Strategy: An Introduction to Game Theory
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Lecture – 58

Hello and welcome to mooc lectures on Strategy, An Introduction to Game Theory.

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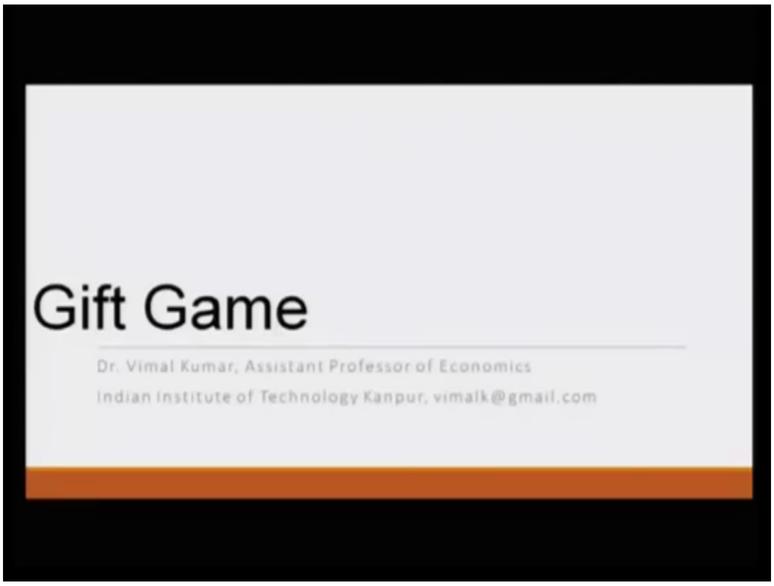

I am going to discuss one more game for which I will obtain perfect Bayesian equilibrium and this module is going to be the last module of this course. We have lot many, many more things in game theory to cover, but given the scope of the course, this is what we thought would be appropriate to cover in this course, so let us start with gift game first. What do we mean by a gift game? Let us look at a very simple game, which looks like very similar to an entry game. What do we have here? Let us say there is a player, a person. A person can decide either to give a gift to person 2 or player 2 or decide not to give any gift.

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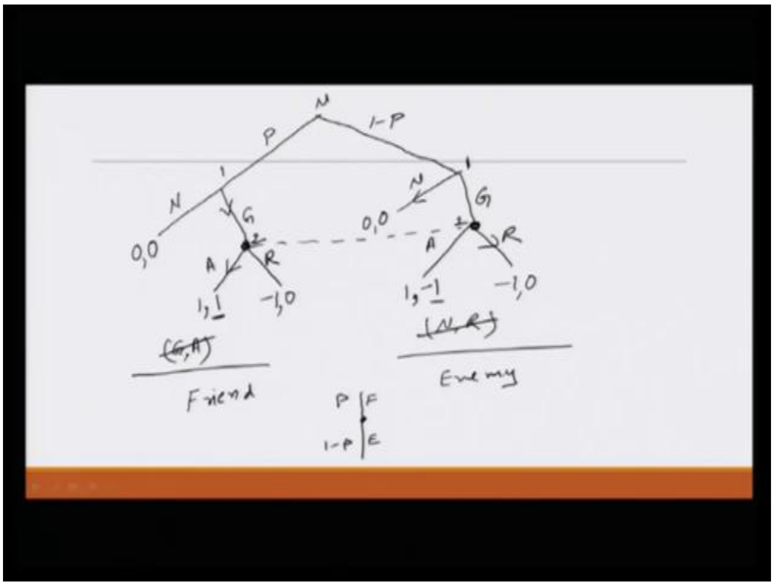

So, if he decides not to give any gift, then game ends, here say no gift and here we have gift, here is player 1 and if game ends and both the players get 0. If player 1 decides to give the gift, then player 2 can either accept it or reject it, if player 2 accepts then both of them get 1 and if player 2 rejects, then player 1 get disappointed and gets minus 1 and player 2 gets 0.

