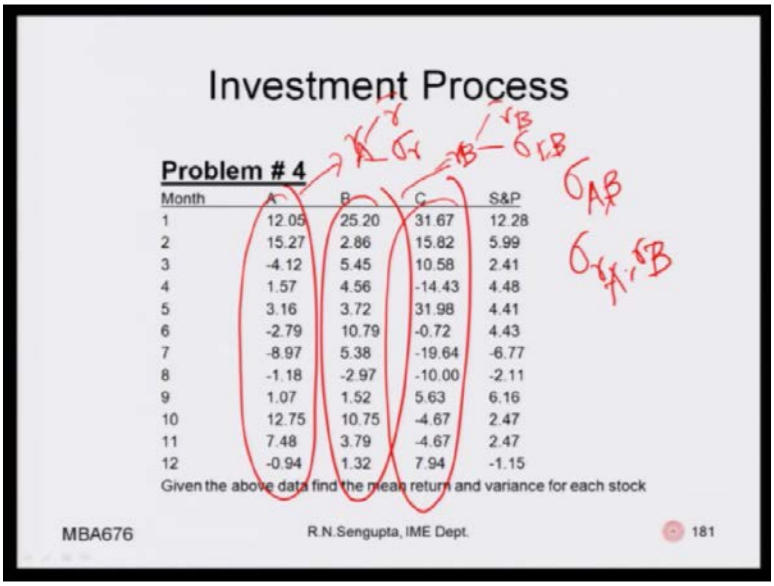

So, again consider for 2006. So, if we have the open, high, low, close of S&P's price; if you see the closed price, which we have; this is the one which you are talking about. So, you want to find out the returns between these two days. It will be basically ln of 3138.65 divided by 3150.95. This would be basically r 1. And then, you can find out r 2, r 3, r 4. So, obviously, r 1 and r 2 can be positive or negative.

(Refer Slide Time: 05:17)




So, we have the rates given. This is the column. So, you see some are negative, some are positive. So, based on that, we plot it.

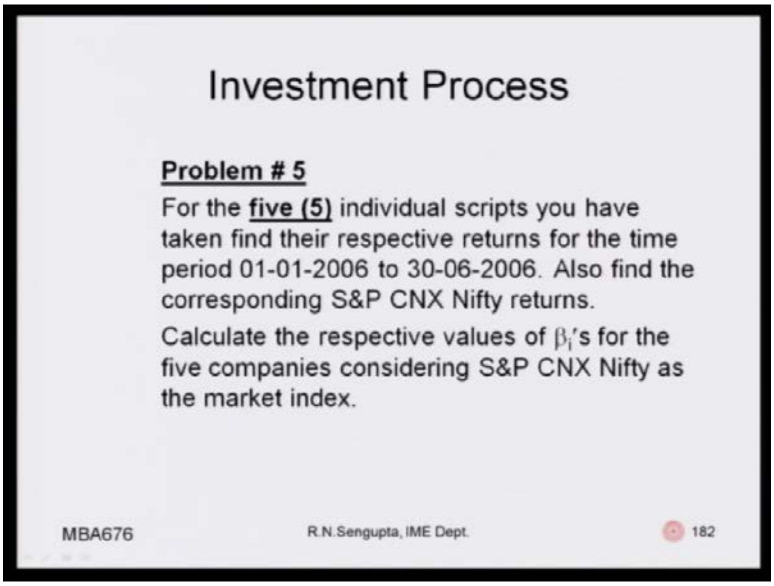
(Refer Slide Time: 05:27)




Now, given… So, we are considering a simple problem; problem where we have A, B, C as a stock. And, we consider S&P prices are again theoretical starting from 12.28 to minus 1.15. So, if we have this; given the data information, you can find out the mean. So, how can you find out the mean? Add them up divided by the number of rating; that will give you the mean for A. So, obviously, you may be thinking that, should we use 12.205? No, these are the prices. So, what you will do is that, first, convert them to r. So, once you convert into them r, you will find out r bar; you will find out sigma of r.

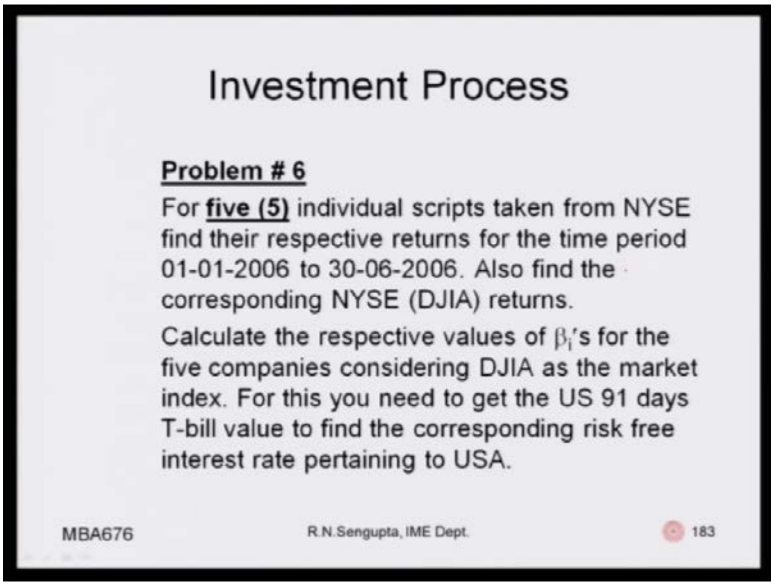
Similarly, if you have basically B, you will first convert it into r, which is the return for AB. And, this is the return of A. Similarly, you will find out r bar, r B. Similarly, say for example, sigma r or B and find out the variance and the expected value of C and continue. So, what we also need to find out is that, what is the correlation coefficient existing between A and B. So, this A and B does not mean the price; it basically means this; that is the returns A and B. They have some random variable and then you find out the covariance's and so on and so forth.

(Refer Slide Time: 06:48)



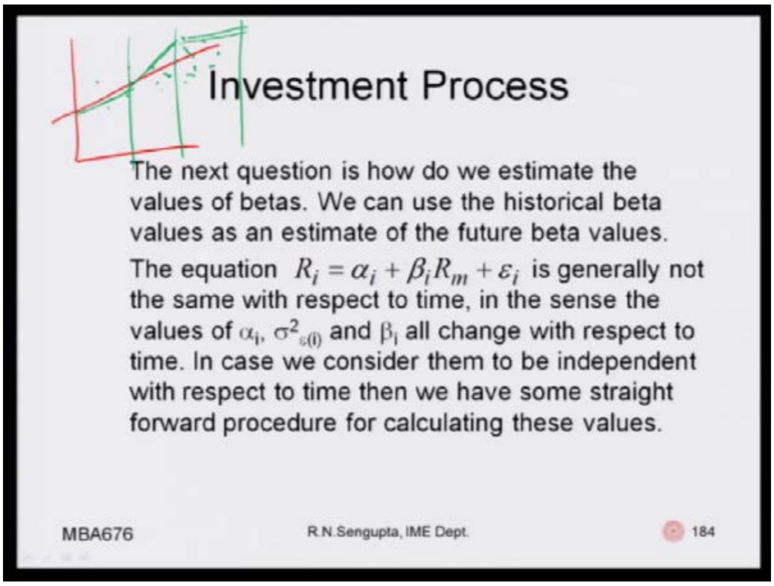

For first five individual scripts, you have to find out for any company… These dates you can change; you can find out the time period between 2000… Say for example, January 2015 to February 2015 and find out the corresponding S&P; and then, try to plot them and try to find out the betas using the equation if you have already done.

