Strategy: Introduction to Game Theory

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Lecture- 09

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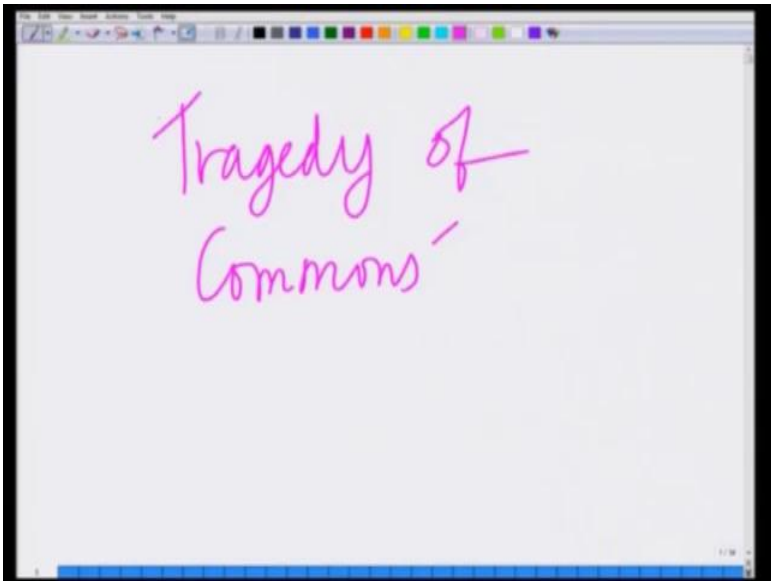

Hello, welcome to another module in this online course Strategy and Introduction to game theory. In the previous module we have looked at an interesting game which was titled the Tragedy of Commons let us continue or a discussion on this game that is the Tragedy of Commons.

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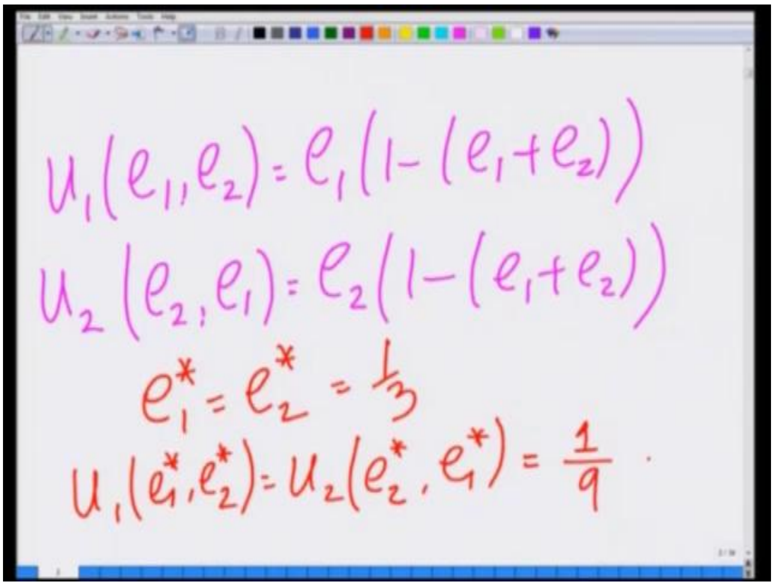

And we had seen the following things that there are two timber agencies which are involved in logging the trees in a particular forest and the payoff each of them that is u 1 of e 1, e 2 was given as e 1 into 1 minus e 1 plus e 2 and u 2 was of e 2, e 1 was given as e 2 into 1 minus e 1 plus e 2 right. And we had analyzed this game and we had found the Nash Equilibrium effort of this game as e1star equals e2 star equals one by three and the Nash payoff u 1 of e1star, e2 star equals u2 of e2 star, e1star equals 1 by 9. So, the Nash payoff what we want to do now is want analyze this game slightly different way and try to see I there is another possibility or if there is another outcome which yields a higher payoff for both the players right.

