Strategy An introduction to Game Theory
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Lecture - 06

Hello, everyone. Welcome to another module in this online course strategy an introduction to game theory. I hope… So, we are looking at different examples of games and some examples of game – examples… So, we are trying to model these games as… and these different games and try to give examples of scenarios, which are as close to real life examples or as close to real life scenarios or as close to real life occurrences as possible. And, what we have seen in the past couple of modules, we have to summarize it comprehensively. We have seen an example of a simple game, which is a prisoner's dilemma and we have seen examples – other examples, which are similar to prisoner's dilemma like a cold-war scenario or a market competition between two firms. And, we understood different concepts. The fundamental concept is of course the Nash equilibrium and also we looked at concept of Pareto optimality. And, we looked at the concept of a dominant strategy equilibrium.

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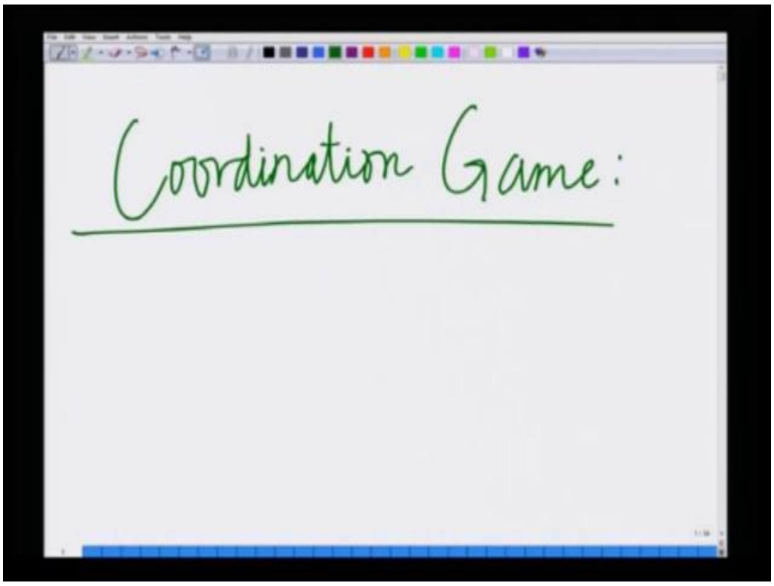

What we want to do now; what I want to do in this module is to introduce a slightly different game is another type of game, which I am going to call… which is known as a coordination game. So, in this module, I want to start talking about a new game, which is a coordination game. And, I want to…

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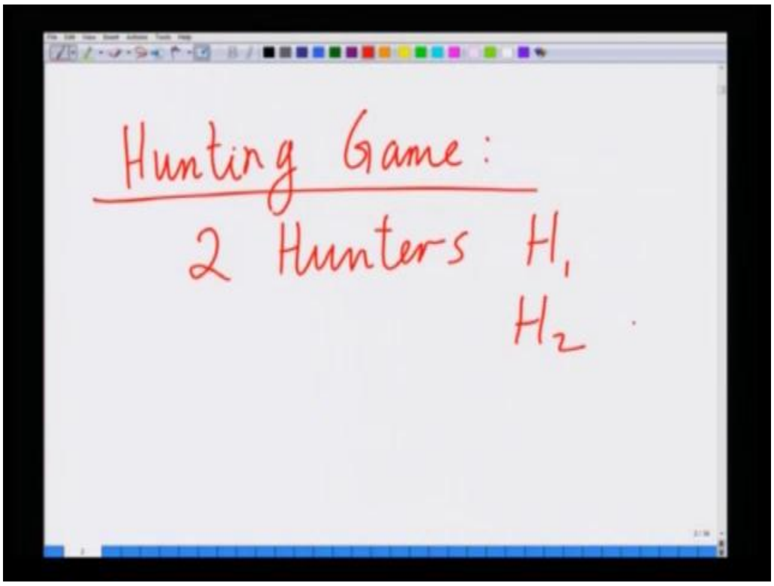

And, I am going to introduce it by talking about a simple example also known as a hunting game. It is a very popular example. It is a hunting game, which involves two hunters H 1 and H 2, which we are going to denote by H 1 and H 2.

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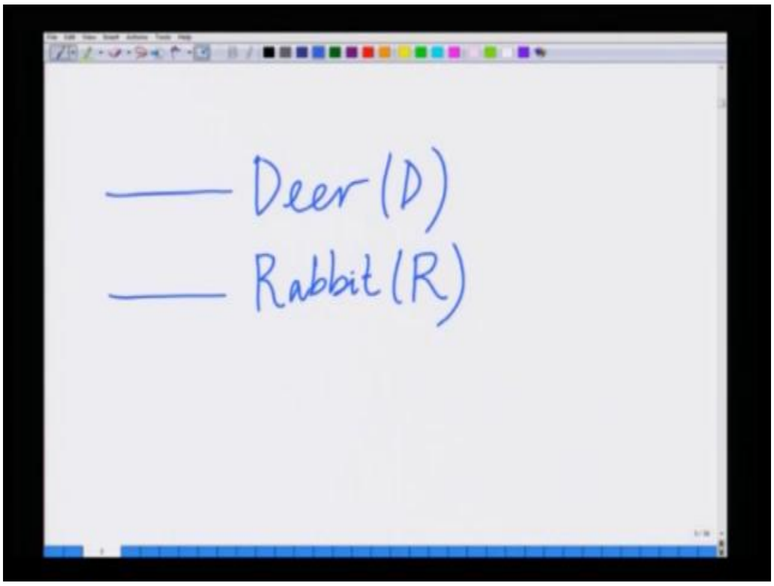

And, those two different hunters have two options either both of them can choose to hunt for the deer, which is D or they can choose to hunt for the rabbit, which is R. Now, the idea is the following: the deer is a big animal. So, it requires a large amount of coordination; it requires multiple hunters to hunt for the deer. Well the rabbit is a small animal. So, each hunter can individually hunt for the rabbit. So, this is a game as you can see which requires some coordination if they want to hunt for the deer. Let us now try to… Let us now draw the game table for this game.