(Refer Slide Time: 07:08)



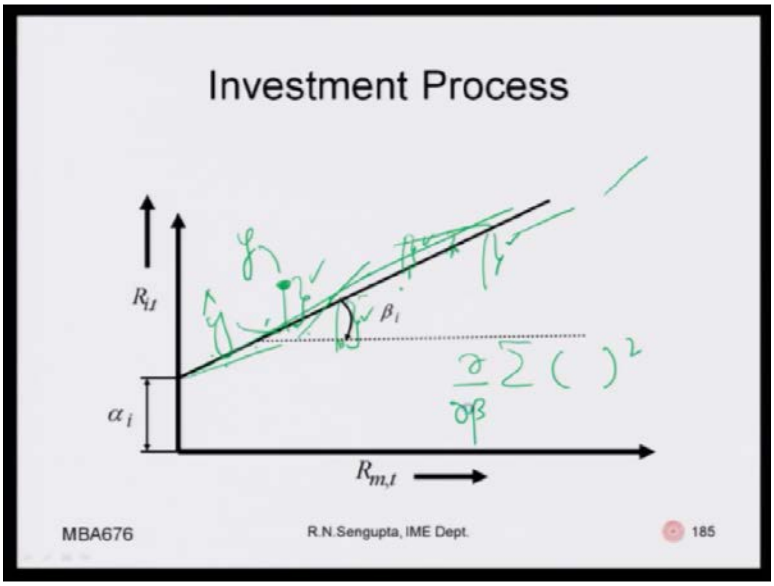

Next, you can basically take the Dow Jones and find out similarly, how the betas are plot. You can basically plot the data points and then basically use this equation to find y is equal to mx plus c, which we have discussed; which is also equal to – exactly equal to r i… r i bar is equal to alpha i plus beta i plus r m bar. Obviously, f 7 is not there at 0.

(Refer Slide Time: 07:32)



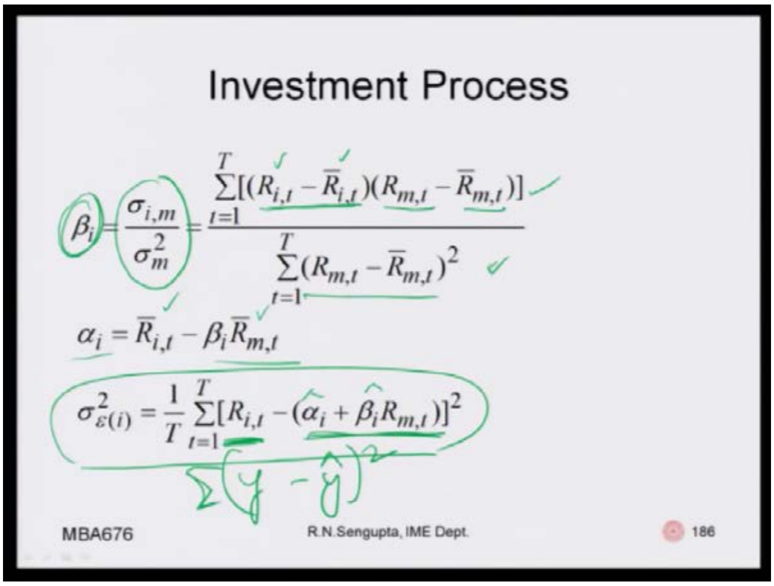

So, the next question is that, how do you estimate the values of betas. So, we have been talking that beta is the slope; it is the tan of the theta and it solves all our problems. Next question is – how do we find out beta? We can use historical data to find out the beta. Now, to estimate the future values of beta; so, the equation, which we see is R i is equal to alpha i plus beta i into R m. But, the white noise is generally not the same with respect to time in the sense that, both alphas, sigmas, then epsilons r’s; obviously, they are changing. Obviously, they are at different fits of the line. In case we consider them to be independent with respect to time; then, we have some straight lines and these straight lines can basically combine together. So, what we actually mean is that, if… So, technically, this is the beta slope. But, what we mean is that, if the sets of points are… If I use some other color options; so, these are the points. So, what we think when we need to do in order to find out the base beta would be; say for example, for the first time period, this is the beta slope; for the second time period, this is the beta; the third time period, this is the beta. So, rather than having a continuous beta, you are taking basically piece wise in order to estimate that.

(Refer Slide Time: 08:54)




So, this is what we have been meaning beta. This is R i's; this is the R m's and this alphas. So, the points are scattered like this. So, the best fit is this; rather than that, we can have the fits of different lines. And, another thing, which I did mention about trying to basically minimize the variance, is that, these points, which are there – we are trying to find out the distance. So, this is the distance between y and y hat. So, basically, these are the actual y's and these are the y hats. We want to find out the distance; square them up; square this up. There is another one, square it up; there is another one, square it up; another one, square it up; and, add all of them. So, once you add, you want to basically find out the variances and then basically try to differentiate with respect to beta or alpha, which we have already discussed.

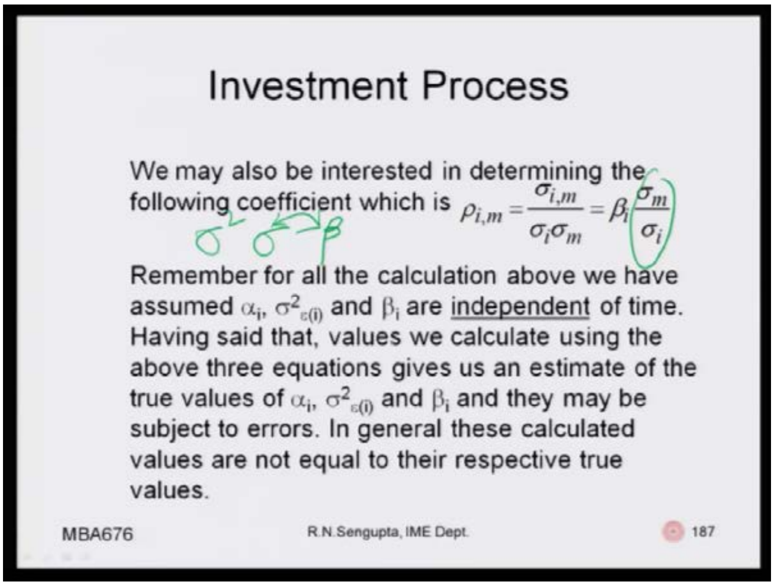
(Refer Slide Time: 09:52)




Now, for investment purposes, as we had mentioned that, if the betas are changing with respect to time and we need to find them; so, if we can easily prove that the values of beta are equal to basically the ratios of the covariance's of the stock with the market divided by the market risk of the variance. So, if we expand that; we can easily prove; that is equal to the sum of the differences, which is the covariance. You see this is basically the first random variable minus the expected value; second random variable minus the expected value; based on that, you can find out the numerators. Similarly, this is the variance; variance is basically covariance of particular random variable with itself. We can find out the variance and then find out the ratio, which give us the value of beta.

