Strategy: An Introduction to Game Theory
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Lecture – 56

Hello and welcome to mooc lectures on Strategies, An Introduction to Game Theory. In this module, we are going to introduce a new equilibrium concept that is a Perfect Bayesian equilibrium.

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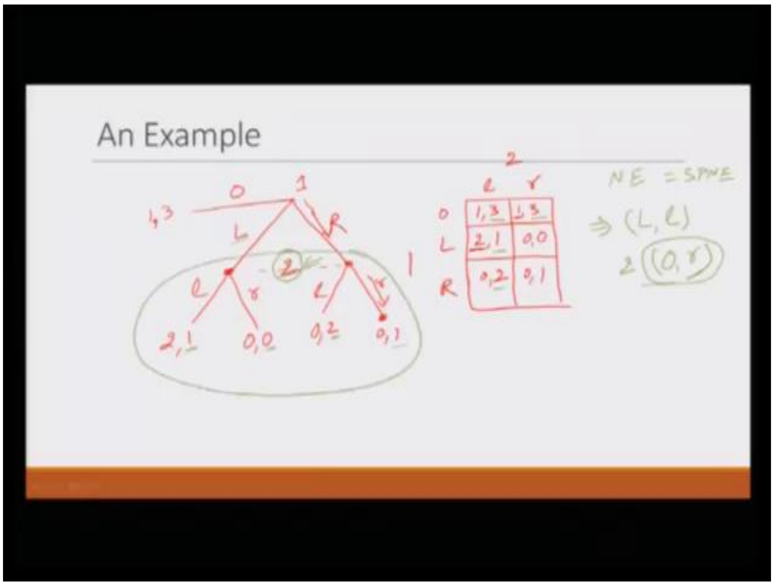

Let us start with an example, so we have a game and you can think of it as a variation of entry game, I could not strict to the payoff that I have, I had used earlier. Just I will introduce that there is a player 1, who can take one of these three actions and these three action are out, left or right. This is player 1, if player 1 takes action out, then game ends and player 1 gets 1 and player 2 gets 3. If player 1 takes action L or R, player 2 does not get does not know, which action player 1 has taken, whether it is L or R.

But, once player 2 takes one of these two action, then player 2 get to move and player 2 has again two actions, I will represent them by small l or r. Remember this is, these two nodes this node and this node, they are in the same information set, so player 2 gets to move. And the payoff here are 2 comma 1, here is 0 comma 0, here 0 comma 2, here 0 comma 1. First of all let us notice how many sub games do we have, this game has only one sub game, the whole game is sub game of itself, we do not have any other sub game.

How about games starting at this node? We cannot have a sub game starting at this node, why, because it is not a singleton, this node is not in a singleton information set, so we have only one sub game in this game. So, let us first find out the Nash equilibrium of this game of course, when we are trying to find the Nash equilibrium we have to ignore the sequential nature of the game, we will assume that players are moving simultaneously.

So, what we have here, we will convert it in to the tabular form and player 1 can take either O, L and R these three actions and player 2 can take small l or r. And here, if player 1 takes action O, then it does not matter, what player 2 is doing that payoff is always going to be 1 comma 3, 1 for player 1 and 3 for player 2 and similarly, we will get the payoff for all the boxes and we will verify for 1.

Like for example, if player 1 moves capital R and player 2 takes action small r, then what happens we reach to, we move in this particular manner and we reach to this terminal node and the payoff would be 0 comma 1, let us first obtain the Nash equilibrium. What would happen? How can we obtain the Nash equilibrium? Let us say player 1 beliefs that player 2 is going to take action l.

What would be the best response from player 1? Let us see if player 1 takes O, then payoff is 1, if player 1 takes action L, then payoff is 2 and if player 1 takes action R, then payoff is 0, 2 is definitely greater than 1 and 0. So, player 1 would take action L, so let me underline it to show that this is the best response. If player 1 beliefs that player 2 is going to take action r, with the same logic we figure out that o is the best response. So, we have obtained the best responses of player 1, given it is given his belief.

Now, let us do the same thing for player 2, let me change the color, if player 2 beliefs that player 1 is going to play o, then whether player 2 plays small l or a small r, his payoff would remain equal to 3, so both would be the best response. Similarly, if player 2 beliefs that player 1 is going to play capital L, then the best response from player 2 would be l, because playing l would give 1 and playing r would give 0. If, player 2 beliefs player 1 is going to play capital R, then l is the small l is the best response.