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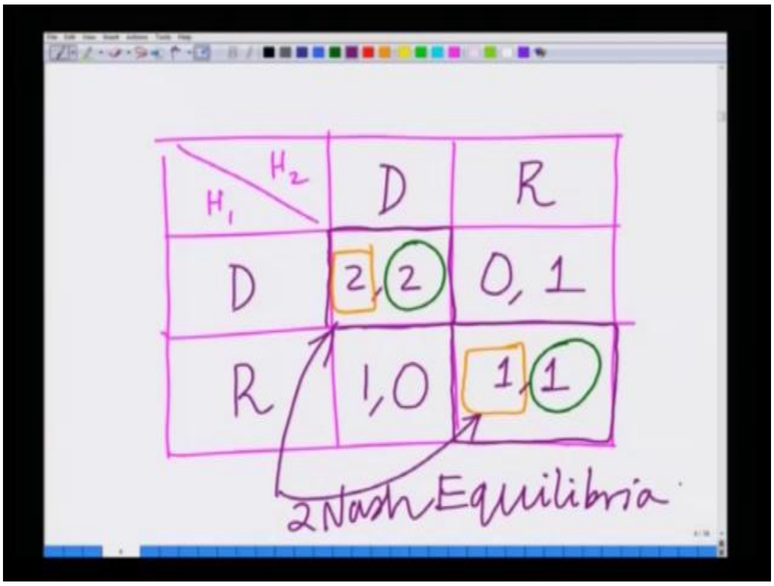

Let us now try to represent this game using the standard game table that we are now familiar with. I have a table, which is a collection of rows and columns. I have hunter 1 and hunter 2. Each hunter – hunter 1 can choose to either go for the hunter 1; can either choose to for the deer or the rabbit. Hunter 2 can also choose to go for the deer or the rabbit. If both of them choose to go for the deer, since the deer is the larger animal; both of them get a payoff of 2 comma 2. Both of them choose to go for the rabbit; then, both of them get a payoff of 1 comma 1 for the rabbit, which is, since it is a smaller animal. However, if hunter 1 goes for the rabbit and hunter 2 goes for the deer; then, hunter one gets a payoff of 1, because he can be successful in hunting for the rabbit individually. While hunter 2 gets a 0 because hunter 2 is into hunting alone and the deer is a large animal, which we said requires coordination; and, hunter 2 cannot hunt for the deer alone. So, he ends up getting a 0.

And similarly, you can see if hunter 1 is going for the deer and hunter 2 is going for the rabbit; then, hunter 1 since he cannot hunt for the deer alone, he ends up getting a 0.

Hunter 2 since he is going for the rabbit, which is a small animal, he is successful and he gets a payoff of 1. This is the hunting game or which as we also said is an example of a coordination game. Let me repeat that again. If both hunter 1 and hunter 2 go for the deer; since they are coordinating, they are successful in capturing the deer. And, since the deer is a large animal, both of them get a payoff of 2 each, that is, 2 comma 2. If they both go for the rabbit; then, since the rabbit is a small animal, both of them get a payoff of 1 each. While if hunter 1 goes for the rabbit; he is successful in hunting for the rabbit. So, he gets a payoff of 1. While hunter 2 – since he is hunting alone, he cannot capture the deer since the deer requires coordination. So, he ends up getting a payoff of 0. And similarly, vice-versa; if hunter 1 is going for the deer and hunter 2 is going for the rabbit; hunter 1 gets a payoff of 0 and hunter 2 gets a payoff of 1.

Of course, similar; of course, now, we know as similar to the previous games that, we can analyze this game using the best response dynamic. So, let us start by looking at the best responses. If hunter 2 is going for the deer; then, the best response of hunter 1 is to choose the deer, because the deer gives him 2, while the rabbit gives him 1. On the other hand, if hunter 2 is going for the rabbit, the best response of hunter 1 is to go for the rabbit since the rabbit gives him 1, while the deer gives him 0. Similarly, if hunter 1 is choosing the deer, best response for hunter 2 is to choose the deer, because the deer gives him 2, while rabbit gives him a payoff of 1. Similarly, if hunter 1 is choosing the rabbit, best response of hunter 2 is to choose the deer; best response of hunter 2 is to choose the rabbit, because the rabbit gives him 1, while deer gives him 0.

And therefore, interestingly, now, you can see that, in this game, there are two intersection of best responses, that is, the two outcomes where the best responses intersect, that is, the deer-deer outcome and the rabbit-rabbit outcome. You can clearly see that, if hunter 2 is choosing the deer, best response of hunter 1 is to go for the deer. If hunter 1 is going for the deer, best response of hunter 2 is to go for the deer. So, deer-deer is indeed a Nash equilibrium. Similarly, you can see that, the rabbit-rabbit outcome is also a Nash equilibrium. So, we have two Nash equilibria in this game or we have multiple Nash equilibria. So, we can see that, in this game, this is an example of a game with multiple Nash equilibria.