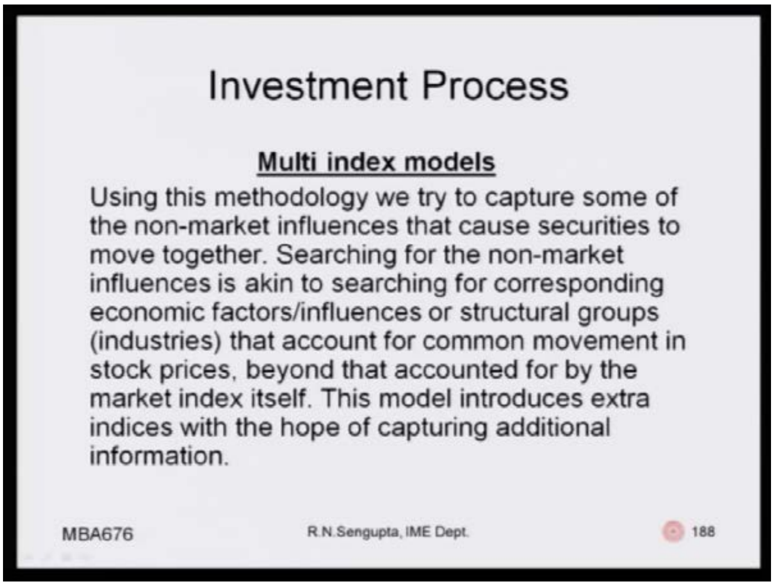
So, if you want to have different betas, consider the time frame; first time frame for the first linear equation would be say for example t – 1 to T – capital T. Similarly, for the variance of the market second can be from small t is equal to t plus 1 to say for example, twice t or some t 2 value. And, we continue doing it such that you can find out different values of beta. Similarly, given the beta, you have found out you already have the return of the market, you have return of the stock; you can find out the alpha's also. And similarly, given all these things, you can find out the white noise, which is sigma epsilon x. So, here this is what; this is the actual predicted value; there technically would have the alpha values and this is the actual values. So, what we have meaning is this. This is… Sum them up; and, this is basically the variances we are talking about.

(Refer Slide Time: 11:38)



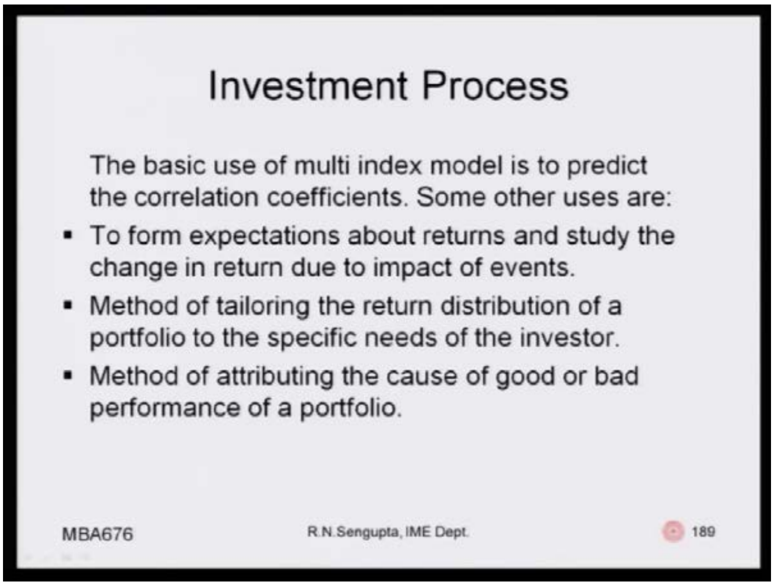

We may also be interested in determining the following coefficient, which is given the correlation coefficient between the i-th stock and the market; which when converted to the concept of beta is given by beta i for that particular stock and the ratio of the market standard division with the stock standard deviation. So, now, we see initially, we had sigma square as a good measure of risk or the standard deviation. We also saw that is, there is a beta, which is basically the slope, which is another value of risk. Second or later on, we found out that, what is the relationship between the beta of that particular stock with its own standard division. Remember – for all calculation above, we have assumed alphas and 6 sigma square and betas independent of each other even though that may not be true.

(Refer Slide Time: 12:29)



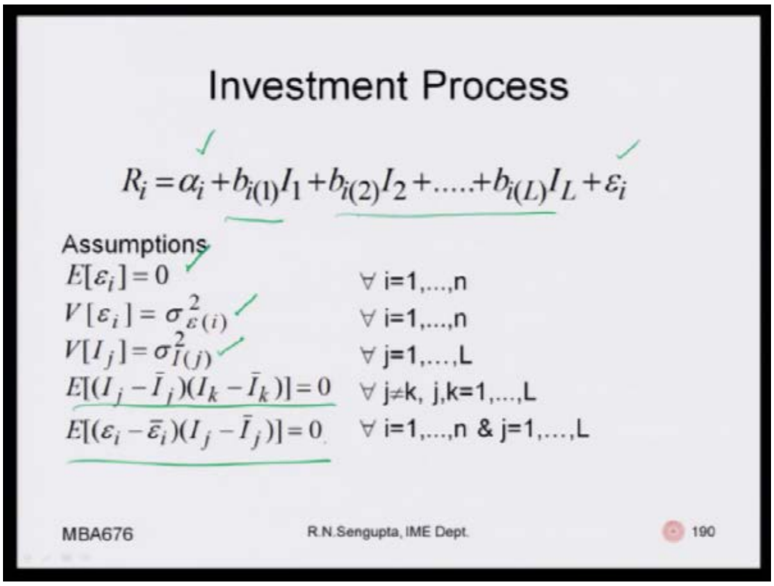

Now, we extend our case for the multi-index model. So, till now, we have been using the concept of single-index model. The single-index model is that, there is only one market, which basically takes the price. In this multi-index model, we will just relax such that more than the market, there is other indices also – other factors also, which basically takes the price.

(Refer Slide Time: 12:50)




So, the basic use of multi-index model is to predict the correlation coefficients; some others are to form expectation about the returns; it can also be method, can be tailor to the return distribution in the portfolio and method for attributing the cause of the good and bad effects, which is happening for any particular change in the stock market for that particular stock.

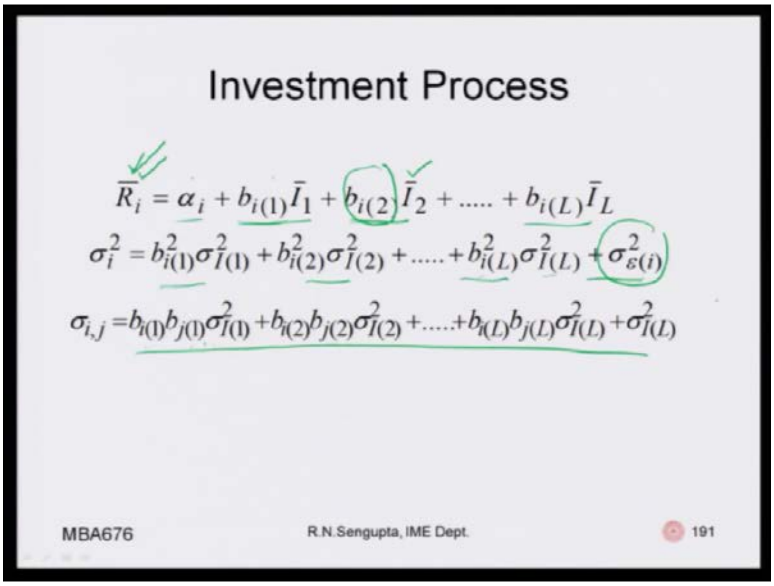
(Refer Slide Time: 13:10)




So, the multi-index model is just an extension of the single-index model. So, in single-index model, we have alpha is you have the first term; this one or that. The third to the last one was not there; the second last one was not there. And, obviously, you have the white noise. Rather than basically having only one effect beyond basically substituting in under… Say for example, more than one effect; where, now, the I 1, I 2, I 3 are the so-called indices or the so-called independent variables which effect the actual variable, which we are going to study is alphas. So, again in the similar line, if we consider the assumptions, the assumptions always, which we have already considered continue expected value of the epsilon, which is the white noise is 0; variance of the white noise is given; variance of the particular stock is also given. So, if we had the single-index model, we have basically sigma square m.

