Strategy: An Introduction to Game Theory
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Lecture – 52

Hello and welcome to mooc lectures on Strategy, An Introduction to the Game Theory. In this module, we are going to talk about Infinitely Repeated Games.

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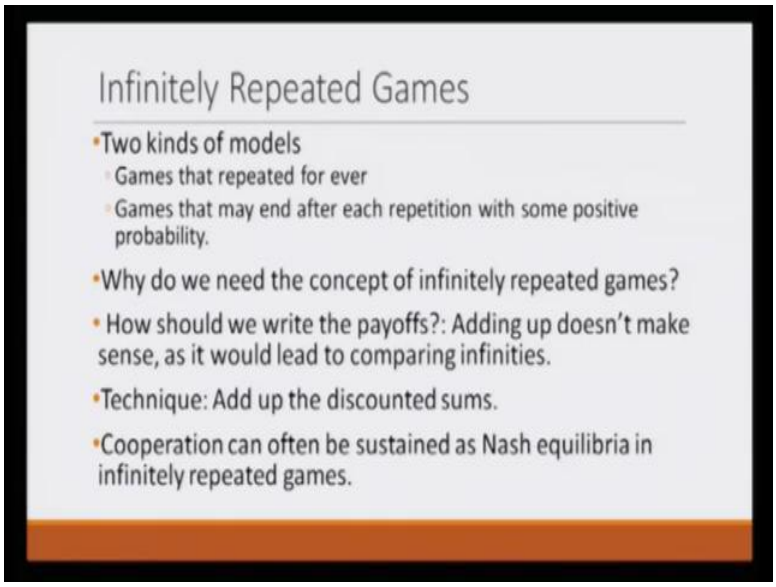

We are already familiar with the notion of repeated games, what we have learnt so far is finitely repeated games. We did prisoners dilemma, we did in which we had only one Nash equilibrium, then we talked about another simultaneous move game in which we had more than one Nash equilibrium. And also while studying chain store paradox; we learnt how to talk about finitely repeated extensive form game.

But, now we are talking about infinite repetition of a simultaneous move game. When do we need the notion of infinitely repeated game in two different cases? One that games that repeated forever, game that never ends, the players keep on playing their stage games forever. The second possibility is that game may end after each repetition with some positive probability.

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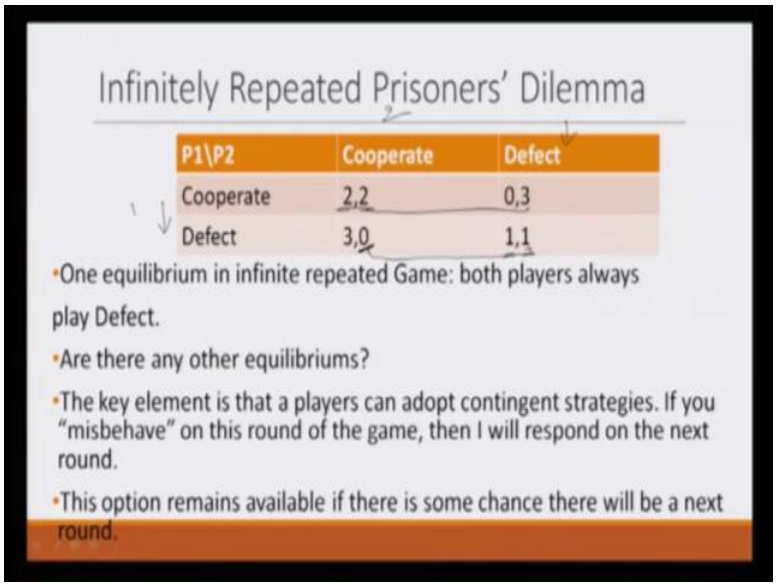

Means, there is some positive probability that game will keep on happening again and again. Now, why do we need ((Refer Time: 01:28)) the concept of infinitely repeated game? The second type that I have talked about is very useful idea that we may not know that how many times we will interact with the other player. In that case, the notion of infinitely repeated game will help us model the situation.