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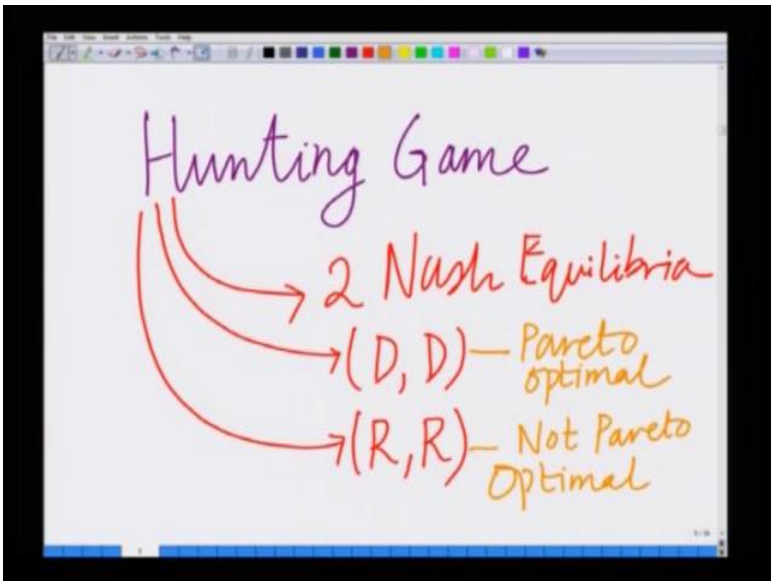

So, for this hunting game, we have two Nash equilibria; either we can go, that is, both the dear-dear outcome, where both of them are going for the deer or the rabbit-rabbit outcome, where both of them are choosing the rabbit. So, both of these outcomes are the Nash equilibria of this game. So, there are two Nash equilibria of this game. And, what is more interesting is if you go back to this game, you can see that, while the rabbit-rabbit outcome is not Pareto optimal; since both of them can simultaneously improve their payoff by choosing going for the deer, that is, going for deer comma deer, the deer-deer outcome or the deer-deer, which is a Nash equilibrium outcome is a Pareto optimal outcome, because both of them cannot improve their payoff by choosing any other outcome; because if they go for either the deer-rabbit or the rabbit-deer or the rabbit-rabbit, they will in fact both be reducing – strictly reducing their payoff.

Therefore, in fact, you can see deer-deer, that is, there are two Nash equilibrium outcomes: deer-deer and rabbit-rabbit. And therefore… And, in this, out of these two Nash equilibrium or Nash equilibria, the deer-deer is in fact a Pareto optimal outcome. The rabbit-rabbit on the other hand is not Pareto optimal; since from the rabbit-rabbit, both the hunters can simultaneously improve their payoff by choosing deer comma deer. So, therefore, this is an interesting game. This is a game that is different from the prisoner's dilemma, because remember – in the prisoner's dilemma, you have one Nash equilibrium and the Nash equilibrium is not Pareto efficient or Pareto optimal. While in this coordination game, we have two Nash equilibria: the deer-deer and the rabbit-rabbit;

and, one of the Nash equilibria, that is, deer comma deer is indeed a Pareto optimal outcome. This is indeed a Pareto optimal outcome.

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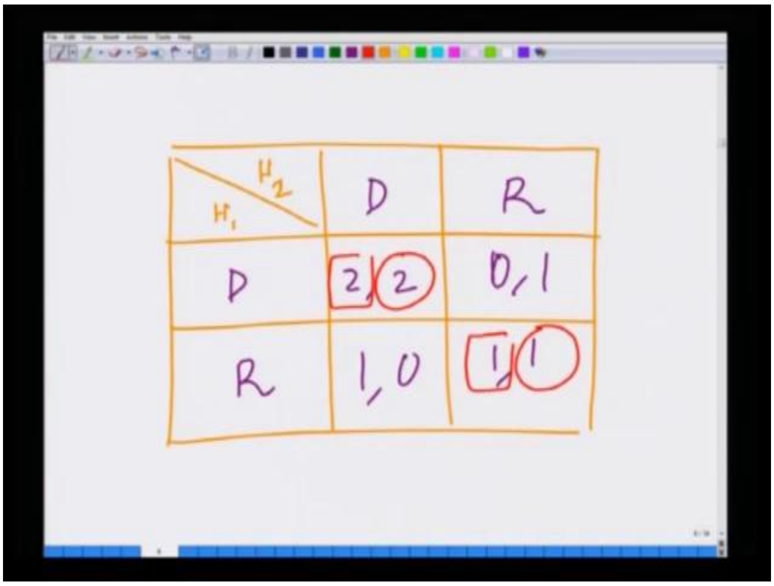

And also, you can see that, if you go back to this game; you can see… Let us draw this game again. If you look at this game, let us try to see if this game has a dominant strategy. We look at H 1, H 2; we have either the deer, rabbit and the payoffs of our… Now, you can see if hunter 2 chooses deer, best response of hunter 1 is to go for deer; and, if hunter 2 chooses rabbit, best response of hunter 1 is to go for the rabbit. So, hunter 1 does not have a dominant strategy, because his best response depends on the action chosen by hunter 2. If hunter 2 chooses deer, best response of the hunter 1 is to go for a deer. On the other hand, if hunter 2 choose – goes for a rabbit, best response of hunter 1 is to go for a rabbit. Similarly, if hunter 1 chooses deer, best response of hunter 2 is to go for a deer. And, if hunter 1 chooses rabbit, best response of hunter 1, hunter 2 is to go for a rabbit. So, therefore, the best response of hunter 1 and hunter 2 depend actually on the action of the other hunter. So, there is no dominant action or strategy, which is uniformly the best response irrespective of the action of the other person. Therefore, there is no dominant strategy; hence, there is no dominant strategy equilibrium; hence, there is no dominants strategy.