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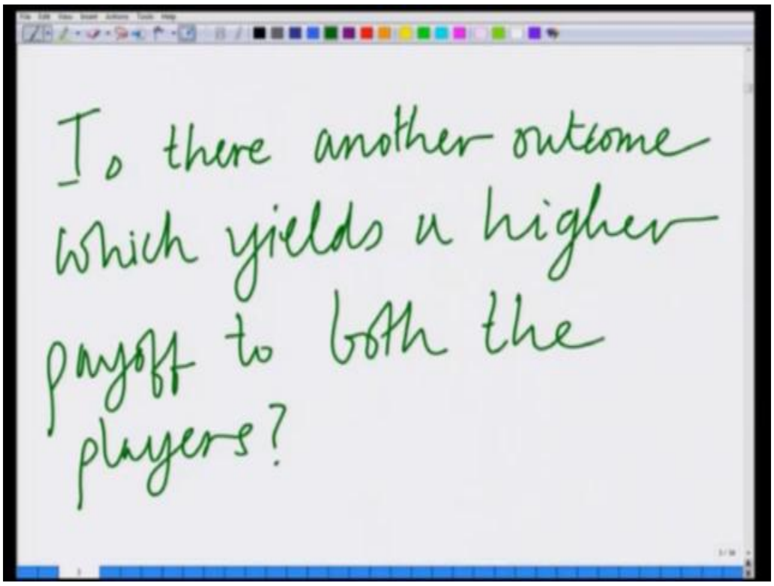

So, we want to see is there another outcome we want to answer this question is this is the best possible outcome or is there another outcome which yields a higher payoff to both the players. Well to characterize this let us try it to look at a scenario which both these agencies collaborate. Let us say both these agencies collaborate to maximize their sum payoffs.

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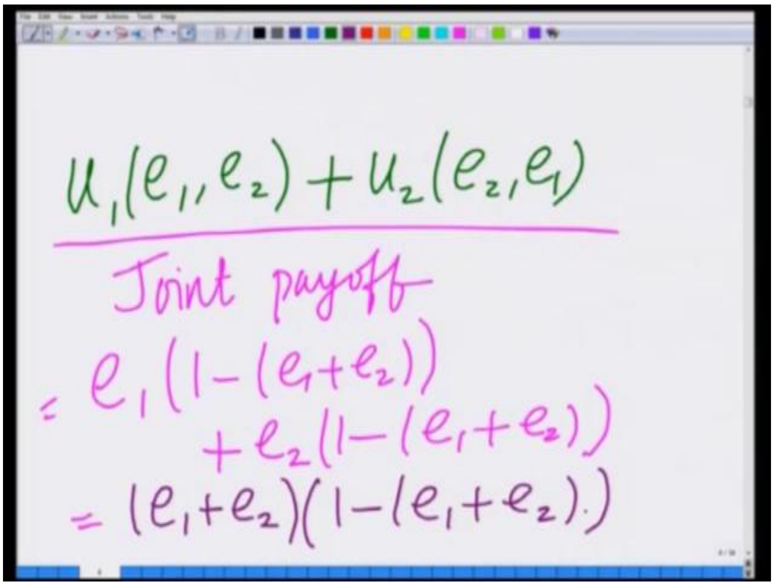

So, we have u 1 of e 1, e 2 and u 2 of e 2, e 1. So, let us look at the joint utility e1, e 2 plus u2 of e2 e1 which we are saying is the joint payoff or the sum payoff. So, let us say these two agencies decide to collaborate and try to improve their joint payoff. Well, what we have is therefore u1 of e1 plus e2 e1, e2 plus u2 of e2, e1 which is e1 into1 minus e1 plus e 2 plus e 2 into 1minus e 1 plus e 2 which can be further simplified as you can see e1 plus e 2 into 1 minus e 1 plus e 2 right. So, the net payoff u1 plus u 2 are given as the net payoff or the total payoff is given as e 1 plus e 2 into 1 minus e 1 plus e 2.