But, here the problem is how should we write the payoffs, because think about let us say, that two players are cooperating forever, what we have defecting forever. So, what we have seen in the prisoners dilemma; that both players are earning 1 and 1. But, let us say that if this prisoners dilemma game gets repeated forever, how should we add up. Because, no matter which payoff we add up, if we add any positive number for the infinite times, we will get infinity.

So, how can be compare infinity with infinity; that is the problem, but this is an easy one, we should be able to solve using the notion of discounted sum, I will come to this topic, I will talk about, what is discounted sum. And again the question is very similar that we are trying to answer in the case of finitely repeated game, can we get any cooperation just because we are having this game repeated infinitely.

We know the defection would sustain on it is own in finite number of repetition as well as in infinite number of repetition, it would sustain at it is own. Because, in all the stages it would lead to Nash equilibrium and we have learnt that Nash equilibrium is self

enforceable. So, again coming back to the prisoners' dilemma game, this prisoners dilemma game, we saw when we studied finitely repeated game also.

So, one outcome is definitely that we will always get defect and defect, but is there any other equilibrium that we can get. The key element is that a player can adopt contingent strategies. What do we mean by contingent strategy? Based on in by contingent strategy, based on some observation contingent upon based on some observation. So, if you misbehave on this round of the game, then I will respond to you in the next round. So, if you cooperate with me, I will cooperate with you.

If you defect, I will also defect and this sort of contingent strategy remains available, if there is some chance that there will be a next round. We saw this did not work in the case of prisoners dilemma, when we were talking about finitely repeated game, because after some point game ends and then, this logic does not work out. And then, when we start doing the backward induction this logic would follow back to the first stage. But, we have some hope in infinitely repeated game, as in this setting the game never ends, it keeps on repeating.

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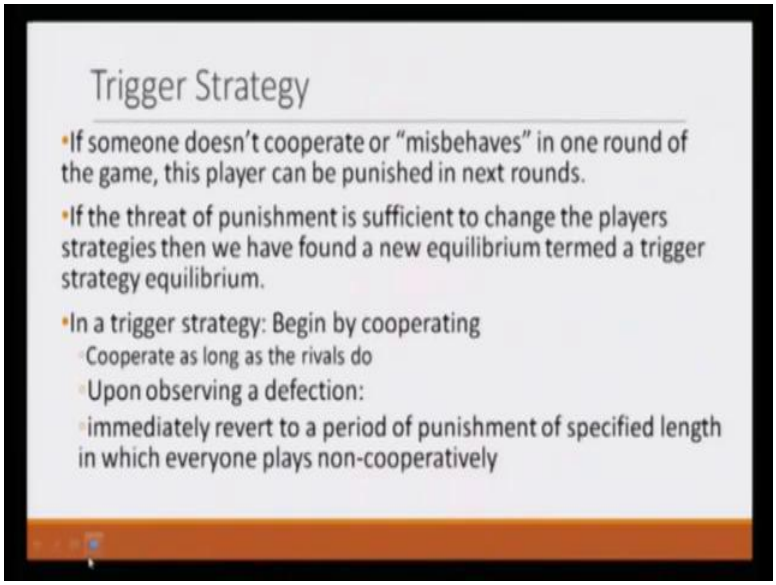

So, let us talk about some of the contingent strategies, one very popular one that we will study today is called trigger strategy. So, if someone does not cooperate or misbehave, let us say we make a deal let us cooperate and if someone does not cooperate or misbehave in one round of the game, then this player can be punished in the next round.

So, if the third of punishment is sufficient to change the players strategy, then we have found a new equilibrium and we can call them trigger strategy equilibrium.

Here, remember unlike finitely repeated game, there is no end of game, so we cannot begin at the end and follow back, the same logic and come to the first stage. So, in a trigger strategy, what should player do, player should begin by cooperating, this would cooperate with each other. So, in the first stage both of them should play cooperate, so both of them would get 2 comma 2.