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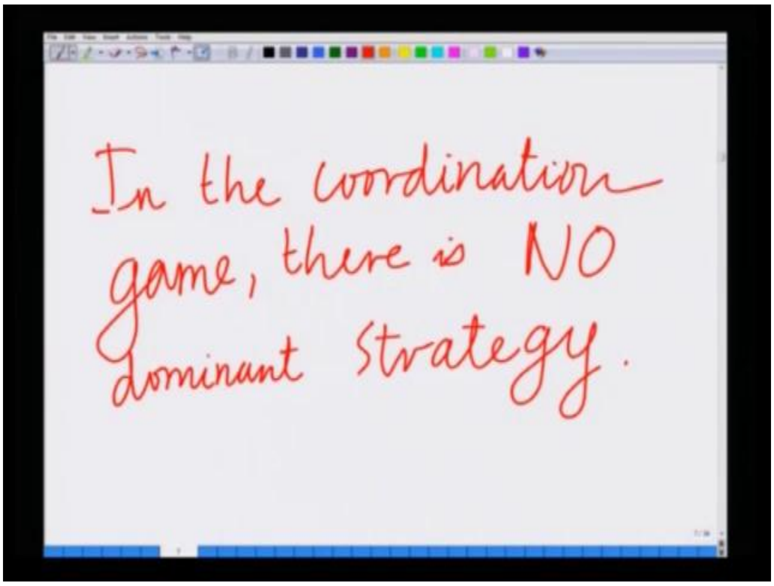

Say in the coordination game, there is no dominant strategy; hence, there is no dominant strategy equilibrium. And now, it is apparent or it is obvious why coordination is important, because this best response depends on the action or the strategy of the other player. You can choose a D only if all the other players or all the other hunters are choosing D, because choose hunting for the deer requires coordination. Therefore, it requires someone to coordinate everyone to hunt for the deer, which is indeed better for everyone, because it is a Pareto optimal outcome and that yields a higher payoff. Therefore, it requires both the hunters to coordinate and come to an agreement to go for the deer-deer outcome than go for the rabbit-rabbit outcomes since the deer-deer outcome yields – it is in the interest of both the hunters to hunt for the deer since it yields a higher payoff for both of them since it is a Pareto optimal outcome. And therefore, this is the coordination.

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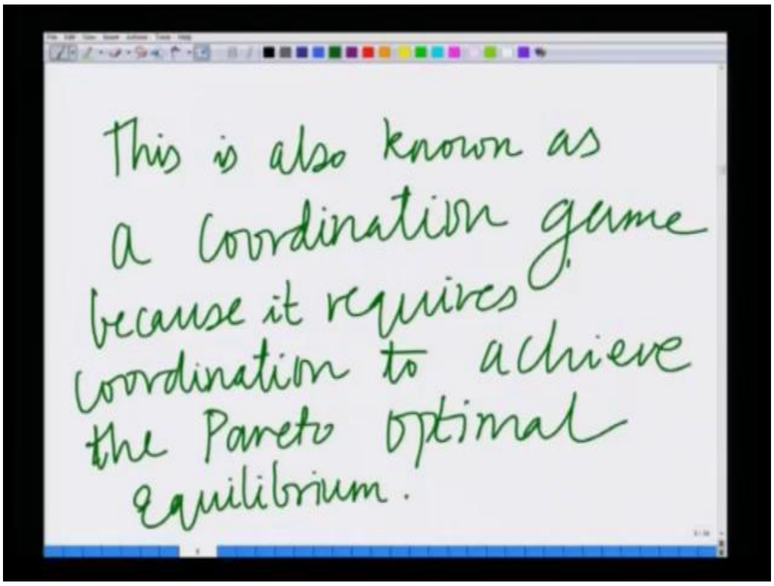

This is also known as a coordination game, because it requires coordination to achieve the Pareto optimal – requires coordination to achieve the Pareto optimal equilibrium; that is, there are two equilibrium deer-deer, rabbit-rabbit. The deer-deer outcome is actually Pareto optimal and it is better for… because it gives a higher payoff for both the players. Therefore, it requires them to coordinate or requires coordination to achieve this equilibrium, which is the Pareto optimal equilibrium. Therefore, this is also known as a coordination game; it is also a leadership game, because it requires someone to take a leadership role so as to coordinate. Therefore, it is of everyone or the coalesce. Therefore, it is of everyone to go for the deer-deer outcome. This is typical of a coordination game. And therefore, it requires coordination to achieve the Pareto optimal equilibrium or what we can ((Refer Slide Time: 15:17)) also call as that good Nash equilibrium; in this game, the good Nash equilibrium because it gives… This Nash equilibrium gives a higher payoff. All the other players compare the deer-deer outcome gives a higher payoff for all the players or hunters compared to the other Nash equilibrium, which is the rabbit-rabbit Nash equilibrium.