So, clearly we have two Nash equilibrium, here we get L comma l and o comma r. Now, let us pay little bit more attention to this game, but let us look at the sub game perfect equilibrium of this game. How can we get the sub game perfect equilibrium of this game? This game has only one sub game, so Nash equilibrium of this game would be also the sub game perfect Nash equilibrium. So, L comma l and o comma r are not only

Nash equilibrium, but they are also sub game perfect N E and SPNE both are the same.

But now, let us pay little more attention to this particular game, let us focus on this information set, where player 2 has a move in the game. When player 2 gets to move in the game, no matter where he thinks player 1 has taken action L or player 1 has taken action R, his action small l strictly dominates his action small r at this information set. How? Let us say, if player 2 believes that player 1 has taken capital L has taken action capital L playing L would give 1, playing R would give 0.

If player 2 believes that player 1 has taken action capital R, playing small l would give him 2 and playing small r would give him 1, of course 2 is greater than 1. So, no matter, which action player 1 has taken, 2 is better off by playing L. So, this equilibrium this Nash equilibrium o out comma small r is little bit fungi, because here player 1 is playing o believe that given a change player 2 will play r.

But, we look at it in this particular information set where player 2 has a move, playing r is not optimal. So, we have to think about this is, this we can think of as a failure of a S P N E to capture this think and why this is happening. Remember, when we are talking about S P N E, what we say, that the selected strategy of all player should be rational not only at the beginning of the game, but also in all sub games.

And of course, S P N E is coming from N E. What is the requirement for N E? That all players choose an optimal strategy given their belief about, what others are doing and second requirement for N E is that those beliefs are correct. S P N E equals one step further, S P N E says that players have to choose optimal strategy not only at the beginning of the game, but also in all the sub games.

But, problem here is in this particular example is that there is only one sub game, which is equal to the whole game. So, S P N E concept face to capture this idea that player 2, if given a chance would play should would never play small r is strategy.

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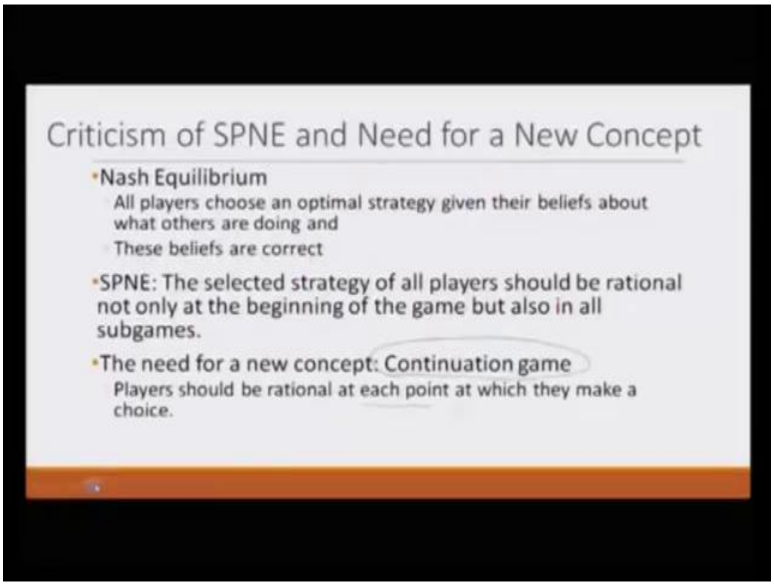

So, what do we need? We need a new concept and the new concept should capture that clear should be rational not only at the beginning of the game, not only at the beginning of all sub games, but also at each point at which they have a, they have to make a choice. And, what do you mean by at each point? I have written a term continuation game. ((Refer Time: 09:03) Let us pay attention to the earlier example that I gave, here we can think that this is, here this part is not a sub game, but we can think of it as a continuation game, that here the game is continuing in this direction.

So, player 2 has to be rational not only at the beginning of sub game, but also at in the continuation game or in other word, player 2 should be rational or not just player 2, all the players should be rational at all the information set where they have to make a decision and being rational means, that they should be making the optimal twice, when it comes with it, when it is about deciding a particular action.