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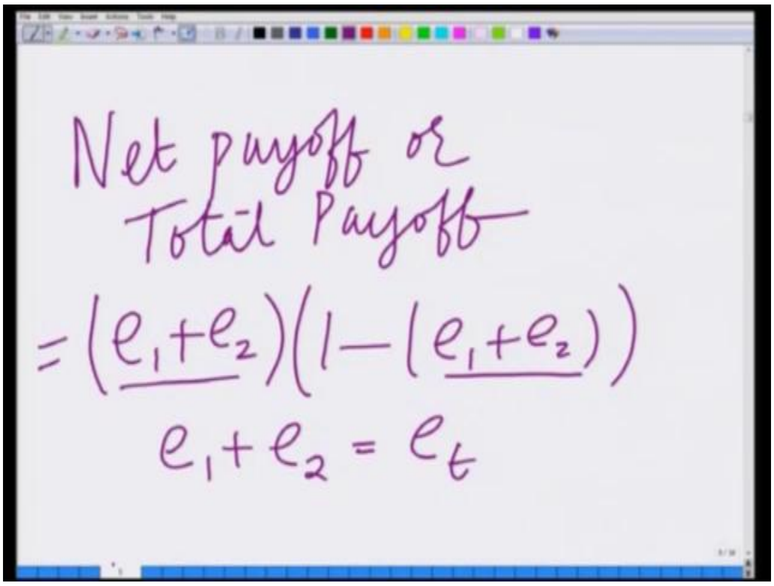

You can see that this depends only on e1 plus e 2. So, if I denote e 1 plus e 2 by e t that is the total payoff. Now, I can write the net payoff as e t into 1 minus e t because the total payoff depends only on e1 plus e 2 which is a sum effort or the joint effort put in by both these timber agencies.

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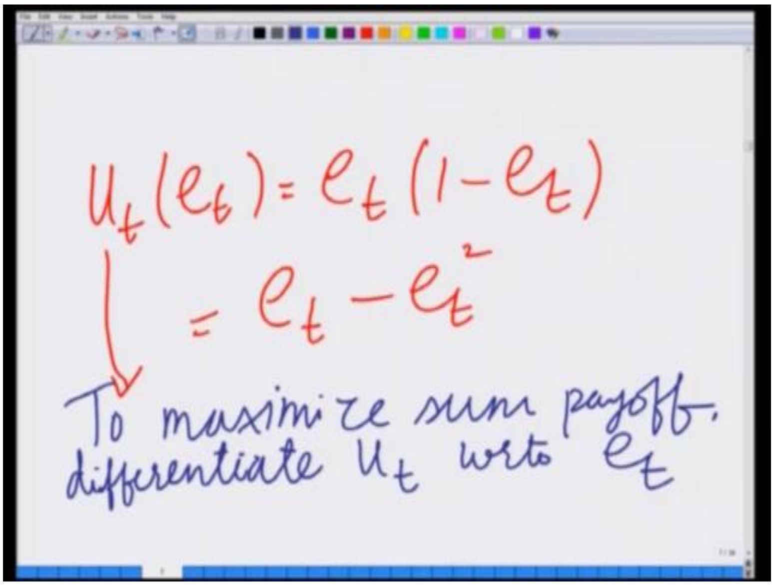

Therefore, I can write the net payoff or the sum payoff of both these timber agencies as e t into 1 minus e t and I can denote this as u t of e t and where t denotes the total payoff and e t equals e 1 plus e 2. So, I am looking at the total payoff which depends on the total effort. So, we considering a scenario with this two different agencies or companies are collaborating to maximize their sum utility or sum profit or sum payoff which can be represented u t of e t. u t of e t equals e t into 1 minus e t which is basically expanded as e t minus e t square. Now, if I have to maximize the sum payoff u t. I have to differentiate u t with respect to e t. To maximize sum payoff differentiate u t with respect to e t. I can now differentiate u t with respect to e t and set it equal to 0. To find the value of the total effort e where this total payoff to both this timber agencies, who are now collaborating or hypothetically collaborating is maximum.

