Strategy: An Introduction to Game Theory
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture – 07

Welcome to another module in this online course, Strategy An Introduction to Game Theory.

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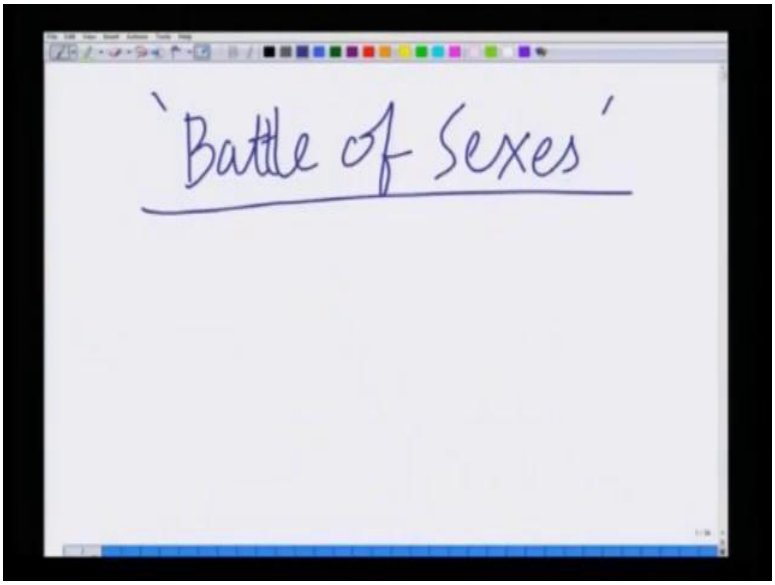

Today, let us look at another very interesting game or rather way colorful game, which is also termed as the Battle of Sexes. It has rather very colorful title, yes, we are going see from it is description shortly. And the game is as follows, the game involves is between a couple ((Refer Time: 00:45)) such as for instant, a wife and husband or boy and a girl. How deciding on the planes for an evening and they have two options, they can either choose to go for a game.

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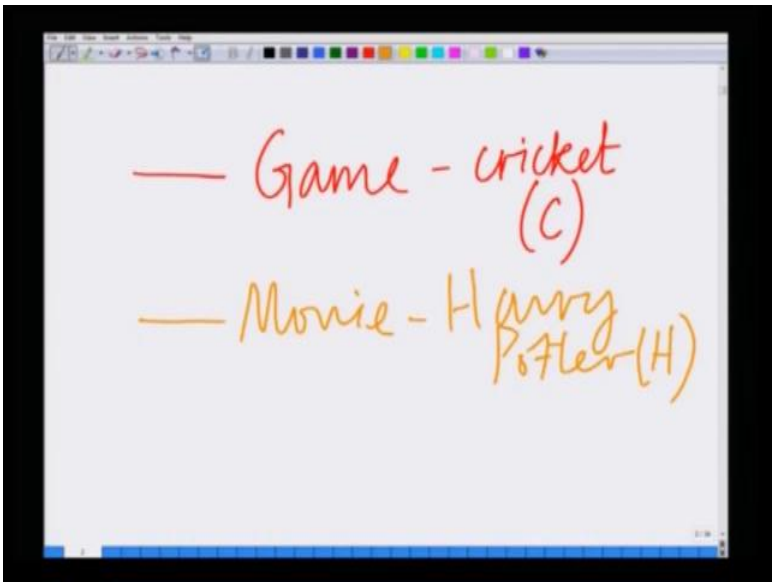

Let say a game of cricket, which I am going to represent by C, which the boy prefers or they can either choose to go for a movie. Just to make it a little bit colorful, let me denote, let say the movie is, it is a Harry Potter movie, which I am going to represent by H. So, there is a couple with such a boy and a girl who are trying to choose or who are trying to decide to choose between watching a game of cricket or a movie; that is Harry Potter.