Then also, continuing the same fact; if we consider the multiple linear regression, where all of them are normal, we do consider that, the relationship between the random variable, which are independent are do not effect each other; that means, there is no covariance existing between the first random variable with the second; second to the third; with the first, second, third are all the independent random variable on the right-hand side of this equation. This means the covariance’s are 0 and, the last one, which we already considered is basically covariance between each and every particular independent variable, which is there with respect to the white noises also 0. So, what we need to find out is basically the expected value with respect to the multi-index model and the variance with respect to the multi-index model. And, this return and risk are with respect to the market or the different type of indices for the particular stock, which you want to find.

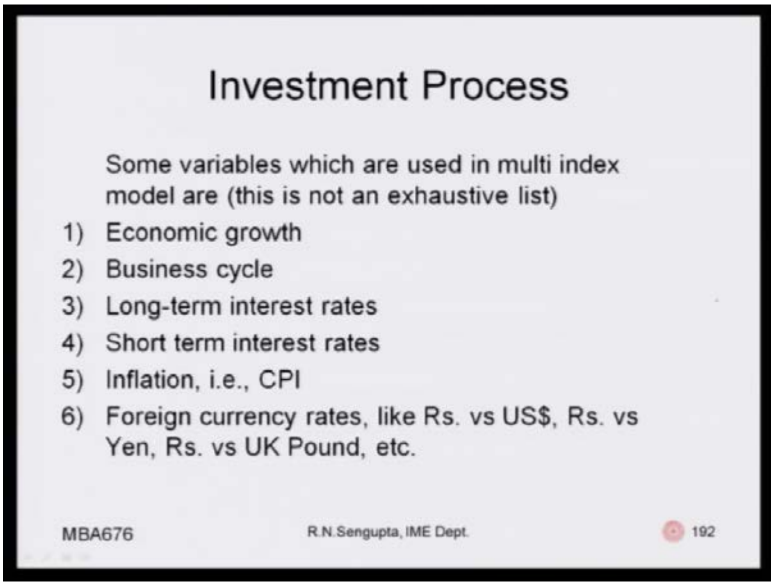
(Refer Slide Time: 15:08)




So, if you expand that, you will have return of the index is equal to alpha i. Again similar way, you had basically beta 1 I 1, beta I 2 and so on and so forth. The last term, where the… Now, if you see in the equation, the white noise is not there because the expected value is 0. So, in the single-index model, which you basically would have alpha i and this beta – b i 1 is basically the beta 1, which we have already considered. Basically, these all means that, they are the tan of this actual rate of change of the dependent variable with respect to the independent variable considering all or others are constant; which means this would be the rate of change of the slope of y, which is the dependent variable with one of the x considering the other x’s are concept. So, if we consider the slope, it is basically the partial derivative. Similarly, if we find out the variance of that particular stock; so, obviously, variance – this is of constant term; it would not have any variance. And, if we consider the variances of all other terms, they do come into the picture. And obviously, this variance of the white noise is also common. But, the interesting fact is that, the covariance is between the terms is 0 because we consider they are independent.

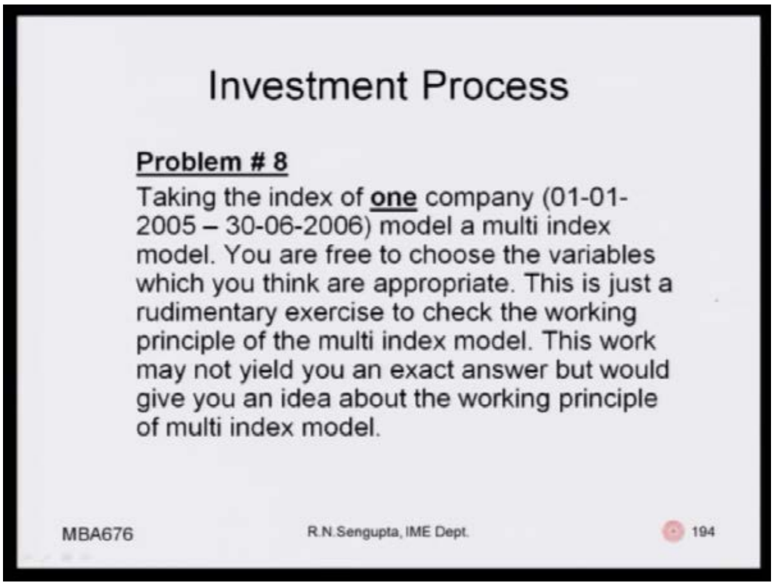
So, now, basically, if we had k number of such variables; it would be k plus the k plus 1, which is basically the white noise. Similarly, if you want to find out the covariances between each and two different stocks or two different financial assets; considering that you are using the multi-index model; then, it would be given by this formula, which is again very simple to understand. These are basically the covariances existing between b 1 1 and b j 1, which is for the first and the second multiplied by the effect of the variance, which is coming from the first dependent one. This is b 1 2 into b j 2 multiplied by the effect, which is coming from the second one and so on and so forth till if you consider K or L number of such independent variables.

(Refer Slide Time: 17:19)



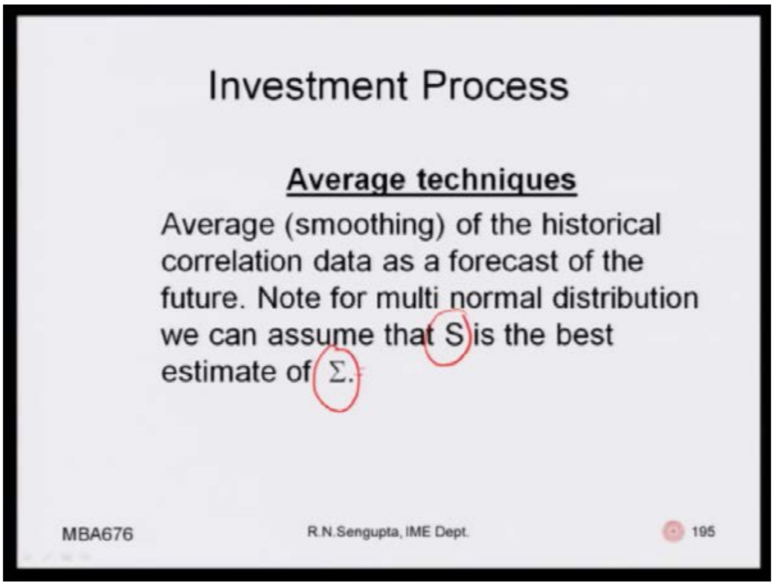

So, some variables, which are using multi index models are the economic growth, what is rate of change of the economic growth; what is the business cycle; long-term interest rate; whether for 5 years, 10 years, for 1 year, 2 years; what is the short term interest rate; what is the inflation, which is the consume of price index; what is the foreign currency rate; what is Indian rupees to dollars rate. Those can also be considered for the multi-index model.