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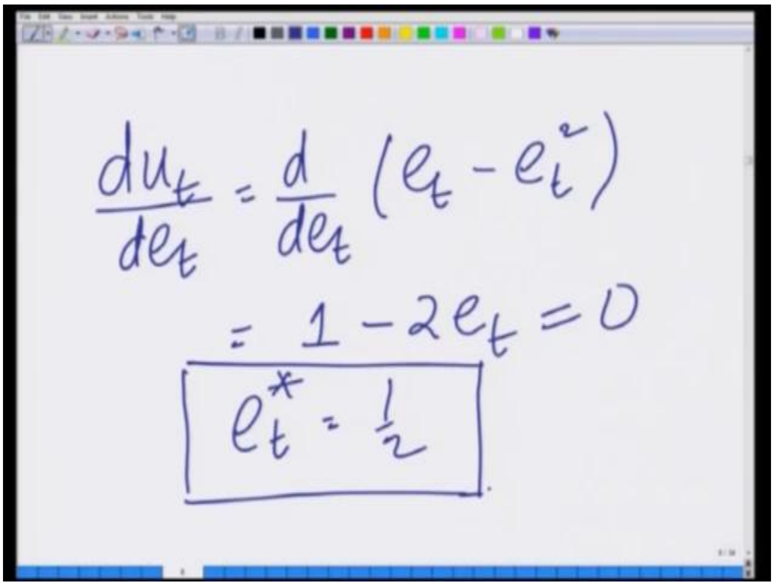

I then have d of u t divided by d e t equals d by d e t of e t minus e t square equals 1 minus 2 e t which I have to set equal to zero which means e t star which is the best response total effort let us not call it the best response which is the optimal total effort where the sum payoffs maximize is e t star equals half. So, what we have achieved so far is that we have found that sum effort e t star which is In fact, e 1 plus e 2 where the total payoff to both of them is maximize. So, we have e t star equals e1 plus e 2 equals half.

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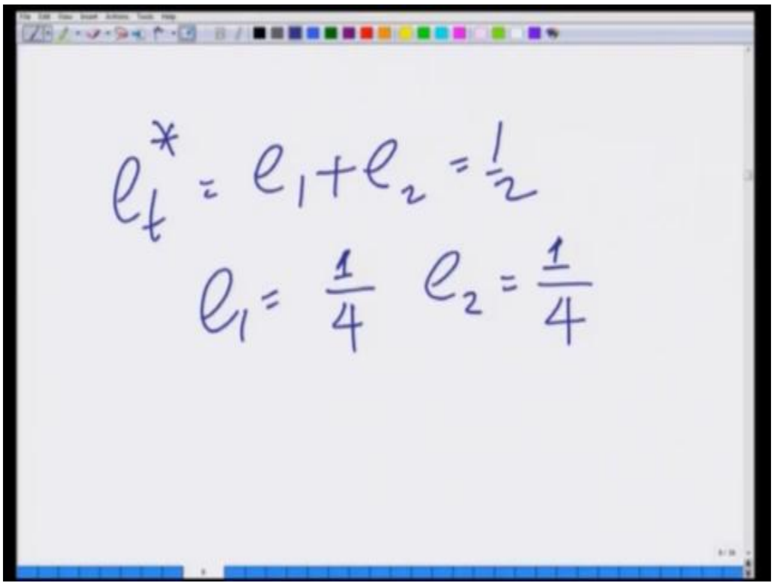

Let us, now set e 1 equal's e 2 a simple scenario that is where each of them is putting equal effort into the total effort which is fair and probably acceptable to both of them. So, we said e 1 equal's 1 by 4 e2 equals 1 by 4. So, if I said e1 equals 1 by 4 e2 equals 1 by 4 the total effort is e t equal to half at which point the total payoff that is u t is maximized.

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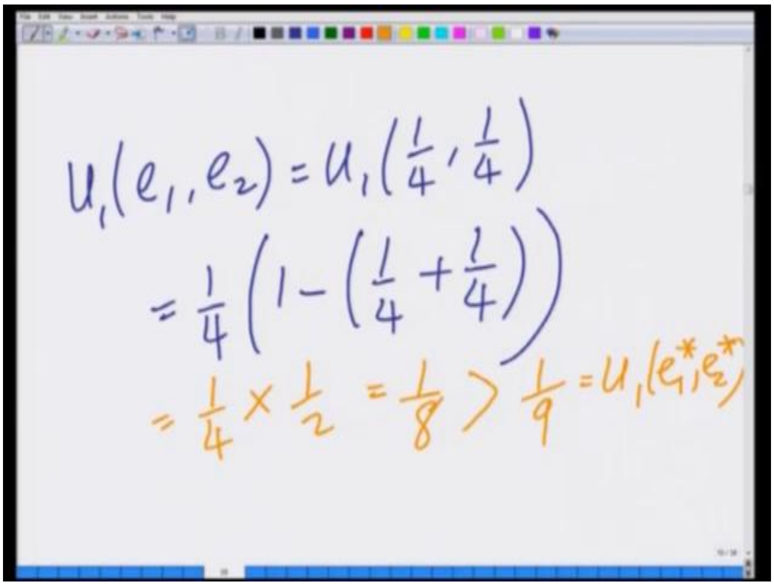