By the way, this game I have taken from a very nice book on Game Theory by Joel Watson, it is called Strategy An introduction to Game Theory very similar to the name of our course. If we obtain the equilibrium, what do we get here? Very simple, if we use backward induction, player 2 will of course, accept and given that player 2 will accept player 1 will gift. So, there is only one backward induction equilibrium or sub game perfect equilibrium that is give a gift and player 2 will accept.

But, let us say there is two different types of player 1, one is friend type that we just discussed and another is an enemy type. What happens how the enemy type is different? The game proceeds in the same manner, but the payoffs are different, here because the gift is coming from an enemy, so it is the assumption is that it is not really a gift, it is some prank. So, player 2 gets minus 1 if player 2 accepts and if player 2 rejects, then player 1 of course, waste his time giving this gift and so player 1 gets minus 1 and player 2 gets 0 as the previous case, the only difference is here.

In the first case player 2 gets 1, after accepting here player 2 gets minus 1, this game is also very simple to solve why, how, because we can use backward induction and here player 2 gets to play. So, between accepting and rejecting let us say, let us call player 2 she, she would be better or by rejecting. So, game will proceed in this direction and player 1, let us call player 1 he. So, he would be better or by not giving the gift, because giving the gift means rejection and rejection will get minus 1 and not giving the gift will get 0.

So, game will proceed in this direction and only one backward induction equilibrium and we will get that is N comma R, this is very simple. So, for very nice, very simple game, but we will add only one twist now and that twist is that player 1 knows his type, whether he is friend type or the in enemy type. Unfortunately, player 2 she does not know whether the person who has approached her with a gift is a friend type or the enemy type.

How should we model this game? We have learnt the notion of dynamic Bayesian games or extensive games with incomplete information. Here player 2 does not know the type of player 1, player 2 does not know whether this game is being played or this game is being played, so of clearly game is of incomplete information. So, we will try to transform this game into a game of imperfect information and how can we do that.

We can say that nature makes some move first and nature selects the type of the player 1. So, let us say nature moves first, here is nature moves first and with probability p nature selects that the player 1 is of friend type, here let me write friend type. Let us get rid of these two and here an enemy type. So, with probability p, nature selects the type of player 1 as friend and with remaining probability 1 minus p, nature selects the type of the player 1 is enemy.

The average assumption here is that this p is known to the player 2. Why? What is the intuition? What is the background story? Background story is simple, that player 2 has some experience and whenever someone approaches have with a gift, she has basic notion that what percentage of people are of friend type and what is the proportion of people, who are of enemy type. This p and 1 minus p, we are getting from that particular notion, so this is known to player 2.

So, let us say right in the beginning both player 1 and player 2, they do not know the type of player 1, but nature moves and natures, this move gets revealed to player 1. So, player 1 knows his type, but player 2 does not know player 1 type. So, now, we also have to connect this, because player 2 does not know whether she is moving at this particular node or she is moving at this particular node. Now, game is fairly complex, in a sense that we cannot use the typical concepts that we have used so far.

We have to use the notion that we just learnt, that is the extensive form game with incomplete information. This game can also be represented slightly differently in many book, it looks tidy to represent in that way, that nature moves first with probability p and with probability 1 minus p. Here, we have friend type; here we have enemy type, F I am using for friend type, E I am using for the enemy type.