(Refer Slide Time: 17:47)



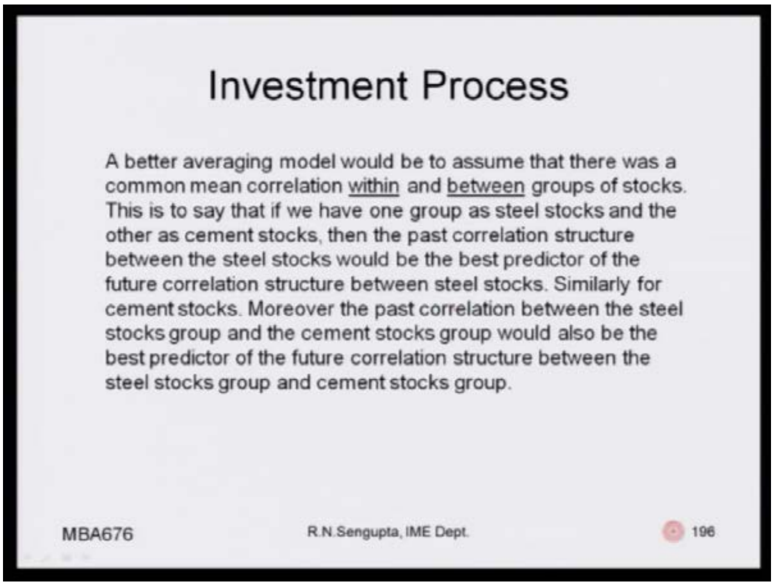

So, let me come to the… So, these are very simple assignments, which they can ((Refer Time: 17:52)) but, we are not going to discuss in our concept. So, now, we will continue our discussion in portfolio analysis related to the averaging techniques.

(Refer Slide Time: 18:04)



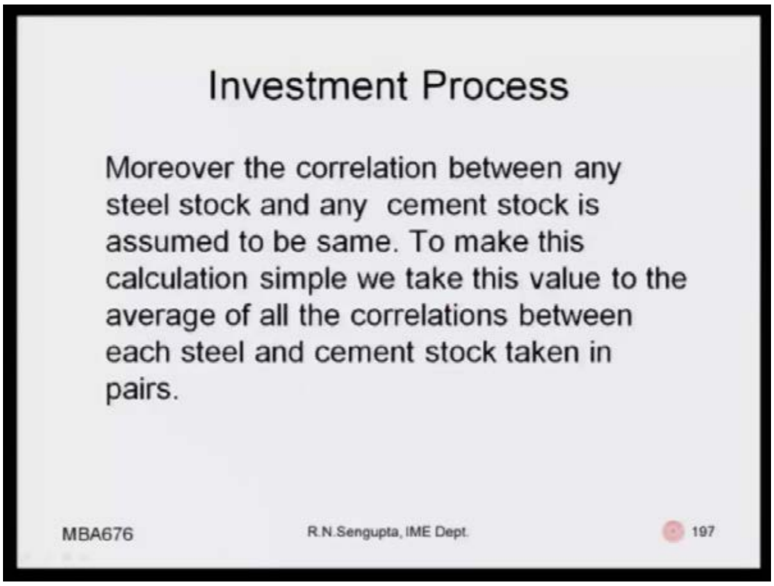

So, averaging techniques or the smoothing techniques of the historical correlation coefficient or the covariance is required in order to basically estimate the covariance-variance matrix, which is this. So, this is basically the covariance-variance matrix for the population. And, this is basically the best estimate we shall try to use on the sample.

(Refer Slide Time: 18:24)



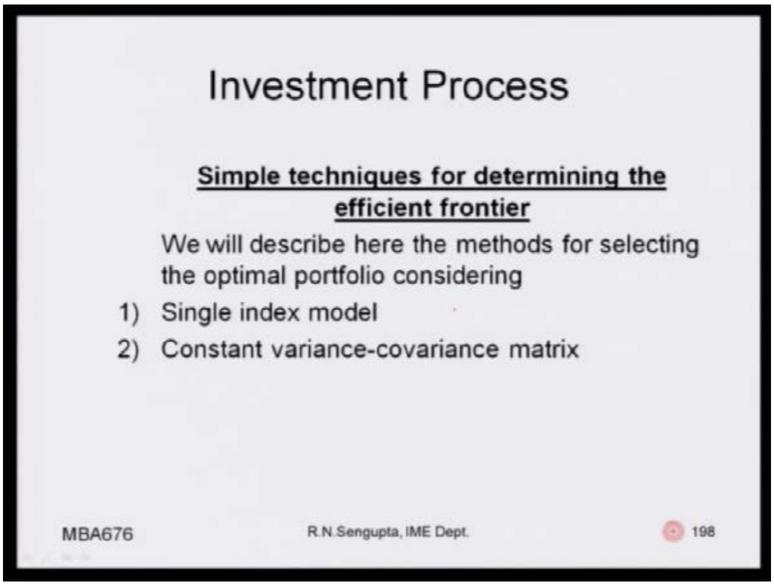

So, better averaging technique would be to assume that, there is a common or mean correlation within and between the groups of the stocks for different type of markets such that it is to say that, if you consider any particular steel stock, where the Tata Steel, Jindal Steel whatever you have or SAIL; it will have basically the same correlation coefficient between any model auto industry like Tata Steel to Maruthi. Tata steel to say for example, Volkswagen. I am considering these stocks are being traded. So, obviously, the correlation coefficient would be the same between as between…. Say for example, Maruthi to Jindal Steel; Or, Jindal Steel to say for example, Ashok Leyland. We will consider that in a very simplistic sense to be true.

(Refer Slide Time: 19:10)



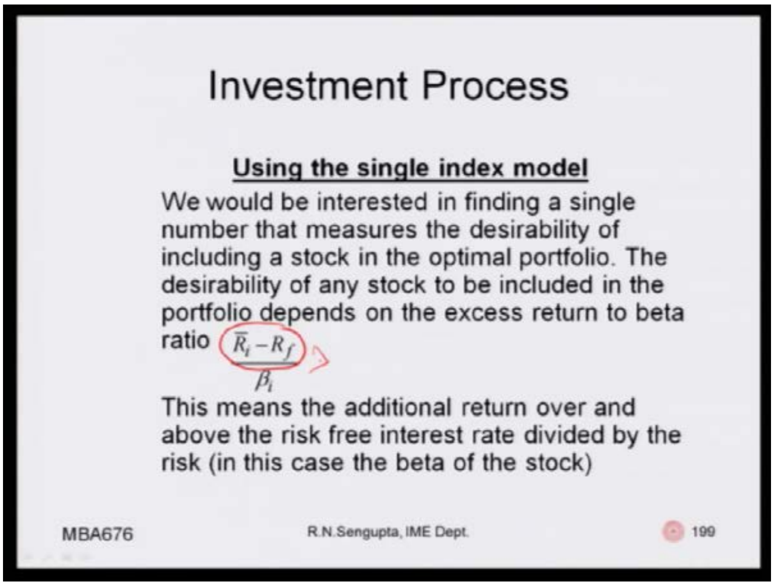
Moreover the correlation between any steel stock and any cement stock assumed to be the same constant. To make this calculation simple, we will take this value as the average of all the correlation coefficients, which is existing between these – all the stocks taking at a time.