And, what is this payoff of each at this point. If you look at the payoff of each at this point that is e1 equal to 1 by 4 e 2 equal to 1 by 4 u 1 of e 1, e 2 equals u1 of 1 by 4, 1 by

4 equals 1 by 4, into 1 minus e 1 plus e 2 1 by 4 plus 1 by 4 which is equal to 1 by 4 into half which is equal to 1 by 8 .You can clearly see that 1 by 8 is greater than 1 by 9 which is the Nash payoff remember u1 of e1star, e2 star. That is in this when both of them are trying to maximize their net payoff u 1 that is the agency 1 is getting a payoff of 1 by 8

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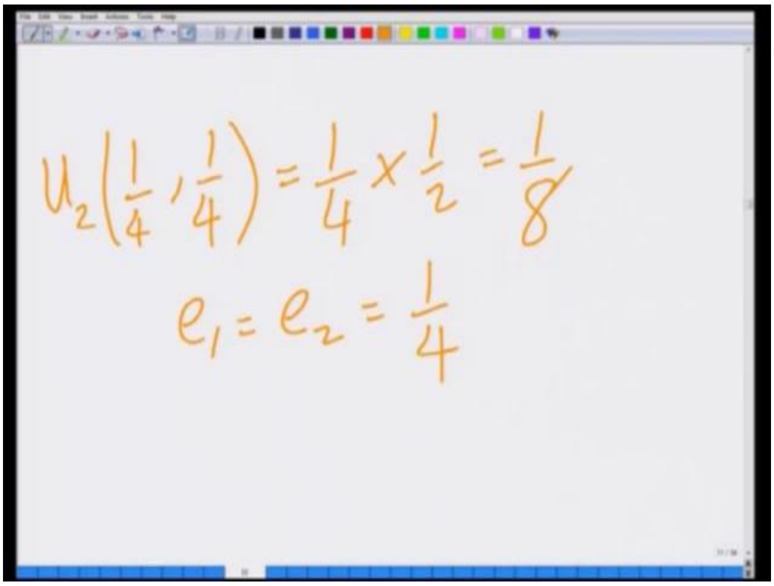

Similarly, you can calculate the payoff of u 2 at this point 1 by 4, 1 by 4 you will find that is again equal to 1 by 4 into half which is equal to 1 by 8. So, at this scenario or in this outcome where both of them are using effort e 1 equals e 2 equals 1 by 4. Both of them are able to receive the higher payoff which is 1 by 8 right. And, still is there is something interesting there is another outcome in which both of them receive a higher payoff compare to the payoff at the Nash Equilibrium right. Because, both of them are using an effort of 1 by 4 the payoff of both of them is 1 by 8 whereas at the Nash Equilibrium the payoff of both of them is only 1 by 9 right. There is another outcome where both of them can simultaneously improve their payoff.

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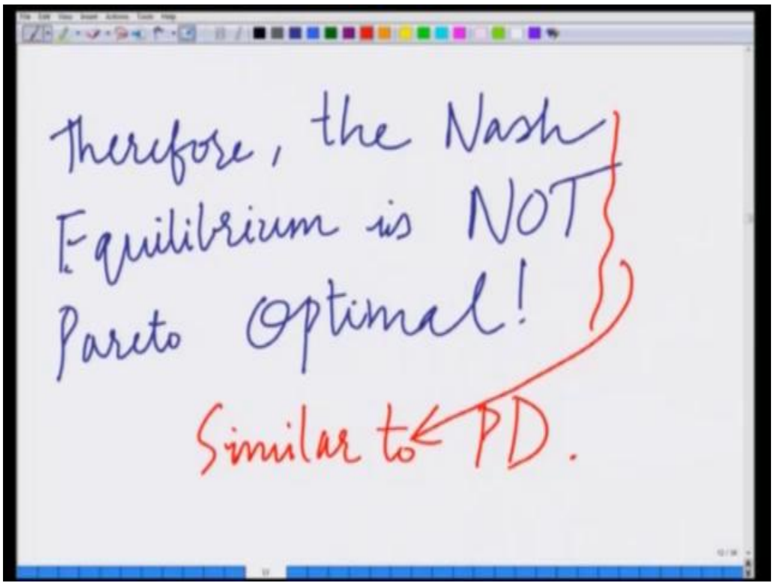