And we are trying to model the strategic interaction between these couple of behavior of a couple in this kind of a game, whether try to make a choice between watching the game or a movie and therefore, the game table for this game can be drawn as follows.

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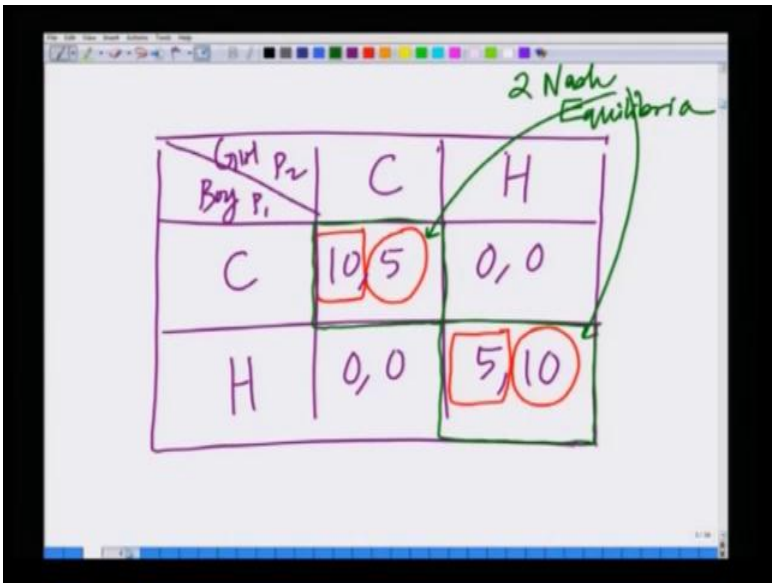

I have a two player game between a boy and a girl, I am going to denote the preferences of the boy, who is player 1 and girl, who is player 2. And both can choose between watching the game of cricket or watching movie and the girl, similarly can choose cricket or a movie and both choose the game procedures as follows.

If both choose different things for instance, the boy chooses to watch cricket and the girl chooses to watch movie or if the girl chooses to watch the game of cricket and the boy chooses to watch the movie and both of them are doing things separately. Both of them are doing things separately, then the point of the evening is lost, so therefore, each gets a payoffs of 0. So, both of them are doing different things; that is, they are doing things separately, then both get payoff of 0.

On the other hand, if both of them decide to do something together for instance, the boy and girl agree to watch either a game of cricket or game of movie. If both of them decide to watch a game of cricket together, then the boy gets a payoff of 10 and the girl gets a payoff of 5. The boy gets a slightly higher payoff, because the boy is doing something that he prefers that the girl is going along, even though she does not prefer it.

On the other hand, if both of them decide to watch a movie together, then in this case the payoffs are reversed. The boy gets a payoff of 5 and the girl gets a payoff of 10, because the girl is watching the movie, which she prefers and the boy is going along. So, this is slightly interesting and a colorful game of an interaction of a strategic interaction between the boy and girl.

And of course, strategic interaction, because by now, you know this can be formulated as a game, because the payoffs of each of them depend not only what they are doing, but it also depend on what their partner or own thing could be together or what their partner prefer so good. And these are the payoffs in this game that is the possible actions are the boy chooses rather watch a game of cricket or the movie Harry Potter and the girl chooses to watch the game of cricket or the movie that is Harry Potter.

Let me just denote this by H, to use consistent notations. Each can choose between cricket or Harry Potter and that can be are as follows. And obviously now one can again analyze this as before as we analyze several games before, I can analyze this game using the best response dynamic. So, if the girl chooses to watch cricket, the best response of the boy is of course to watch cricket, because he chooses cricket, he gets stump, if he chooses, it was the Harry Potter movie, he get 0. So, therefore, the best responsible is

goes to cricket.