Notice, what happens if it is finitely repeated game, then they always defect and get 1 comma 1. So, in trigger strategy, they both begin by cooperating. Now, the next thing is cooperate as long as the rivals do, all the rivals do. And upon observing at defection, let us take keep on cooperating as long as the rivals are also cooperating. And when you see a defection from the next period onward, revert to a period of punishment of a specified length in which you play defect. So, this is specified length can be forever or can be for five period or can be for one period.

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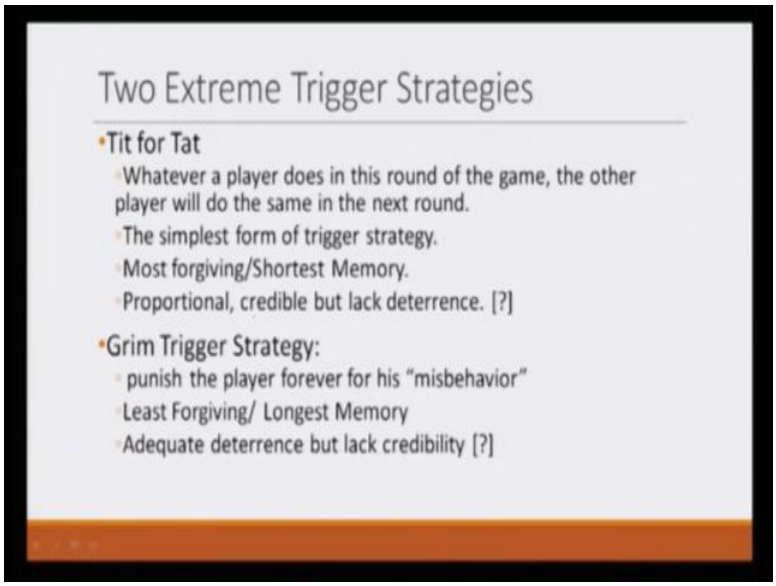

So, let us talk about two extreme cases of trigger strategies, one is called tit for tat; that we talked about right in the beginning, when we started talking about repeated games. So, whatever a player does in this round of the game, the other player will do the same in the next round. So, you are just mimicking your opponent, this is the simplest form of the trigger strategy and this is most for giving or in other word, we can say shortest memory.

Why, because punishment if someone defects for one period, he gets punished only for one period and immediately, they can go back to the cooperation. It is proportional and it is credible, but it lack deterrence, I will emphasize this term credible, but lack deterrence little later. As oppose to tit for tat, we have other on the other extreme, we have grim trigger strategy, grim trigger strategy extreme says that punishment period is forever.

If a player misbehaves, if a player defects, then you punish that player forever, this is least forgiving and it has a longest memory, means one someone defects someone misbehaves with you, you keep on punishing that person forever. But, remember in the prisoners' dilemma game, punishing other player also means punishing yourself. So, this is not very credible, because the player would be not only punishing the other player, but also himself. But, this has adequate deterrence in a sense, that it would better other player from cheating.

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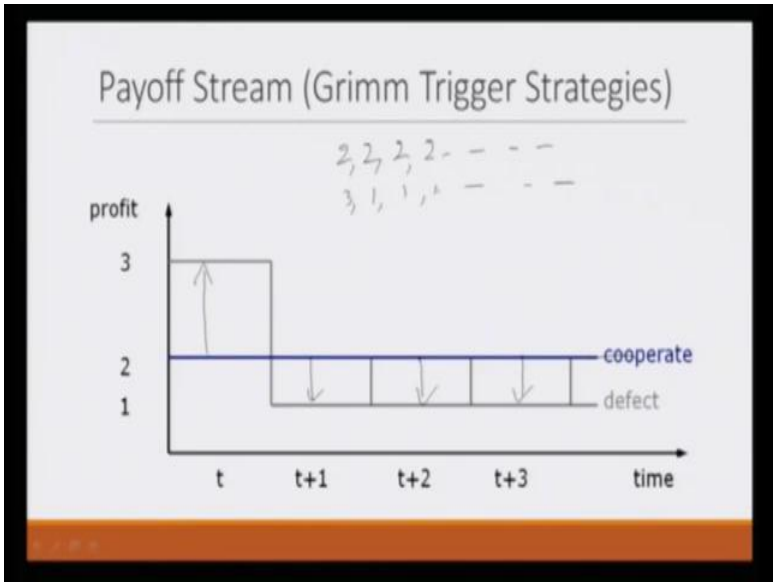