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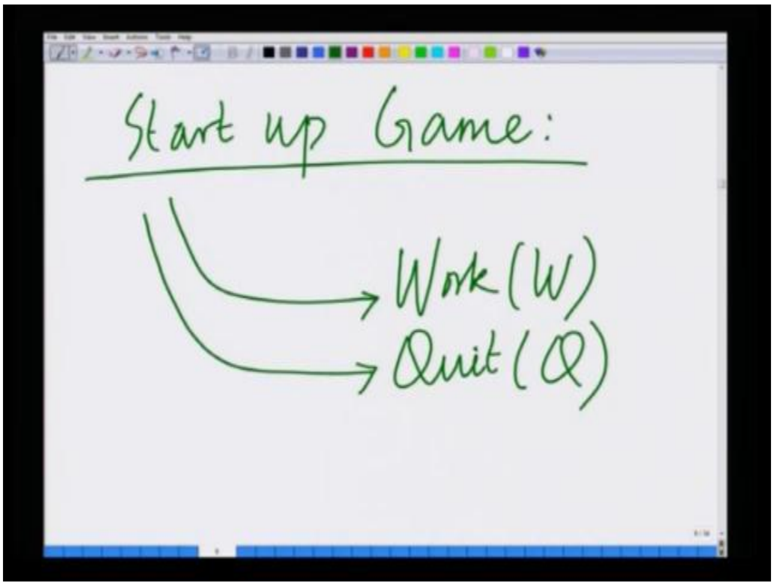

Another way to understand this is in terms of a startup game; that is, one example, which is similar to a coordination game is a startup game or which is ((Refer Slide Time: 15:53)) game involving startup. A game involving a startup can be model as follows. Let us say we have a startup. And, in each worker, a startup is a small company. So, there is some uncertainty about its survival, some uncertainty about the pay packages employees receiving in a startup. So, each employee has two options either to keep working, that is, work which we represent as W or to quit, which we can represent as Q. So, each employee has two options: either to work or to quit. And, of course, if the employees work together – all the employees work together, the startup can be successful and can yield a higher dividend for all the stake holders. However, if all the employees quit and then they join a different company, then it yields a slightly lower payoff for each of them. Let us say they join a larger company, which yields a slightly lower payoff. But, if few workers remain working while the others quit, those who quit get a higher payoff. But, those keep working can get a payoff of 0 because with only a few workers working, the startup is not sure take off or the startup eventually founders a flops and everyone is left. The people who continue to – the people continue to remain working are left with a very low payoff. So, this is similar to a coordination game.

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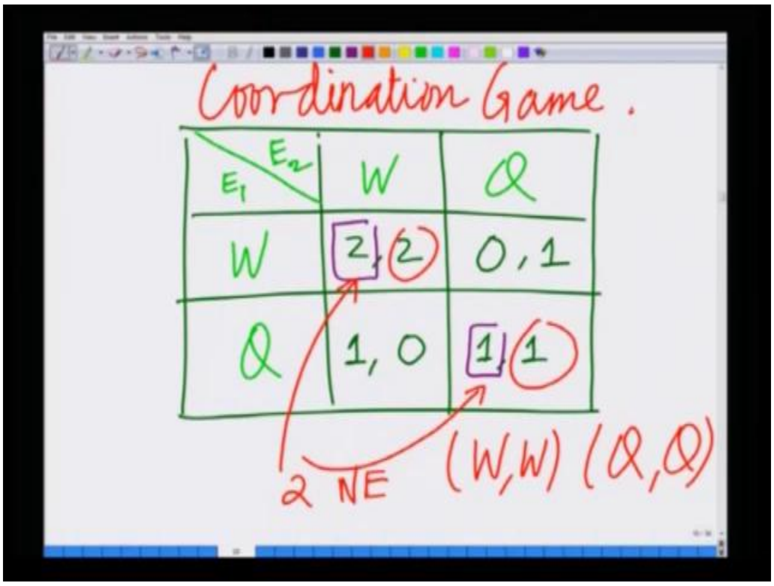

Let us draw a simple game table to sort of reflect this scenario. Again we have a startup game in which the employees have the option of either working or quitting – working or quitting. Let us say we have E 1; we have E 2. E 1 can either work or quit; E 2 can either work or quit. If they both work, they get a payoff of 2 each, because the startup is multiply successful. If they both quit, they join a different company; they get a lower payoff of 1 each. However, if E 1 is working and E 2 is quits, the person who has quit gets a payoff of 1; the person who continues to work eventually gets a payoff of 0 because the startup is not successful, because the startup again requires coordination of the efforts of a group of people to make it work. Similarly, if E 1 quits, he gets a payoff of 1; and E 2 is working, he gets a payoff of 0, because the startup eventually is not successful, because the workers have quit.

