Strategy: An Introduction to Game Theory
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Lecture - 57

Hello and welcome to mooc lectures on Strategy, An Introduction to Game Theory. In this module I am going to obtain perfect Bayesian equilibrium for the example that I had given in the last module.

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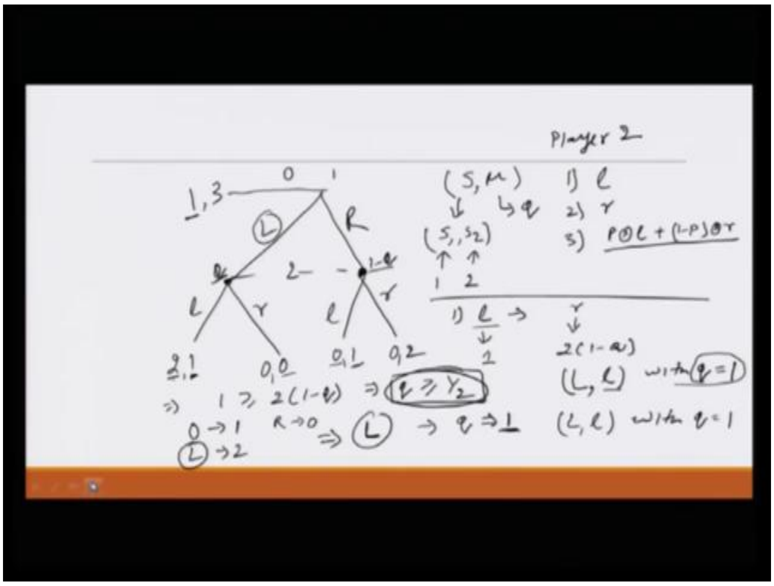

Let me just quickly draw the game, the game is we have player 1, player 1 can take one of the three actions O, L, R and then, player 2 can take one of the two actions l comma r and the payoffs are 2 comma 1, 0 comma 0, 0 comma 1 and 0 comma 2 this is the example we had in the last module.

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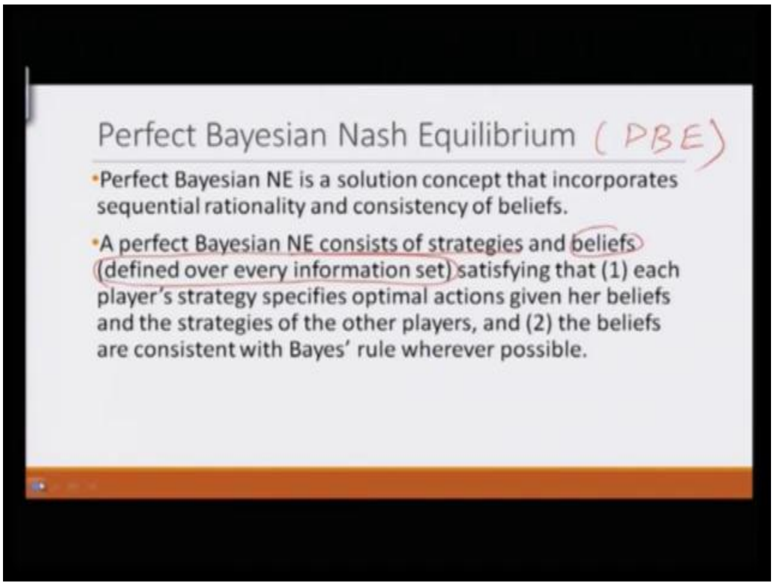

Now, we have learned the notion of perfect Bayesian Nash equilibrium, just to recap give recap what is perfect Bayesian Nash equilibrium. Perfect Bayesian Nash equilibrium is a solution concept that incorporates sequential rationality and consistency of beliefs, operationally a perfect Bayesian Nash equilibrium consists of a strategies and beliefs. Remember, we call it an assessment satisfying two conditions that each players strategies specifies optimal actions given her beliefs that the strategy of the other players and second, the beliefs are consistent with Bayes rule whenever possible.

Coming back to the example ((Refer Time: 01:48)), let us say what would be the perfect Bayesian equilibrium here, theoretically speaking perfect Bayesian equilibrium would be something S comma mu. What is this S? S is the strategy profile S 1 comma S 2, where S 1 is the strategy of the player 1 and S 2 is the strategy of player 2 and what is this mu, mu here is, because there is only one information set, which is not single term, so belief needs to describe only at that information set.