On the other hand, if the girl chooses to go for the Harry Potter movie, then the best response of the boy is to again go for the movie, because if he chooses the movie that is Harry Potter he gets 5, while he chooses to go to the game of cricket, because it is not going them together they really get 0. So, therefore his best response is to go for H. Similarly, if the boy chooses cricket, then the best response of the girl is to go for cricket, because she choose a cricket gives a payoff 5, while choosing the move gives us a payoff of 0.

And with the boy, she is choosing to go for the movie that is Harry Potter then; obviously, the best response of the girl is to go for the movie, because going for the movie gives us a payoff of 10. And therefore, from this game, you can see now where the best response is intersect; there are again two boxes, where the best responses intersects. So, I have two boxes for this game, where the best responses are indeed intersecting and therefore, there are two Nash Equilibria for this, there are two Nash equilibrium.

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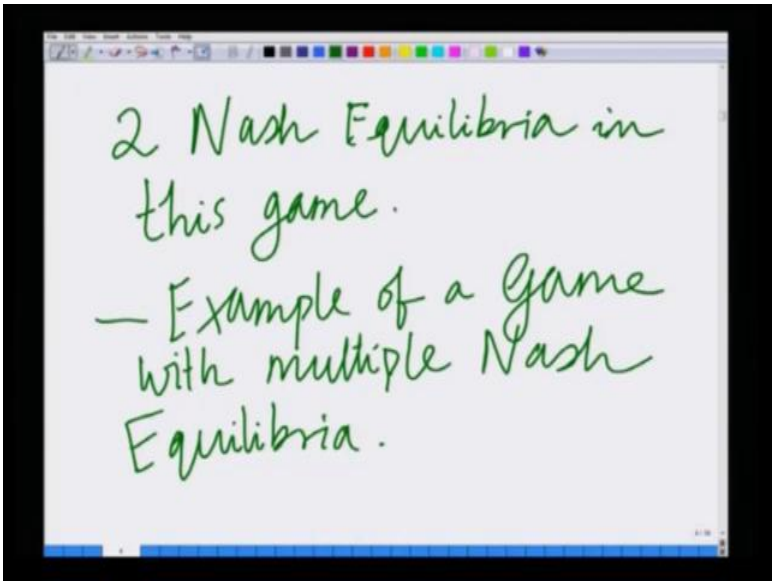

So, this is a game again with multiple Nash Equilibria, so there are two Nash Equilibria in this game. So, this is an example of a yet another game, because remember we also saw the coordination game in which there are two Nash Equilibrium. So, the coordination game is also an example of the game in which there are multiple Nash Equilibria. This is also another example of a game with multiple Nash Equilibria.

And what are the Nash Equilibria? As we let seen ((Refer Time: 07:31)), both the C C;

that is where both of them are watching a game of cricket or H H; where both of them are watching Harry Potter movie or Nash Equilibria.

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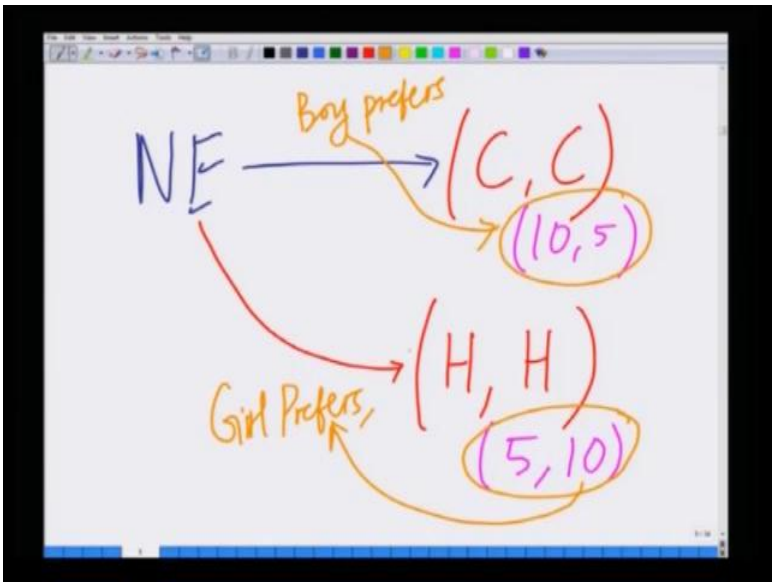

So, a Nash Equilibria are N E of this game or both of them are watching the game of cricket or both of them are watching the movie Harry Potter. So, this is multiple Nash Equilibria, there are two Nash Equilibria, where both of them are watching the game cricket or both of them are watching the movie; that is Harry Potter and the payoffs corresponding to these different Nash Equilibria.