And again, if you can see this game, this game is very similar to the coordination game; because if E 2 is working, the best response of E 1 is to work; if E 2 is quitting, best response of E 2 is to quit. If E 1 is working, best response of E 2 is to work. If E 1 is quitting, best response of E 2 is to quit. And therefore, you can see there are two Nash equilibria – either both of them work or both of them quit. And of course, a good Nash equilibrium is… So, there are two Nash equilibria – 2 NE, that is, the work, work or quit comma quit. As you can see, work-work is Pareto optimal Nash equilibrium. If all the employees stick together and work towards the ultimate success of the company, then that company sort of takes off and yields higher dividend for all the employees or all the stake holders in comparison to a situation, where everyone is quitting. So, again it requires a leadership role – someone to assume a leadership role so as to keep the employees – different employees from quitting the company or make the employees work towards making this startup effort successful. So, this is startup game, which is an example of – another example of a coordination. This is a startup game, which is another example of a coordination game. This is…

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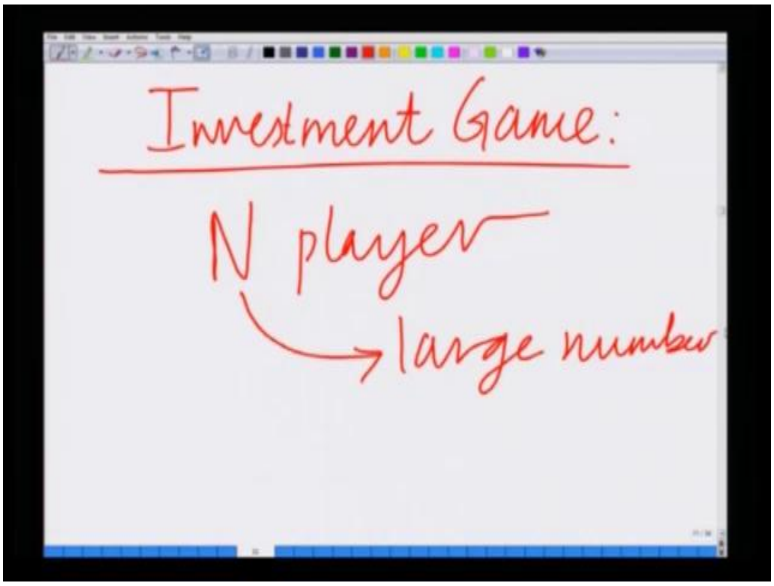

And, let us take another example – another example that is again close to a real life scenario. Let us take a look at an investment… Let us take a look at an investment game – an investment game. Let us now generalize a game – this kind of a coordination game to an N player game. Let us take a look at for instance an N player game; where, N is let us say a large number. We are looking at investors probably across a city or across a country or so on.

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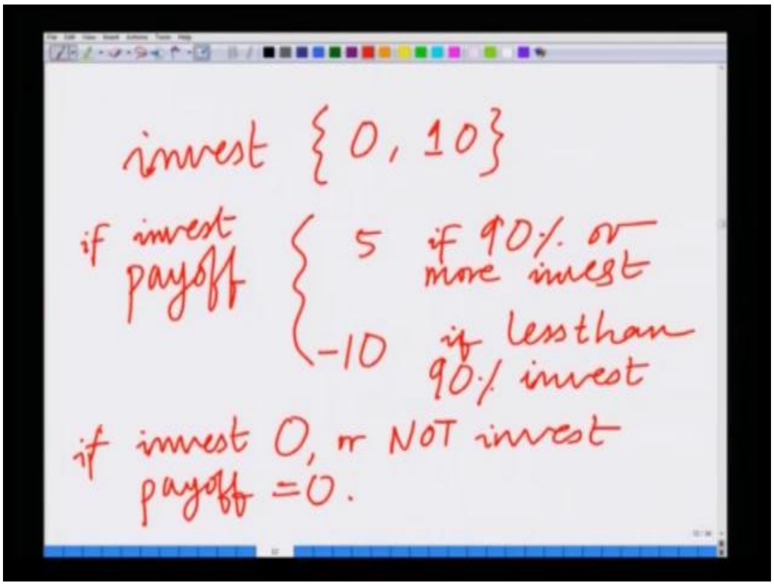

And, the investment game works as follows; that each can invest either 0. Or, let us take as a simple example. Each can either choose to invest 0 or 10. And, the payoff is if you invest – if you invest – if invest, payoff is 5 if 90 present or more invest. And, the payoff is minus 10, that is, you lose all your investment if less than 90 percent. So, in the investment game, you can either invest 0 or 10 rupees. Let us say a simple investment game. And, if you invest 10 rupees, your payoff is 5 rupees, that is, you get 10 plus 5 – 15. If 90 percent or more of the people invest, that is, investments are only successful when the large number of people invest. On the other hand, if less than 90 percent people invest, then your payoff is minus 10, that is, you lose your original investment. If you do not invest, that is, if you invest 0… If you do not invest or you invest 0, then your payoff is obviously 0. If invest 0 or not invest… If you invest 0 or if you do not invest anything, basically the payoff is 0.