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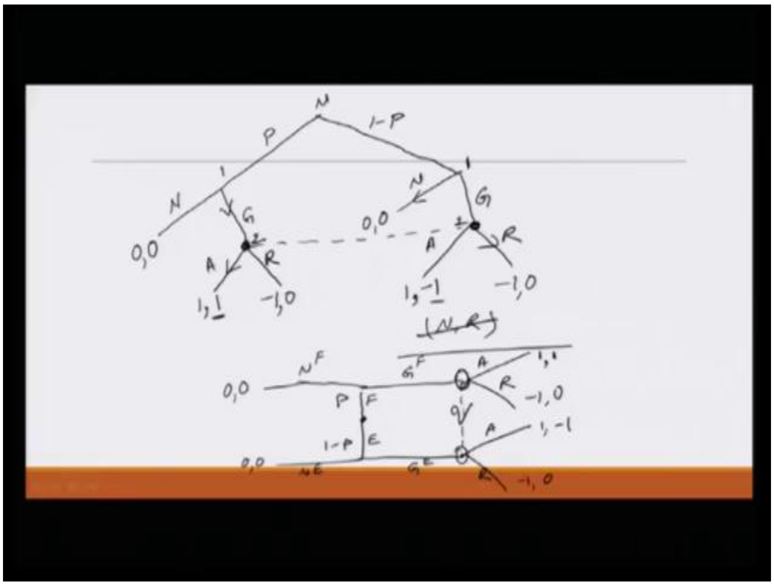

Let me erase this and friend type can take one of these two actions, give a gift. Let me say, I put a superscript F just to indicate that this gift is coming from friend type. Let me also erase it here and player and this friend type can also decide not to offer any gift, so no gift from friend type and in this case, game ends. Similarly, here enemy type no gift from the enemy type, here game ends and they get 0 and enemy type can also offer a gift, so here G E.

And of course, here player 2 can either, accept or reject, here accept or reject and this should be connected, because player 2 is not able to distinguish between this node and this node. And the payoff we already have from the earlier game, both representations are fine, I am just copying the payoffs, so this can also be used, let us try to solve this game. How did we solve in the earlier case? We started with one of the strategies of one of the players.

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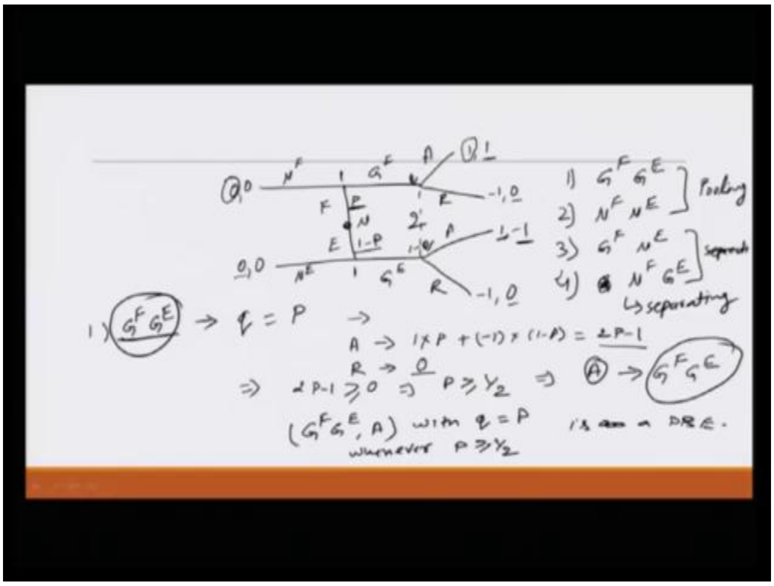

Let us say, let us start with the strategies of player 1. What are the strategies of player 1? We should have here G F and G E, so strategies of player 1 would be player 1… Player 1 is of two different types, so we will have to describe an action for each of the different types of player 1. So, possibility is G F G E that both the types decide to give a gift to player 2; second would be N F N E in this, both the types decide not to give a gift to player 2.

Third would be G F N E; in this only friend type decides to give a gift to player 2 and similarly, we will have a fourth one in which the friend type will decide not to give a gift and the enemy type will decide to give a gift. One thing I should mention it, that you know it does not sound plausible that why would enemy type give a gift and friend type would not give a gift. When we are describing the game, we do not have to think about what would actually happen in the game, we only have to describe it, so that is what we are trying to do here.