Therefore the Nash Equilibrium is not Pareto optimal which is an interesting observation. So, this has one Nash equilibrium e1star equal to e2star equals 1 by 3 the Nash payoff is 1 by 9 however, when e1 equal to e2 equal to 1 by 4 in that outcome both are receiving the higher payoff which is 1 by 8. Therefore, since both of them can simultaneously improve their payoff the Nash Equilibrium is not Pareto optimal in that this game is similar to the Prisoners Dilemma. Similar to PD which is our acronym for the Prisoners Dilemma therefore, now we have an interesting example of a game and that also explains the term the Tragedy Commons because the optimal effort the one that yields a higher payoff for both of them is e 1 equal e 2 equal to 1 by 4. But, instead each is using an effort e 1 equal to e 2 equals 1 by 3 which is greater than the effort 1 by 4.

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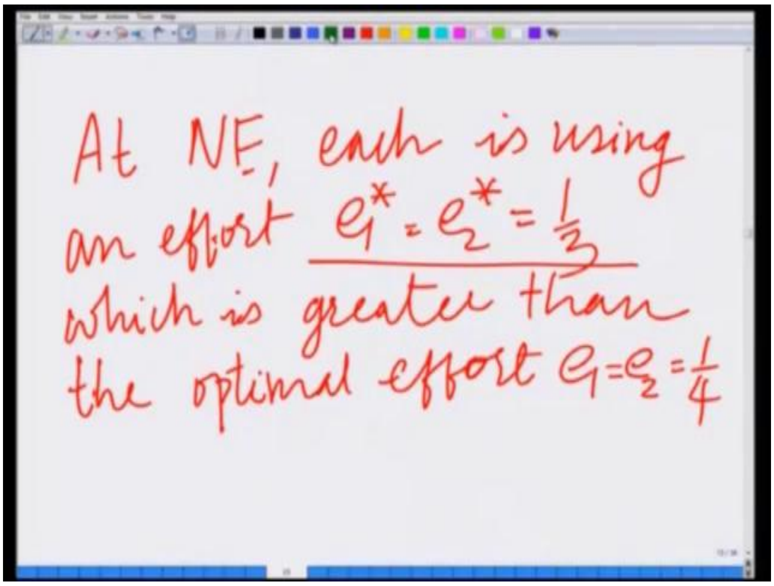

There by, both of them are leading to a faster depletion of the resource eventually leading to a lower utility right because, each of them at Nash Equilibrium NE is using an effort e 1star equals e 2 star equals 1 by 3 which is greater than the optimal effort of the Pareto Optimal effort e 1 equals e 2 equals 1 by 4 and that this effort the Nash Equilibrium effort the payoff is lower. Each is cutting more trees by using more effort that is 1 by 3 which is more that the optimal effort 1 by 4 this is leading to a faster depletion of the resource.

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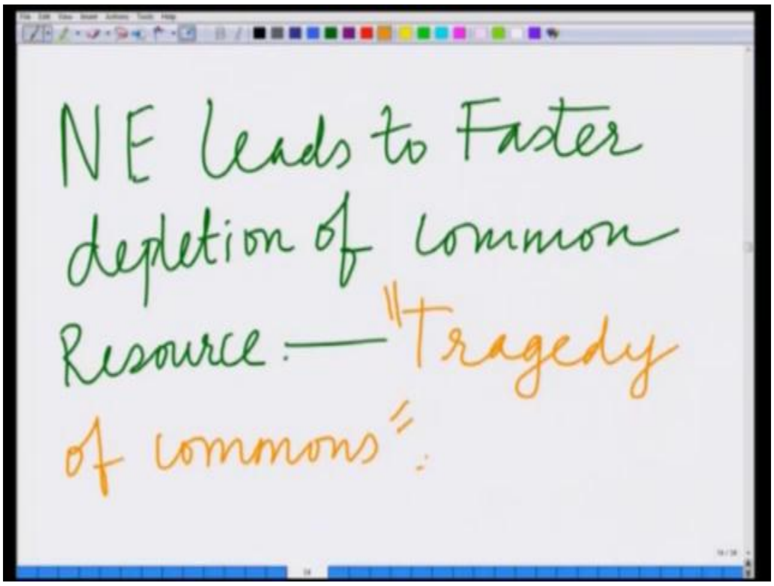