So, mu is basically, but we were calling last in the last class q, so let us say let us hypothesis that in equilibrium player 2 would play l. What are the possibilities for player 2? Player 2 can either play l or let us say or player 2 can play r or player 2 can play a combination of l and r. So, let us say l with probability p and r with probability 1 minus p, this is mixed strategy from player 2 of course and these two are pure strategy.

And let us say, that first player 2 place l, so player 2 place l, what does it mean, let us say that q is the belief that player 2 assigns. But, this node as restore player 1 has taken action L and 1 minus q is the probability that this particular node has reached R, player 1 has taken action R. So, if player 1 player 2 is playing small l, what does it mean, that his belief is such that playing l gives at least as much benefit as playing r, what it means, how you much he gets from playing l, by playing l he gets 1 multiplied by q and plus 1 multiplied by 1 minus q.

So, playing l gives 1 and playing r gives 2 multiplied by 1 minus q this we had calculated in the last module also. So, when our hypothesis is that player 2 plays l small l, then clearly what we think that it means that 1 is greater than or equal to 2 multiplied by 1 minus q, it means that q is greater than or equal to half. So, q is greater than or equal to half, this is what we obtained as the belief of player 2.

Now, let us look at the strategy of player 1 and what would be the strategy of player 1, player 1 can either play O or player 1 can take capital L or capital R or combination of one of these, some of these pure strategy or pure actions. So, let us say if player 1 plays O, then how much he will get, player 1 plays O player 1 will get 1. If, player 1 plays L, because we are already we have hypothesized that player 2 plays small l, so if player 1 plays capital L, then his payoff would be 2.

And if player 1 plays capital R, then again his payoff would be 0 as we have hypothesis that player 2 is going to play small l. So, what should be the best response from player 1? Player 1 should of course, play l, so best response from player 1 is L. Now, we have to check, what we have done, this is what we are doing in the Nash equilibrium, we were taking a strategy and seeing whether it is best response of the other players strategy or not and we are doing for all the players, but here we have to take care of the belief system also.

So, if player 1 is playing L in the equilibrium, what should be player 2's belief in the equilibrium, player 1 is always playing L in the equilibrium. So, his belief should be that player 1 this node each reached with certainty that is the only consistent. If we think that player 1 is playing L and player 2 beliefs that this node is reached and by playing this node is reached, then it would not be consistent. Why, it would not be consistent? Because, playing L will lead to this node, not in this node.

So, this means that q should be updated to 1, but this is consistent with, what we obtained as q should be greater than or equal to half, so belief is consistent, so this is a perfect Bayesian equilibrium. So, let me write and then we will check L comma l with q is equal to 1. Let us, see if player 1 is playing L, can player 2 do better by deviating, instead of playing l player 2 plays r, what would happen, his payoff decreases from 1 to 0.

So, player 2 is not better off by deviating. How about player 1? Player 2 is playing l, should player 1 deviate, player 1 should also not deviate and this is the consistent belief. So, we obtained one perfect Bayesian equilibrium, we should not stop here, we should try to find other equilibrium in this.

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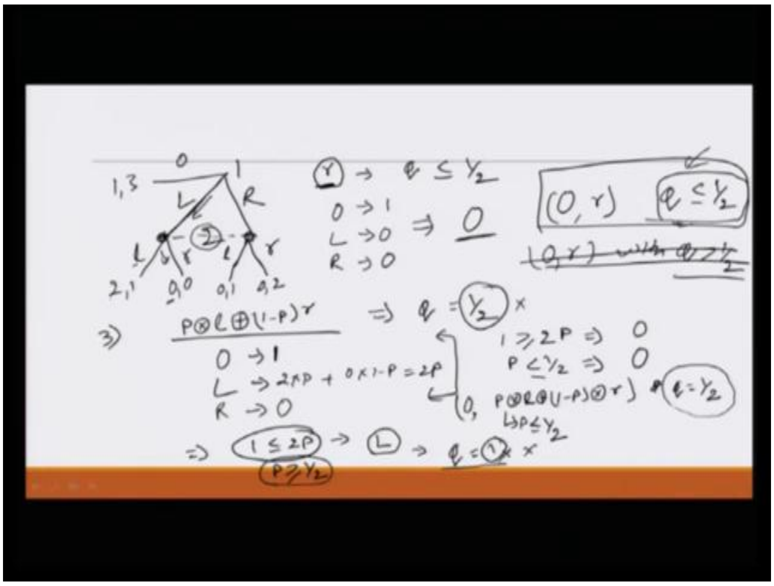

I will again quickly draw the game, so that we will continue from there. What, how did we start? We started with that player 2 plays small l. Now, let us start this time that player 2 plays small r, so hypothesis is that player 2 plays small r, if player 2 plays small r, then Q has to be less than or equal to half, Q should be less than or equal to half. What should be the best response from player 1, given that player 2 is playing r, let us see if player 1 plays O, then player 1 gets 1.