So, the fourth one is N F G E; if you pay attention to here in first to both the types are taking same action, this is also called pooling and here both the types are taking different action, it is called separating. So, we were taking about if good type will try to signal, although it is not built here, try to signal how if somehow he is the only one who decides to gift and enemy type is not able to give a gift, then it would be a credible signal to player 2, that player 2 should accept the gift, but we let us solve the game.

So, let us start with G F G E from player 1, so player 1 play G F G E and let us say player 2 beliefs that this node is rest with probability q and this node with probability 1 minus q. Here what happens, both the types are taking the same action, so this is prior known to both the player, this is prior if this is known to player 2 that up priory, player 1 is of friend type with probability p and enemy type with probability 1 minus p.

So, if both types offer gift in the equilibrium, then player 2 does not learn anything new, so in this case q is going to be equal to b. Now, let us calculate what should player 2 do and to figure out that, we have to see what would be the payoff of player 2, if she plays A or if she plays R. If she plays A, her payoff could be 1 with probability p and minus 1 with probability 1 minus p, so 1 multiplied by p plus minus 1 multiplied by 1 minus p and this is equal to 2 p minus 1.

And if player 2 decides to play reject, in this case no matter which no ((Refer Time: 12:37)), she always gets 0, so it is 0. So, now, it depends on the value of p, so let us say again then we will have to make a jump, we pick we will have to think about what is the value of p, but here we are solving for a general case. Again, the solution the equilibrium would depend on the prior belief of player 2, let us say that prior belief is that 2 p minus 1 is greater than or equal to 0.

It means p is greater than or equal to half; that means, player 2 will play A. So, player 2 will play A, then let us see what is the best response from player 1, player 2 is going to accept. So, player 1 if player 1 is of friend type, player 1 knows his type, player 1 does not have to worry about, player 1 knows his type. So, player 1 if player 1 is a friend type, he can either play, he can either decide to give a gift or decide to withheld the gift.

If he decides to give a gift, it will be accepted and his payoff would be 1 and if he decides not to give a gift, his payoff will be 0. So, friend type is better off by giving the gift, so best response from the friend type is to give a gift. How about the enemy type? Enemy type giving the gift means, because player 2 is going to accept, so it will means again of 1, the payoff would be 1 and if player 1 decides, enemy type decides not to give a gift, then payoff would be 0, so of course, best response would be to give a gift.

So, we started with G F G E and we see that G F G E is the best response. So, what do we obtained? That G F G E comma A with q is equal to p, whenever p is greater than or equal to half is a is a P P E. So, what we saw? If player 2 prior belief about the type of player 1, is that most of the time more than half of the time player 1 is of friend type, then player 2 would accept and with the belief equal to that prior probability and this would be a perfect Bayesian equilibrium. So, this is a pooling equilibrium, both types do the same thing.

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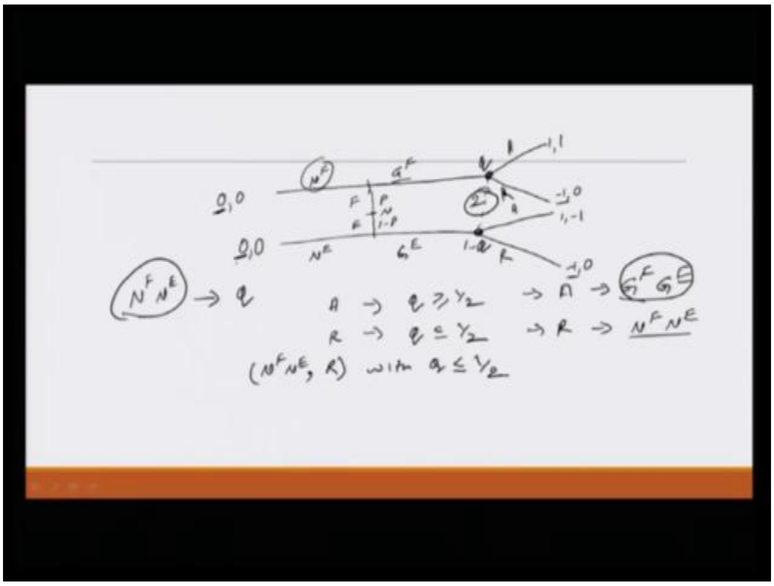