Now, if you look at the payoff corresponding to these different Nash Equilibria, the payoffs corresponding to cricket are well 10 comma 5 and the payoffs corresponding to both watching the Harry Potter movie are 5 comma 10. And now, you can see, why both these of even though this game has two different Nash Equilibria, why this game is different compared to the game, compared to coordination game. Because, these two different Nash Equilibria are different in the sense that each prefers a different Nash Equilibria.

For instants, the boy prefers a Nash Equilibrium in which he is getting a higher payoff of 10. So, the boy prefers this Nash Equilibria of C comma C, while the girl prefers a Nash Equilibria of H comma H, since the girl is receiving higher payoff of 10 in this Nash Equilibrium compared to the other.

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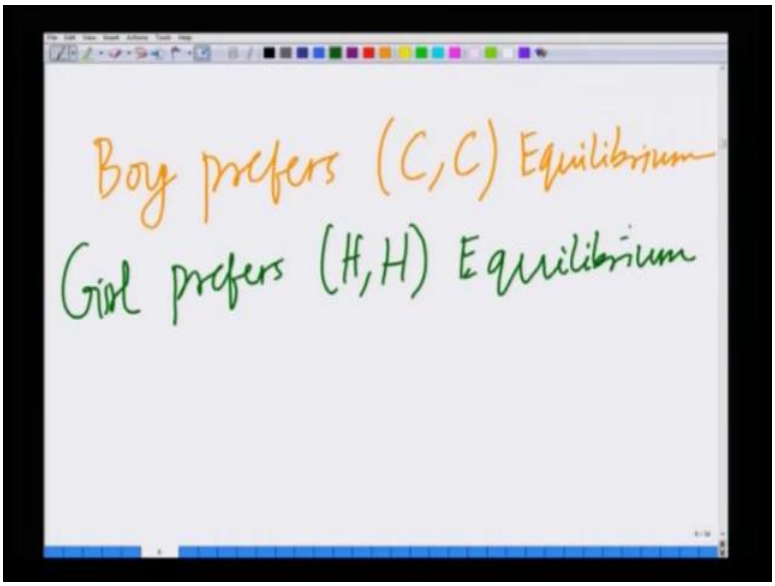

And therefore, we have the boy prefers the C C equilibrium and the girl prefers the H H equilibrium and therefore, this is not a coordination game. Because, remember in the coordination game every one prefers one Nash equilibrium versus the other, because one of the Nash equilibria is a higher payoffs of every one. However, in the Battle of Sexes game, the Nash equilibrium C C gives higher payoff of the boy. So, the boy prefers this Nash equilibrium, while the Nash equilibrium H H gives a higher payoff of the girl and therefore the girl prefers the H H Nash equilibrium.