If player 1 plays L, then player 1 gets 0, because now notice in this hypothesis this time we are assuming playing 2 is playing r. So, if player 1 plays L, game will move in this direction and the payoff would be 0, if player 2 plays, if player 1 plays capital R, then again payoff is 0, so it means the best response is O. So, what do we get? We get the best response of player 1 is O and player 2 is playing r and what did we get, that Q is less than half. You might be one ring that, what happened to belief of half of other update of belief.

Here notice that player 1 plays O, so here Bayes rule cannot be applied, because player 2 does not get any more information, game never reaches to this particular information's set, which contains this node and this node. So, we cannot update it, so any belief that player 2 starts with is fine as long as it is consistent. So, if player 2 starts with this particular belief, where Q is less than or equal to half, this is a perfect Bayesian equilibrium.

We cannot say o comma r with Q greater than half is a perfect Bayesian equilibrium. Why not? If Q is greater than half, then player 2 should play l not r as we have obtained earlier, so this is not possible. Now, third possibility also we should look at that player 1, player 2 is mixing between l and r. Let us say, that mixing probabilities is that player 2 plays small l with probability p and small r with probability 1 minus p.

So, I will write it in a particular manner, this is for multiplications saying that it is not normal, multiplication I will this is what player 2 plays, so this is the mixed strategy. So, clearly in this case Q has to be equal to half, because if Q is not equal to half, then player 2 would be either better off by playing l or r. Only at Q is equal to r, player 2 gets the same payoff from playing l or playing r. So, if player 2 is mixing, then Q has to be equal to half.

Now, let us look at the payoff that player 1 gets by playing O, L and R, if player 1 plays O, then payoff remains the same in the earlier cases, it is 1. If, player 1 plays L, how much is the payoff, l happens with probability p and R happens with probability 1 minus p. So, if L happens the payoff is 2, so 2 multiplied by p, if R then payoff is 0, so 0 multiplied by 1 minus p, so it is 2 p. And if player 1 plays capital R, then no matter what player 2 does, his payoff remains equal to 0, so this is the payoff.

So, what should player 1 do? It again now depends on the value of p, so if 1 is greater than 2 p, then player 1 should play O or in other word, if p is less than or equal to half, then player 1 should play O. In this case, what is happening, let us say player 1 is playing O, player 2 is playing a mixed strategy between l and small l and small r and what is the, with probability with player will, let us indicate it that probability should be less than half.

And, what should be the Q here? Q here should be equal to half, again I did not use the Bayes rule for this simple reason, that game never reaches to this information's set, player 2 gains no you information, so player 2 has absolutely no reason to update himself. Now, we should also consider the case, what happens when 1 is less than or equal to 2 p or in other word player 1 decides to play L. Because, player 1 will play L, if and only if 2 p is greater than or equal to half or p is greater than half.

If player 1 plays L in the equilibrium, then player 2 has no information to gain and Bayesian updating will take place, because player 1 moves in this deduction. So, in that means, in this case Q has to be equal to 1, but this Q is equal to 1 and here we are getting Q is equal to half. Because, Q is equal to half is required, for player 2 to mix between small l and small r, but here if p we take greater than or equal to half, then Q is equal to 1, so this is not consistent.

So, there is no equilibrium, in which p is greater than or p is greater than half. So, that is it we got how many equilibrium three different times of equilibrium. We obtained one we obtained L comma l with Q is equal to 1 second we obtained o comma r, where Q is less than or equal to half this is true for all such beliefs not just any will as long as player 2 start with this beliefs that, if this node reach its reached with probability less than or equal to half, then we get this as an equilibrium.

But, let us say write in beginning to player 2 has this believe that this if this node is reaches this is, what we called probably is where Q is equal to 1 then this equilibrium will not sustain. And, in the third type here we are player 1 plays a mm pure strategy o and player 2 mixes between l and r the only requirement here is that he should mix in such a way that he should not put more than half weight on small l in this case the beliefs is Q equal to half. So, thank you will do in one more examples in the next module to make these things clear to you.

Thank you very much.