Now, let us start with the other pooling equilibrium that is a possibility that in which player 1, both the types of player 1 decides, not decide not to offer any gift. So, what we have, we start with N F N E, as we are taking about N F N E game will never reach to this information set which had these two nodes. So, player 2 can have any prior belief on q, so let us say that belief is q. We have already figured out that player 2 plays A, if q is greater than or equal to half and player 2 will play R, if q is less than or equal to half.

So, let us say that player 2 plays A, best response from player 2 is A. If player 2 plays A, then the best response from player 1 would be depend, it would depend on the type of player 1. If player 1 is of the friend type, in the previous side we figured out that friend type would decide to offer a gift and so would the enemy type. So, if player 2 is going to accept the gift, both types would offer a gift, so we will have here G F G E, here we start with N F N E and we reach to G F G E.

So, there cannot be any equilibrium of, in which player 1 is playing N F N E and player and there cannot be any equilibrium in which player 1 is playing N F N E and player 2 is playing A. How about player 2 playing R? If player 2 is playing R, again it depends on the prior belief appear 2, if player 2 beliefs that q is less than or equal to half, then player 2 will play R. So, if player 2 is playing R, what would be the best response from player 1?

If player 1 is a friend type, then not giving a gift would give him 0 and giving the gift would give him minus 1. So, the best response would be not to give a gift and if player 2 is of enemy type, again not giving a gift gives a payoff of 0 and giving a gift gives a payoff of minus 1, so best response is N E, so because 0 is greater than minus 1. So, we start with N F N E, we reach back to N F N E, so here we found an equilibrium and let me write it, it is N F N E comma R with q has to be less than or equal to half.

So, this is another pooling equilibrium that we obtained, we may have more equilibrium. Deliberately, I have selected an example which has many equilibrium, so you understand how equilibriums are obtained in case of perfect Bayesian games.

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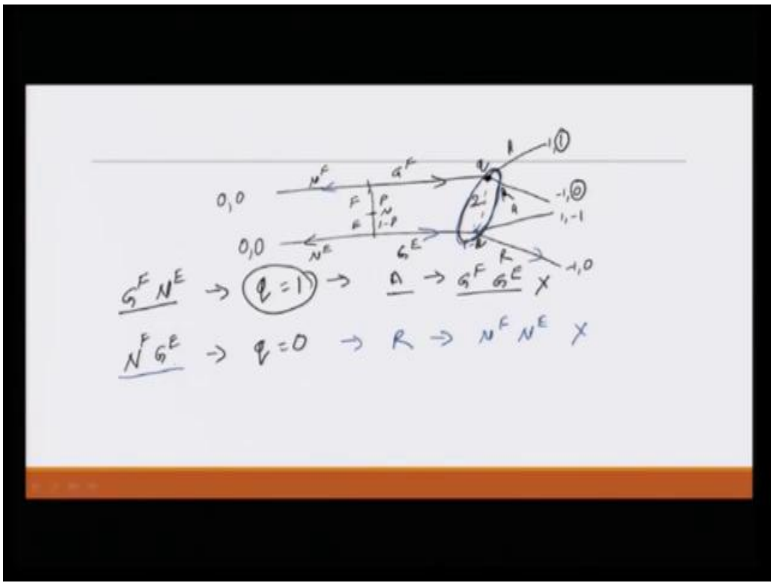

So, now we will move to the separating one. What do we have? Let us say, in which we will start first with the friend type offers a gift and enemy type does not offer a gift. In this case, again let us say that player 2 will start with the prior belief of q, but only friend type is offering the gift, player 2 will update his belief in the equilibrium. So, in this case, because friend type is moving in this direction and enemy type is moving in this direction.