(Refer Slide Time: 19:25)




So, simple techniques for determine the efficient frontier will be – we consider the single-index model to be true and we will consider the constant variance-covariance matrix to be true.

(Refer Slide Time: 19:34)



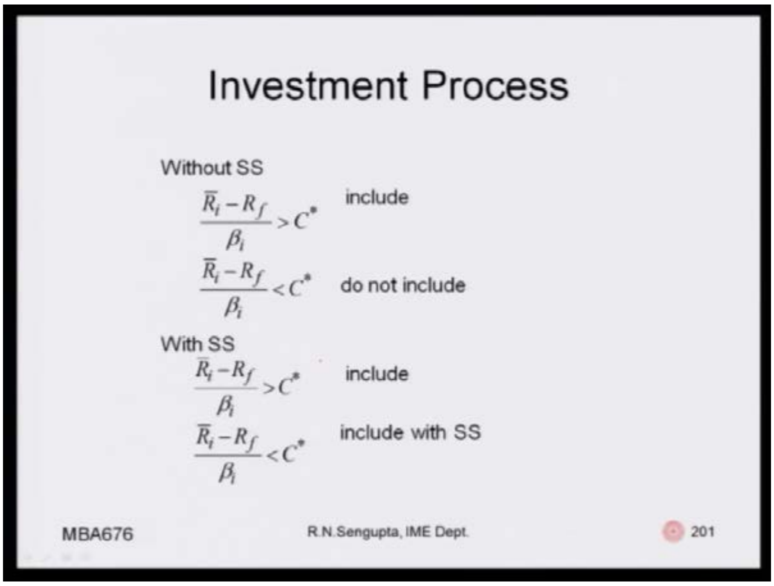

So, we would be interested to find a single number, which is a measure of the desirability of including a particular stock. But, now, we will consider that, over and above the particular stock of the set of stocks, which are risky; we will also have the risk-free interest rate, which is given by the concept of R suffix f. So, this means the… If you see the difference on the top, which is the numerator; it gives you that, how good or bad that particular stock is with respect to the risk-free interest rate, because if the stocks returns are not as good as the risk-free interest rate; obviously, I would not invest in this stock; if it is better, obviously, I will definitely like to invest.

Now, the second question is that, if you are only concentrating on R i minus R f; this only gives one-sided loop or loop-sided or off-sided view of the market. In the sense, it only gives the return. What about the risk? So, we know that we consider risk as say for example, sigma; we can also consider the risk as beta. So, what we want to find out is basically the ratio been the overall return difference, which is there in the numerator and the risk, which is given for the particular stock in the beta. This means the additional return over and above the risk-free interest rate divided by the risk of the market of that particular stock with the market would basically be the best measure; that how good or bad it would be to include that particular stock in our portfolio.

(Refer Slide Time: 20:54)

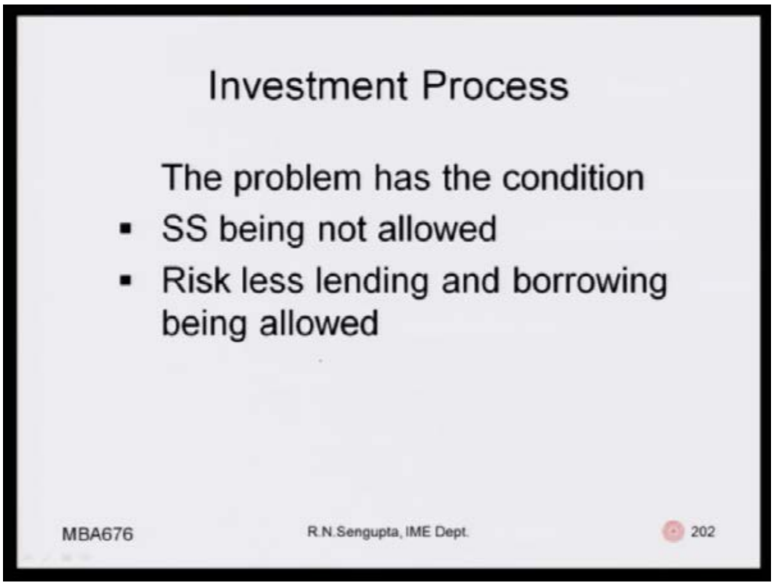

Now, if there is no short selling; then, if this ratio is negative or 0; we will not include that. If there is short selling; if the ratio is negative; obviously, we would include that; that means, we are going back to the concept of Lintner’s definition and all those things. We will see in a simple case.

(Refer Slide Time: 21:09)



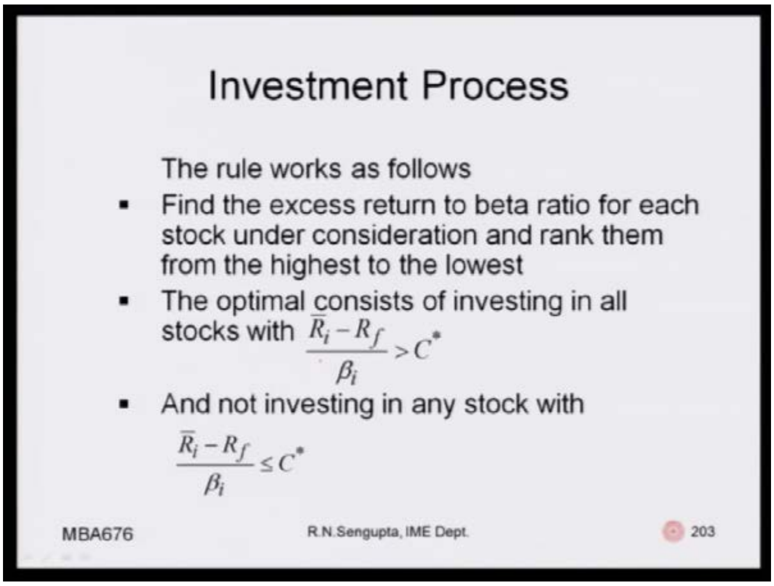

So, without short selling, if the ratio is greater than some C star; we include. If it is less than C star, which is negative; we do not include; that means, all would be 0. The values of that weights would be 0. And, with short selling being there; if they are greater, we include; even if they are less, we include, but in this short selling sense.