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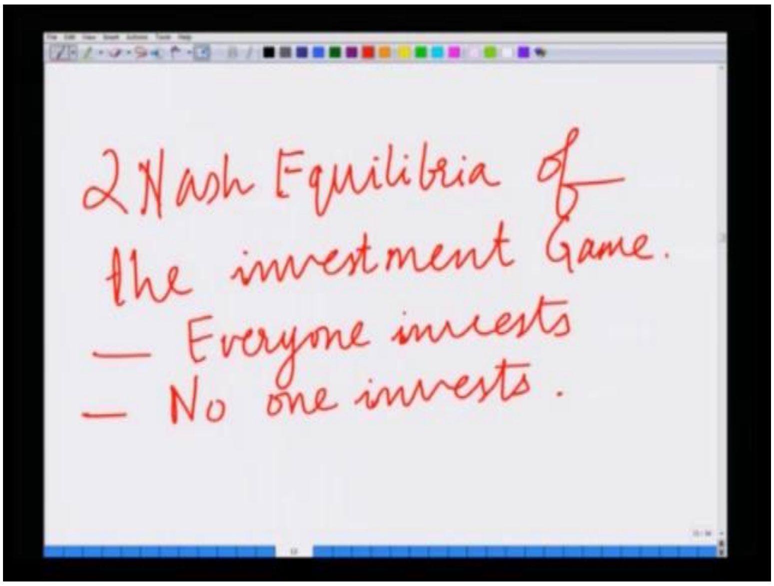

Now, let us try to understand and let us try to analyze the Nash equilibrium of this game. My claim is the Nash equilibrium of this game is where everyone invests or no one invests. There are 2 Nash equilibrium – 2 Nash equilibria of the investment game. Everyone invests or… Everyone invests or no one invests. There are 2 Nash equilibrium – the Nash equilibria of this game, where everyone invests or no one invests.

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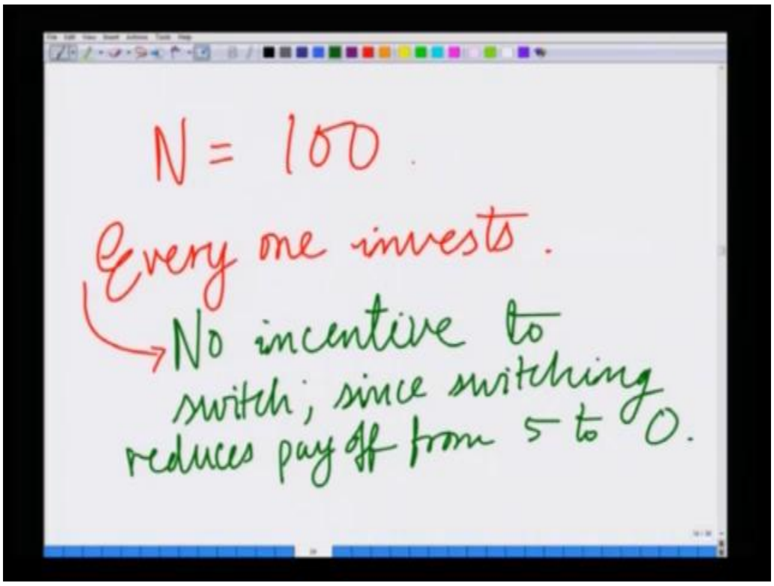

Let us take a simple example. For instance, let us look at N equal to 100. Let us take the Nash equilibrium, where everyone invests. Let us try to argue that, it is indeed everyone invests. Let us take the example, where everyone is investing; that is, all the 100 people are investing. Is it a Nash equilibria? Well, let us take or let us say I am one of the investors; I am investing 10 rupees; and, everyone is investing along with me. If everyone is investing because greater than 90 percent of the people are investing; that is, if all the 100 people are investing, 90 percent is 90 people. Since greater than 100 are investing, then I am sure to get back a return of 5 rupees. So, at the end of the period, I am going to get 10 plus 5, that is, 15. But, if I do not invest, that is, if I switch from investing to not inversing, then I am only going to end up getting 0. So, my pay off is 5; if I continue to invest 0, if I do not invest anything. Therefore, there is no incentive for anyone to switch. Therefore, no incentive to switch since switching reduces payoff from 5 to 0. If everyone is investing and I am investing… Since greater than 90 percent is investing, I am getting a payoff of 5 if I switch. Since I will not be investing anything, I get a payoff of 0.

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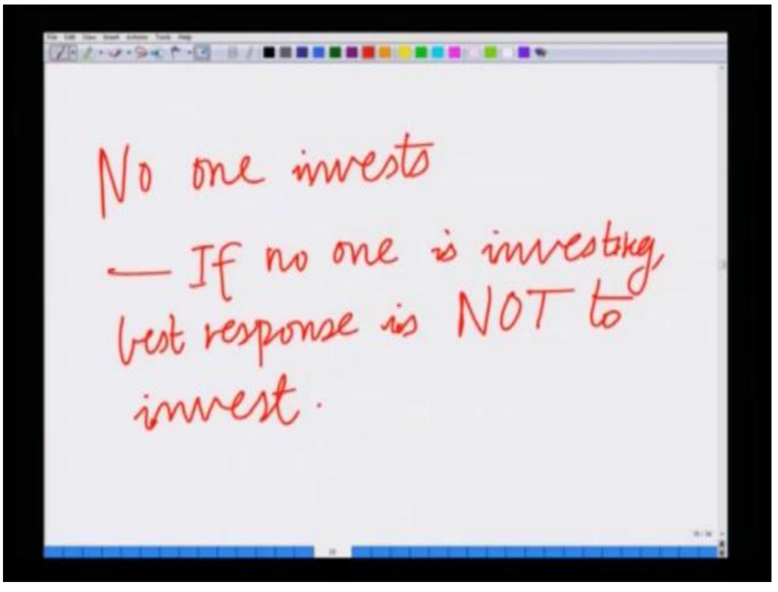