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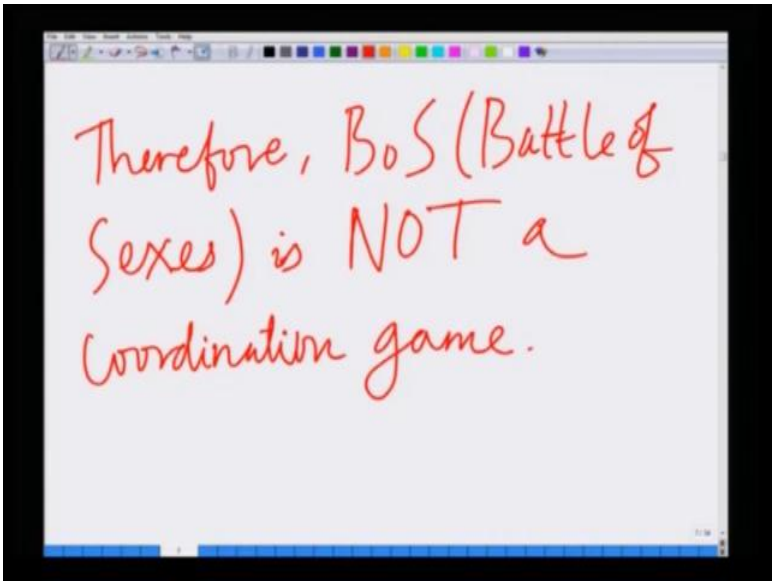

Therefore, even though there are the two Nash Equilibria, this is not a coordination game. Therefore, the battle of sexes, hence therefore, battle of sexes which abbreviate as

B O of S, B O S. Therefore, the battle of sexes is not a coordination game, because each of the players prefers a different Nash. Even though, it has multiple Nash Equilibria, each of the players prefers a different Nash equilibrium therefore, this is not a coordination game.

So, naturally now this is an interesting example, where each prefers a different Nash equilibrium. So, what do you see happen in practice? In practice, you find that every time, they are choosing one Nash equilibrium was is there? No, rather what you find is that several times they are compromising on choosing one action versus the other. Several times they are finding practice, the couples are compromising, on other words watching the game cricket together or going out for a movie together.

So, there is another compromise dynamic which can be a possible solution to the inherent competition, to the inherent ambiguity between these two Nash equilibrium and that is therefore, can be mode and therefore, there is another Nash Equilibria. Another Nash Equilibrium, which is a sort of compromise Nash equilibrium, where each of them is choosing either cricket to watch the game of cricket or to watch a movie with assert in frequency.

But, each of them is using a randomized or a mixed strategy and this will be introduced in another module, when we talk about mixed strategy. So, what you are saying is because each players prefers different Nash equilibrium, this gives raise to different dynamic, where each one is compromising a certain set of time; that is another compromise alternative. But, they are choosing between cricket and watching the Harry Potter movie with a certain frequency.

So, in this game; that the third Nash equilibrium hidden, which is going to be explored in a future point. So, right now what we have seen is basically there are two Nash Equilibria in this game and each one prefers a different Nash equilibrium.

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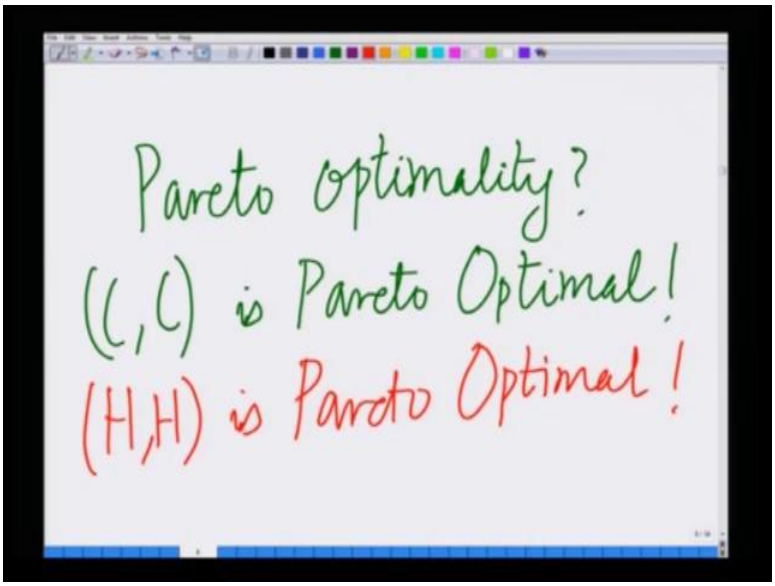

What about Pareto optimality? Let us look at Pareto optimality, what we can say about Pareto optimality. Remember we define Pareto optimality as an outcome, an outcome is Pareto optimal, if there is no other outcome, where both the players can simultaneously improve their payoffs. Now, let see if these outcomes if these Nash Equilibria are Pareto optimal, let us take a look at C C. The C C outcome heals a payoff of 10 comma 5; that is 10 to the boy and 5 to the girl.