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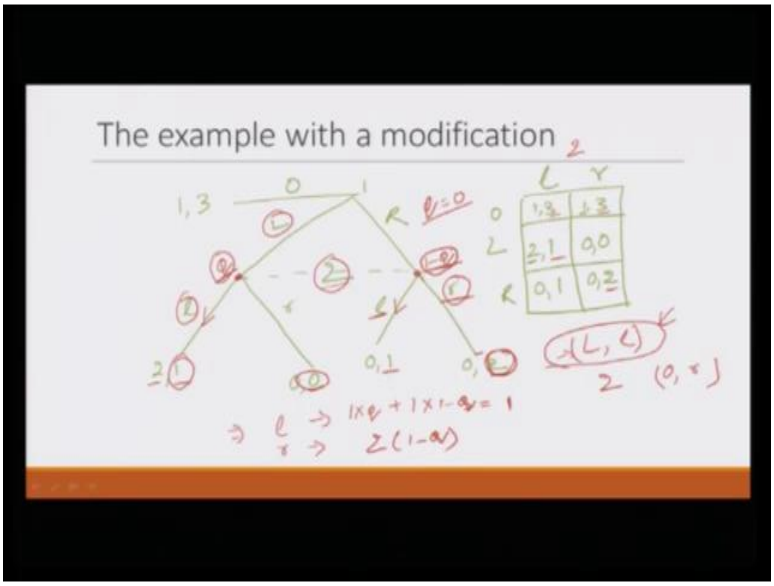

Now, let us continue with the same example with slight modification, what do we have, now let me just point out the modification. Here we have player 1, here we have player 2, again the same strategies same actions for player 1. Here payoff remains the same, here we have again the same payoffs. If you notice, what we have done we have extends the payoff in R comma l and R comma r this is the only one modification that we have made.

Let me get rid of this, what would happen, let us look at the Nash equilibrium in this case, what happens to the Nash equilibrium. Again we will repeat the same process and if we obtain the Nash equilibrium, what are the Nash equilibrium in this case, let us see again here this will be the best response. If player 1 beliefs that player 2 is going to play small l and what would be the best response from player 1, if player 1 beliefs that player 2 is going to play small r that will be to play o.

And similarly we have to think from player 2's perspective also, again let me change the color and if player 2 beliefs that player 1 is going to play out, then the best response would be both l and r. If player 2 beliefs that player 1 is going to play capital L, then the best response would be L and if player 2 beliefs that player 1 is going to play capital R, then the best response would be small r.

So, what do we get two Nash equilibrium again L comma l and o comma r same as the earlier case. But, what we have to think, now what we are saying that player 1 in this Nash equilibrium what does it suggest, that player 1 plays capital L and player 2 plays small l. So, let us say player 1 plays capital L and player 2 plays small l, payoff to player 1 is 2 and player payoff to player 2 is 1.

But, why should player 2 play small l at this point? Player 2 is not aware, whether player 1 is playing capital L or capital R. Because, playing r and if player 2 beliefs that player 1 is going to play capital R, then playing R would give 2, which is greater than 1. So, the idea is the focus here is the beliefs that player 2 should have about, what player 1 is doing. We have been talking about belief right from the beginning, when we started talking about normal form game.

We said that players form belief about, what other players are going to do. We said right there that when we talked about Nash equilibrium, that all players choose an optimal strategy given their beliefs about what others are doing and second that those beliefs are correct. But, now here that is not good enough, we have to bring that belief in the fore front. Because, given a player 2 belief say that player 1 is going to play R with full, with probability 1, then playing small r is better. There is no way to rule out that, so we have to bring that belief system in the fore front.