Now, consider the other scenario – other scenario, where no one is investing – no one invests. Let us argue that, this is also a Nash equilibrium. If no one is investing; so, less than 90 percent are investing; no one is investing; everyone gets a payoff of 0. If I switch and invest since no one else is investing, I will lose my investment. So, I get a payoff of a minus 10. So, it is better for me not to invest. So, if I am invest… So, if no one is investing, my best response is not to invest. If no one is investing, best response is not to invest. So, there are 2 Nash equilibria if… And therefore, even in this scenario, there is no sense to… It does not make sense or it does not… So, switching only reduces my payoff. If no one is investing, then switching to investing ((Refer Slide Time: 26:38)) reduces my payoff. So, there are 2 Nash equilibria: one is where everyone is investing; another is where no one is investing. So, there are 2 Nash equilibria. And of course, the one where everyone is investing, everyone is getting a return of 5. And, the one where no one is investing, everyone is getting a return of 0. And therefore, there are 2 Nash equilibria in which one of them is Pareto optimal; the one where everyone is investing is a Pareto optimal Nash equilibrium. Therefore, this is again similar to a coordination game.

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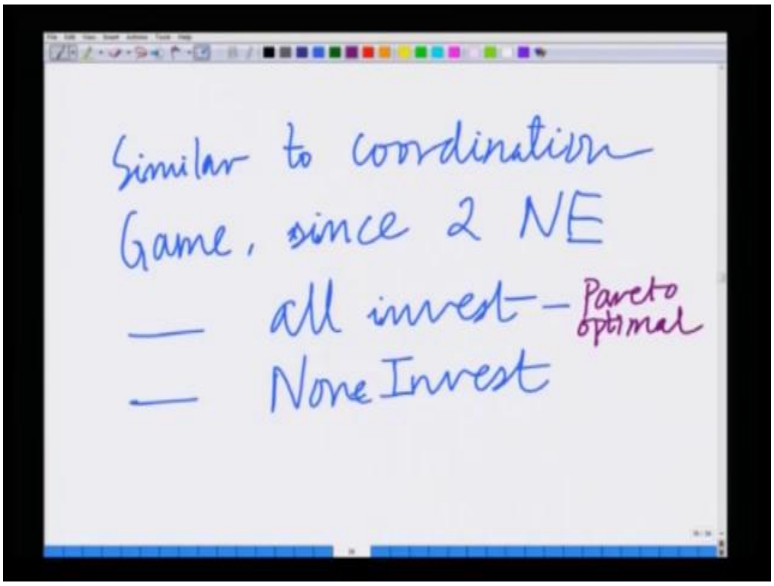

Therefore, similar to a coordination game, since 2 NE, that is, all invest and none invest. And, this all invest is a Pareto optimal Nash equilibrium. And, you can clearly argue – again argue – easily argue that, any other outcome is not a Nash equilibrium. For let us say 50 percent are investing – 50 people are investing; 50 people are not investing. If only 50 percent is investing, then since less than 90 percent people are investing, those who are investing will get a return of minus 10. So, all those who are investing have an incent to switch or unilaterally deviate from this position to not investing; thereby go from minus 10; that is, losing their money to not getting return – any return that is getting 0. So, therefore, those who are investing can switch. Similarly, if let us say 95 percent is investing; let us say 95 people are investing, 5 are not; now, since 95 percent or 95 percent or greater than 90 percent are investing, those who investing are getting a return of 5; those who are not investing are getting a return of 0. So, those who are not investing have an incent to switch to investing and basically increasing their payoff. So, you can see clearly, in a society, everyone will drift; it will drift towards one equilibrium or the other, that is, one where everyone is investing or one where everyone is not investing.

And clearly, one of the Nash equilibria, that is, everyone investing is more desirable, since it is a Pareto optimal, that is, it yields a higher payoff for all the stake holders. And therefore, it is again a coordination game, where it makes sense… where it makes… when it gives an incentive for the regulator to the engross or to make sure that, proper economic conditions prevail that, everyone is investing. Again, this example is very similar to a bank run. Again you can understand a coordination as there was a bank – works only when everyone keeps their money invested in a bank. If all of us decide to withdraw the money from the bank, simultaneously, the bank is going to collapse and everyone is going to get a 0 return or no return at all. So, there are again 2 Nash equilibrium: one is where everyone keeps their money invested in a bank and ultimately everyone gets high return; two – where no one keeps their money in a bank and everyone gets a 0 return.

And therefore, it is the interest or it is in the interest of all of us to avoid a bank run, so that the bank does not crash because that is a bad Nash equilibrium, where everyone is trying to simultaneous withdraw his money and the bank is not able to reproduce the money of all the people; and therefore, the bank collapses and people end up losing money. So, therefore, all these things – these – these examples such as the investment game, bank runs, etcetera, which are very close to again real life examples can be understood as coordination games; where, there are multiple in fact 2 Nash equilibria: one of them, which is a Pareto optimal Nash equilibrium is a desirable Nash equilibrium. And therefore, it requires coordination or a leader or the appropriate prevailing conditions – economic and so on to make people coalesce or to make people converge towards the good Nash equilibrium, which yields a higher payoff for everyone. So, that explains the background and the theory of a coordination game. Hope you appreciated this module.

Thank you very much. And, we are going to look at the other games in the next module. Thank you.