If the both shift to H comma H in the boys payoff is been reduce from 10 to 5. So, therefore, the boy is not improved each payoff. If they go to C comma H, then go to receive a payoff 0 comma 0 that is the payoff both of the decreasing an also H comma C, then the payoff both of them decreasing 0 to 0. Therefore, this no other outcome there both the boy and the girl can simultaneously improve the payoff, which means an Nash equilibrium C comma C is Pareto optimal.

Because, there is no other outcome with simultaneously gives a higher payoff both the boy and girl C comma C is a Pareto optimal equilibrium. What about H comma H, look at this again H comma H, if we going to see C comma C, then the payoff of the girl is decreasing from 10 to 5. And any other process just C comma H or H comma C, if payoff both of them decreasing to 0 comma 0. So, there is no other outcome, where both simultaneously improve payoff, which means that H comma H is also optimal Pareto optimal equilibrium.

Therefore, this is very interesting game, where you can see, there are multiple Nash

equilibrium; that is there are two Nash Equilibria and each of them is Pareto optimal. This is unlike the prisoners dilemma, where that is one Nash the equally, remember in the prisoners dilemma; that is one Nash equilibrium, it is not Pareto optimal. In the coordination game, there are two Nash Equilibria, only one of them is Pareto optimal.

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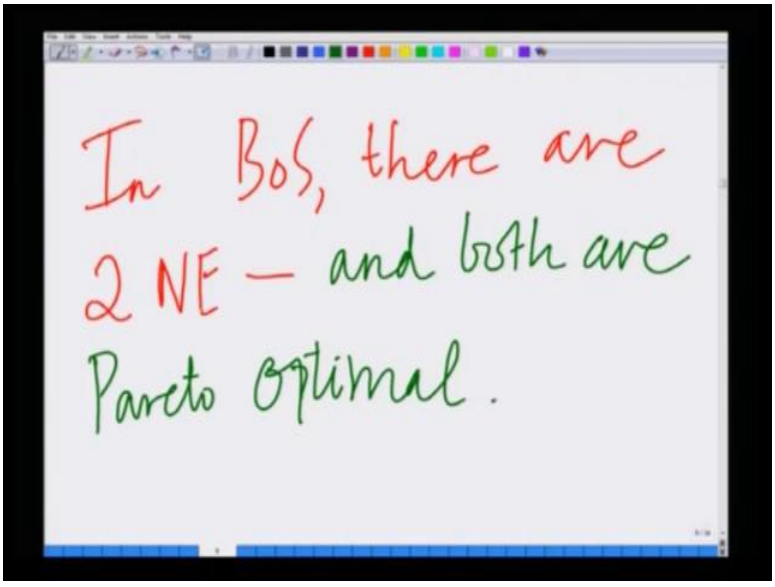

And in the battle of sexes, there are 2 N E and both of them are 2 N E are Pareto optimal. So, battle of sexes in B O S, there are 2 N E and both of them are Pareto optimal. So, the battle of sexes ah which you are interesting colorful game in a couple is also interesting in terms of it is behavior of the game theory perspective, because it is multiple Nash Equilibria.

And each layer preference a different Nash equilibrium and further, both the Nash Equilibria a Pareto optimal in a such that, a there exist no their outcomes, where both simultaneously improve they are. So, this is yet another example for a kind of game that in occur in the context of game theory. So, we will end this module here and we will consider other games in different modules.

Thank you very much.