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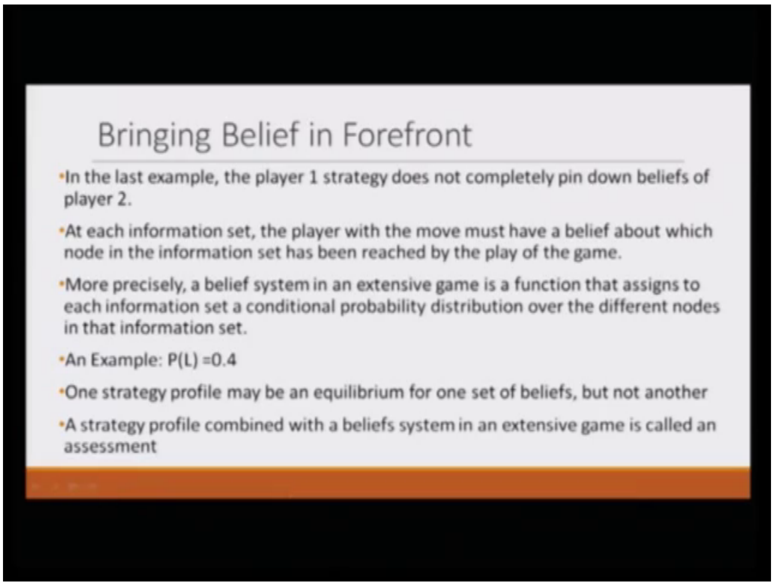

So, as I said that in the last example the player 1 strategy did not completely pin down beliefs of player 2. At each information set, the player with the move must have a belief about which node in the information set has been reached by the play of the game. As I am talking about that, what we have to say that precisely a belief system in an extensive game is a function that assigns to each information set, a conditional probability distribution over the different nodes in the information set.

So, what I am saying basically in this game, player 2 should assign a probability to these two information set, these two nodes in the information set, because these are the only two nodes present in the information set. So, we can say player 2 assigns probability small q to this particular node that this, when this information set is reached, then this node is reached with probability q and here this node is reached with probability 1 minus q and this can be anything.

We will of course, talk about that later on that it has to be, earlier we were talking about correctness of belief, here we will talk about consistency of belief. So, why it is important? That one strategy profile may be an equilibrium for one set of belief, but it may not be for some other set of belief. So, when we are talking about equilibrium we have to not, we have to talk about not only the strategy profile, but also we have to talk about the belief system consistent with that strategy profile being the optimal outcome. By the way a strategy profile combine with belief system in an extensive game is called an assessment.

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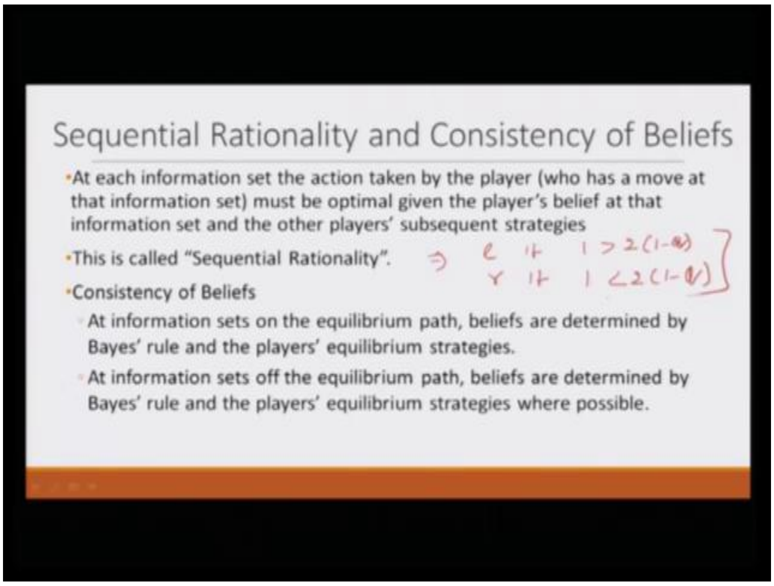

But, before we talk about the equilibrium, let us talk about two important notion, which are sequential rationality and consistency of beliefs. At each information set the action taken by the player, who has a move at that information set must be optimal given the player's belief at that the information set and the other players' subsequent strategies this is called sequential rationality. So, let me go back to the example ((Refer Time: 16:38)) and it would be clear.

Let us say, for example, if player 2 beliefs that this node is reach to its probability q and this node is reach with probability 1 minus q. So, what it means that if he plays l small l, then his playoff would be this path will be followed with probability q, if he chooses l. So, 1 multiplied by small q. And this path will be followed with probability 1 minus q and that would be in this case the payoff is 0s ((refer time17:10)) not 0 here that would be the payoff of player 1 here is again the playoff to player 2 is 1.