If the Nash Equilibrium; Nash Effort NE leads to faster depletion of the common resource. In this case the common resource the forest using more effort at the Nash Equilibrium leads to a faster depletion of the common resource and that is the tragedy of Commons. So, to summarize what is the tragedy of the Common we have looked at this strategic interaction with these between this multiple agents which are trying to use a common resource such as a forest and although there is an optimal outcome at the Nash Equilibrium each one is using more effort than is required which is leading to a faster depletion of the common resource leading to a lower leading eventually leading to a lower utility or a lower payoff to all the stake holders or all the population. And that is the tragedy of Commons which leads to a faster depletion of resources and lower utility or lower payoff to all the players and this is termed as the tragedy of the commons and.

In fact, this can be use to model not only the depletion of common resource such as forest, but as I indicated in the previous module that can be used model the depletion of any common resources such as fisheries or pollution of the environment or passer lands or mines over exploitation of mines over mining and so, on and so, forth. So, this is an interesting example a simple example which tells us that because of this game or the strategic interaction between this multiple players who are suppose to use the resource instead because of this game or the strategic interaction they are over exploiting that resource eventually leading poor payoff; eventually leading to a worst payoff for all this stake holders or all the players. This is known as the tragedy of commons and can be model used to model several resources; several scenarios in real life with depletion of natural resources.

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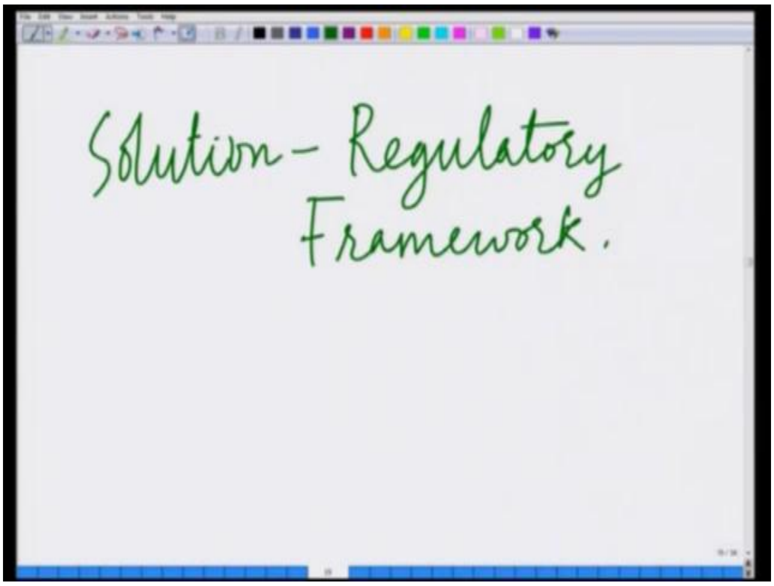

So, what is the solution in this scenario the solution is naturally to impose a regulatory frame work. So, the solution is to impose to impose a regulatory framework to curve over exploitation of any particular resource. Such as to curve over mining or fishing or over cutting of trees in a forest to allow for possible re generation and it replenishment of the natural resource and usage of these natural resource in an efficient and an optimal fashion in such a way as it is beneficial. So, this is an important game and tells as what happens when a resources over exploited and so, on. And the Nash Equilibrium provides us a game theory, provides interesting framework and study of such an interaction and behavior of such a behavior of multiple competing agency in such a scenario. So, with this we will end this module and we look at other games in the upcoming modules.

Thank you.