(Refer Slide Time: 21:27)



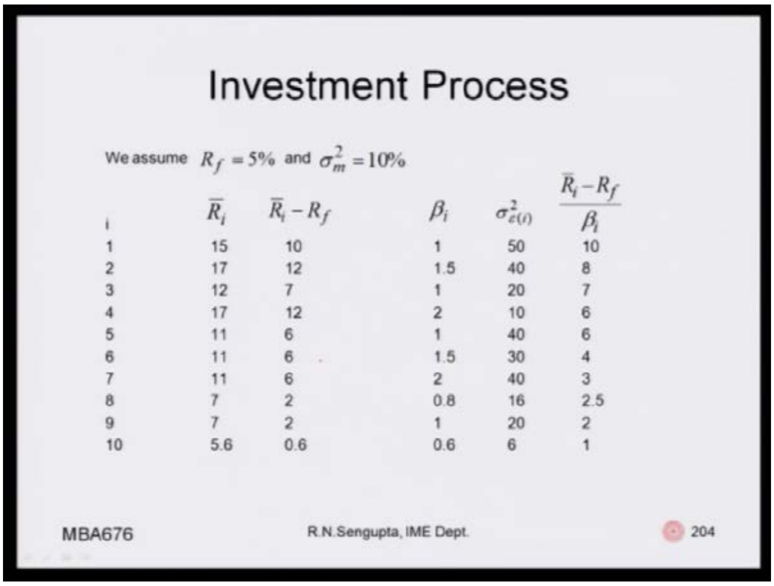

The problem has the condition that, short selling being not allowed and the riskless lending and borrowing being allowed. That is the first instant.

(Refer Slide Time: 21:34)



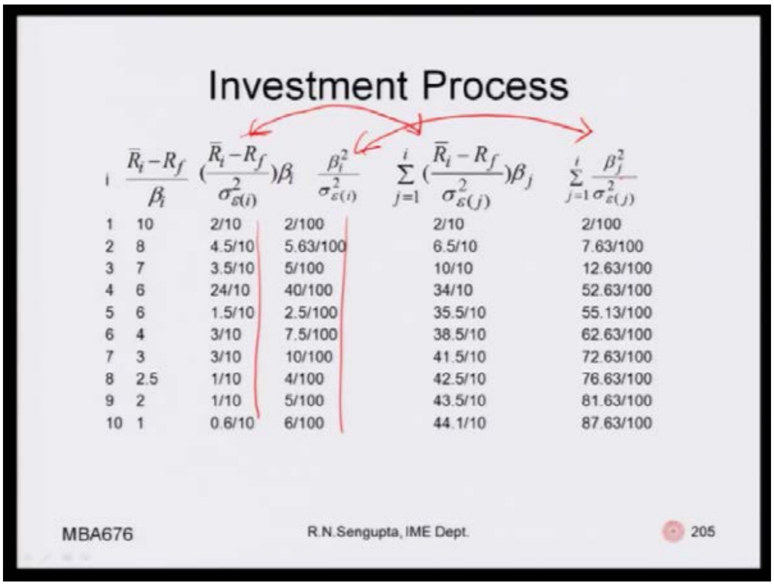

The rule works as follows. Find the excess return to beta ratio for each and every stock; rank them; obviously, we will rank them. And then, from the highest to the lowest, the optimal consists of investing in all the stocks such that the ratio is greater than C star; not investing in those particular stock whether the ratio is less than C star. So, we need to find out what is C star and I am going to come to that within few minutes.

(Refer Slide Time: 21:57)



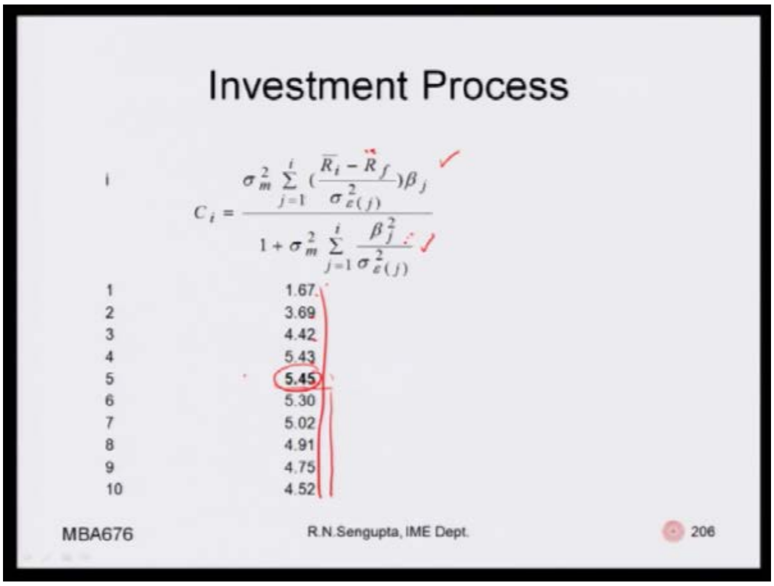

So, consider a hypothetical data. The first column are the stock numbers – 1 to 10. R i's are given from 15, 17, 12, so on and so forth. And R i-f’s which R i minus R f; R f, which is the difference between the risk-free interest rate and the particular stock return; where, the risk-free interest rate is given as 5 percent is given. So, here rank them. We have already ranked them according to the maximum-minimum. And now, we have… Maximum to the minimum in the sense that, this if we… Our actual aim is to find out the ranking with respect to the ratio. So, I am saying that, with respect to the ratio the ranking has been done. We have the betas, which is already given and we have the sigma square, which is the white noise for that particular stock. And, on the last column, we will basically have the ratio. So, what we see is that, the ratios of the first is 10 and the last one is 1. So, we want to basically make a decision that, at what stage should we cut-off such that, that particular stock was included; and, the stock would not be included considering short selling is not there.

(Refer Slide Time: 22:58)




So, we just go to this concept or the rule. This is the extension of the excel sheet, which we have. If you see it; the last column was basically the ratio. So, here the first column is basically the ratio. So, we find out the ratio between the difference in the returns by the white noise multiplied by beta. So, they are ranked. Then, we find out the beta square into sigma square. So, what we are doing is that, we are trying to find out the ratios of the internal risk of the particular stock with the white noise. And then, we start summing it up; that means, we are trying to sum up the third column. The first column being basically the i-th stock number. And, in this case… And, the last column if you see, will basically sum up or find out the cumulative values of the ratio, which is beta square j by sigma square j.

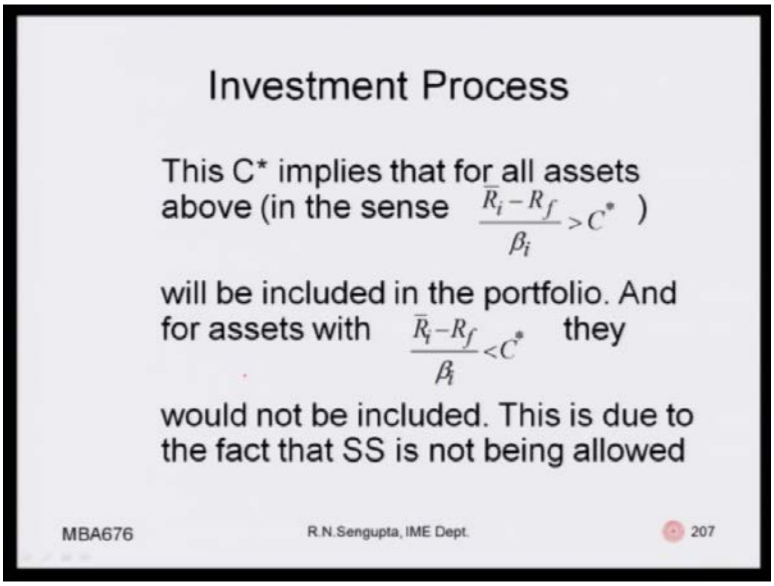
(Refer Slide Time: 23:49)




Now, the C i values what we want to find out based on which we will cut off is given by the formula; without the proofs is given by this. So, C 1 would be for the values of j is equal to 1 to i. So, you make a subtle difference between the index of j and i. So, C 1 would be j is equal to 1 to 1. And then, you find out this ratio, which is given, which you already found out plus 1 by sigma square into the ratio, which is basically already found; that means this ratios, which you find out on the numerator – this one and the next one are already found out and given in the table in the slide number 2 and 5.