So, it is one multiply by one minus q and that gives us total of 1. It player 2 plays r, what would be the payoff with this particular belief system q this small q multiplies with this 0 and 1 minus q multiply by 2, so this would be equal to 2 1 minus q. So, it is very, very clear, that it we are talking about con we are talking about sequential rationality, then what does it mean player 2 should play l if 1 is greater than two multiplied by 1 minus q.

And, if player 2 should play r if 1 is less than 2 multiplied by 1 minus q this is followed from the example. So, this is a requirement that new requirement that we are talking about that player should be sequentially rational. Arrear also, we wanted players should be sequentially rational, but it was not in the forefront we were not talking in terms of the explicit mention of belief.

Now, we are we are bringing in belief system on in the forefront now after sequential rationality we are also have to talk about consistency of belief it is not like any belief system is acceptable. The belief system has to be consistent, what does it mean that at information set on the equilibrium path beliefs are determined by Bayes rule. This is the same rule that you must have learn while talking about dynamic game in static setting when you learn be any Bayes Nash equilibrium.

And, it should be consistent with players equilibrium strategy at information sets of the equilibrium path beliefs are determined by Bayes' rules and the players' equilibrium strategy wherever possible fine. So, these are the two requirements sequential rationality and consistency of belief, let us look at the consistency of belief again in the example. Let us say, in this example if equilibrium that we are talking about is capital L comma small l a belief system cannot have, where q is equal to 0, why when q is equal to 0.

Then, it means player this particular node is reached player 2 beliefs this particular nod is reached in that particular case player 2 should plays small r not small l, why because small r gives 2 and small l gives 1 2 is greater than 1. So, having belief system small q is equal to 0 with this particular equilibrium is not acceptable that violates that consistency of belief.

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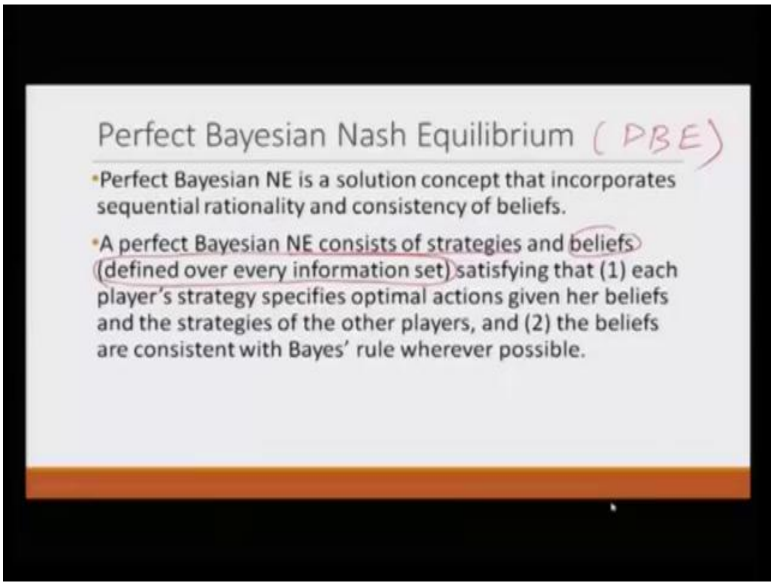

So, now we are ready to give the equilibrium definition of perfect Bayesian Nash equilibrium in solve we call it P B E perfect Bayesian Nash equilibrium is a solution concept that incorporates sequential rationality and consistency of beliefs as I had already mentioned. Now, a perfect Bayesian Nash equilibrium consists of a strategy and belief defined over, every information set. Notice in the Nash equilibrium consist of only strategies this part the beliefs defined over every information set is the new requirement.

So, a perfect Bayesian Nash equilibrium consist of a strategies and beliefs satisfy some conditions, which are that each player's strategies specify optimal action given her belief and the strategies of other players. The second requirement is the beliefs are consistent with Bayes' rule whenever possible. If, we contrast it with Nash equilibrium, what is the Nash equilibrium Nash equilibrium consist of the strategies.

In, which each player's strategy specifies optimal action given her belief and the strategies of other players and the requirement is next requirement is that her belief is correct one. So, this is the definition of perfect Bayesian Nash equilibrium in the next module. I am going to solve the example that we started with and we will solve some more example to that you understand this notion this concept in well define in good manner.

Thank you.