So, if this information set is rest, it means this node has rest, because only friend type is offering a gift. So, q is equal to 1 and if q is equal to 1, accept is better than reject. So, player 2 will accept and if player 2 will accept, we have already figured out what is the best response from player 1 that is G F G E. So, this cannot be an equilibrium, why, because best response of G F N E is A and best response of A is G F G E.

So, it is very similar to what we were doing in case of Nash equilibrium N F N E, the only thing is that we have to take care of how the belief system that is coming here A. So, what we saw, there cannot be any perfect Bayesian equilibrium in which the friend type offers a gift and enemy type does not offer a gift. Now, let us look at the other possibility in which friend type does not offer a gift and enemy type offers a gift.

In this case q has to be equal to 0, why, because now the game is let me use the blue coloring. Game would move if it is enemy type, player 1 is of enemy type, then game would move in this direction and if player 1 is of friend type, then game would in this direction. So, clearly if this information set is rest, then this node has rest and their 1 minus q has to be equal to 1, implying q has to be equal to 0..

So, in this case player 2 will reject and if player 2 rejects, we have calculated earlier the best response from player 1 would be N F N E. So, again we do not have any Nash perfect Bayesian equilibrium, in which player 1 plays N F G E and we have covered all the possibilities. What we saw, that in this game we have pooling equilibrium, but we do not have separating equilibrium. So, before we close let me give you the guidelines, how to figure out a perfect up, all the perfect Bayesian equilibriums of any dynamic extents, dynamic incomplete information game, dynamic Bayesian games.

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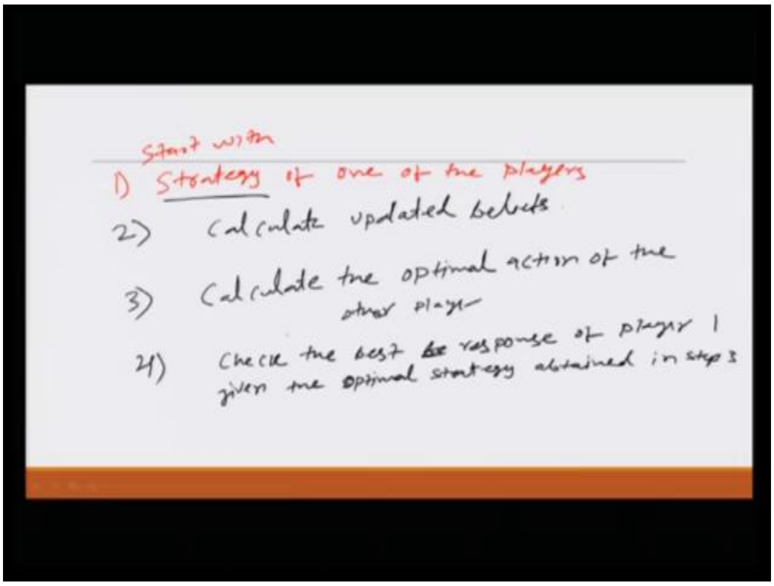

So, first step that you have seen, what I have been doing that I am starting with strategy of one of the player, strategy of one of the player, players start, this is the first step. The second step is that if possible, calculate the updated belief of the, of at all the information set which are not single done, so calculate updated belief. If possible, because we saw in some of the cases we could not calculate the updated beliefs.

In those cases what should you do, you should start with any prior beri belief and any arbitrary prior belief and keep it. Third step would be that given the updated belief or given that taken belief, calculate the optimal action of, calculate the optimal action of the other player. The fourth and the last step would be, let check whether the player 1's strategy, whose strategy we started with strategy, let us check whether player 1's strategy is a best response to player 2's strategy, if so, then you have found a perfect Bayesian equilibrium.

Check the best response of player 1 given the strategy, given the optimal response strategy obtained in step 3 and if it matches with the first one, then you have obtained the perfect Bayesian equilibrium. And this is the way; you can calculate perfect Bayesian equilibrium in any of the games, in which you have dynamic Bayesian setting.

Thank you very much.