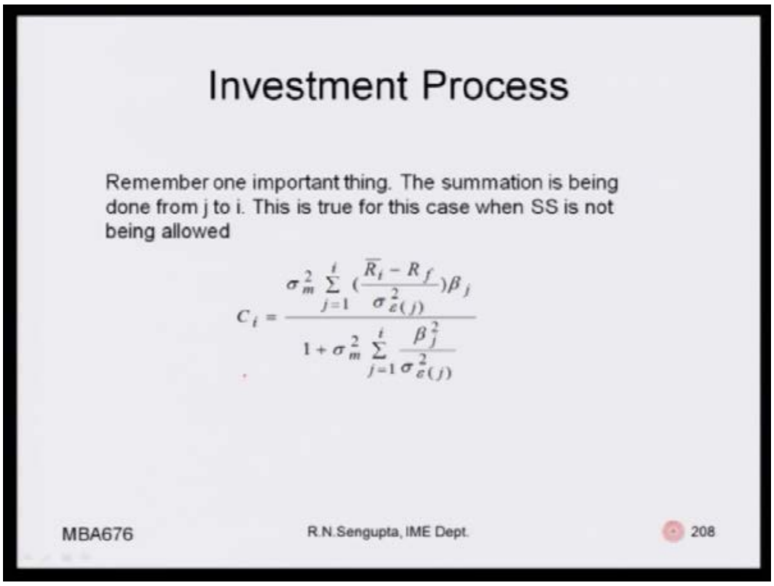
So, now, when we rank them; so, as we are ranking them; you will see a very nice thing. The C i value starts increasing. At a certain value of 5.45; after that, it starts decreasing; which means that, if you keep adding the stocks, which are 6, 7, 8, 9 and 10; basically, it means they would give me negative values if short selling is not there; that means, we will only consider in our portfolio the first, second, third, fourth, fifth; and, the ways would be decreasing; that means, the first one would be the highest one; similarly, the last one will be the lowest one considering the cut-off is say for example 5.45. So, this 5.45 is basically the C star value, which we are talking about.

(Refer Slide Time: 25:13)



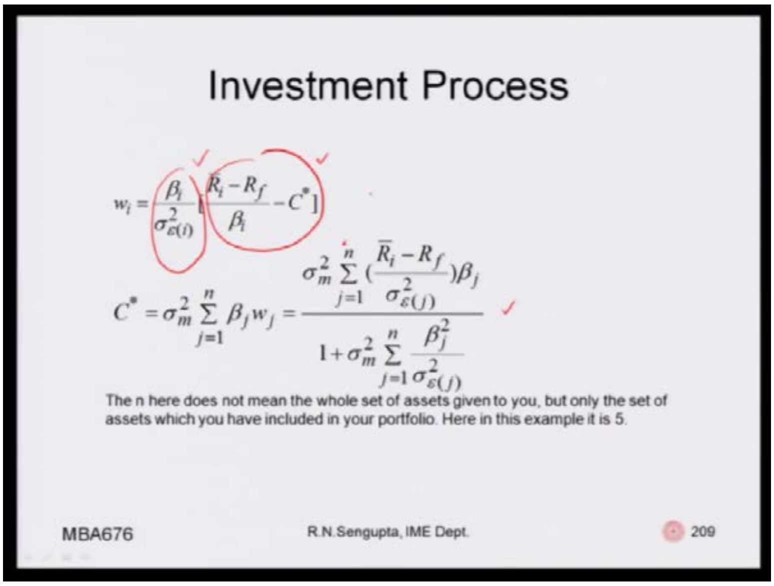

This C star implies that for all stocks, if they are greater, include; if they are less, do not include.

(Refer Slide Time: 25:20)



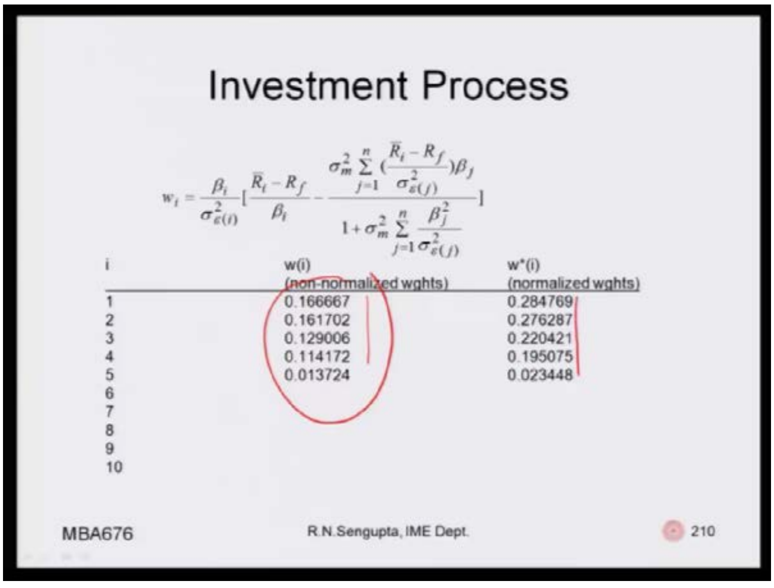

So, remember one thing. The summation is being done from j is equal to 1 to i, which I does mention. So, be very careful when you do the summation for C 1, C 2, C 3, C 4 whether j index would change only from 1 to that value for which you are trying to find out the C value.

(Refer Slide Time: 25:35)




So, w i’s are now found out using this equation. So, these are the differences, which you already found out. Greater it is, more it is invested; less it is, would not be invested to that ((Refer Time: 25:46)) as extended. w i's will be given by the formulas, which you can check and book by this one. And, a C star value, which we have already found out is given by the equations till the n-th stocks, where it is positive. So, this end does not mean the whole set of stocks as I was mentioning. So, it will be some small subset of this. Whole set of assets is not included; but, only the set of assets, which have been included in the portfolio. Hence, in our example, it is 5. So, if you go back, first, second, third, fourth, fifth are included; hence, they would be n is equal to 1 to 5.

(Refer Slide Time: 26:22)



So, once you find out the weights, these are the weights; but, very interesting thing note – if you add them up, they are not equal to 1. What we need to do? We need to normalize. One we normalize them. Using the symbol normalizing concept, we have already done. Then if you add up, the weights add up to 1; which means that out of this 10 stocks; you will include the first five with weights 28.4, 27.6, 22.0, 19.5 and 2.3. And, once you have found out the weights, you can find out the overall return of the portfolio, sigma of the portfolio and then proceed to find out the efficient frontier accordingly. If short selling is not included, we have considered the case, where short selling is not product.