So, this is the payoff stream in grim trigger strategy, what happens they cooperate and cooperate in the first period and then, keep on cooperating. So, when they are cooperating, what is happening, let us look the payoff, ((Refer Time: 08:30)) they both are getting 2 comma 2. But, let us say in a particular period a player defects, since other player is cooperating, let us say player 1 keeps on cooperating and player 2 defects, then player 2 gets 2, 3 and player 1 gets 0, but this is grim trigger.

So, player 1 would now punish him forever, so now, he is going to play defects forever. So, in that case what would happen, the player would always earn 1, 1 because the other player is also going to defect, other player cannot cooperate now, this is known to him, if he cooperates he gets 0. So, he would like to have as high pay as possible, so he would also defect.

((Refer Time: 01:28)) So, now, if one defects in one stage, payoff would increase from 2 to 3, but in all remaining stage, it would decrease from 2 to 1. So, that is what I am showing here that let us say in period t, it goes from 2 to 3, but in all remaining period, it decreases to 1. Now, the idea is, if we see this is the payoff continuous stream of 2 and here let us say which one is larger, how should we determine that this stream has higher value or the stream of 3 and then, 1 comma 1 has the higher value.

How should we calculate? For that, we are going to use a notion of discount, what happens that a money that you have in your pocket is more available to you, that money which is promise to you a year later. Why, because several reasons, one reason could be that you can put this money in the bank and you can earn interest. So, someone tells you that, I would give you 100 rupees 1 year from now, so for that 100 rupees would be equivalent to less than 100 rupees today.

Why, because that value let us say if it is 95 rupees, you put that 95 rupees and in a year time, bank will give you 100. So, you earned interest that can be one reason, other reason could be that you do not know, whether would survive, you would be alive 1 year later or not. Third, the human psychology is the things that you like to con consume now would be more available. So, this is several reasons for having that discount factor.

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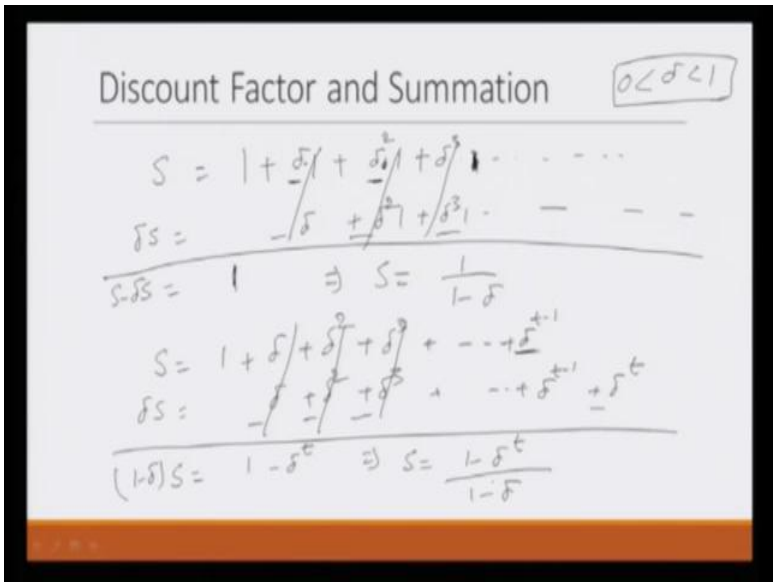

So, let us say a person is earning, there is a summation that we have 1 delta, because let us say 1 is earning in this period, 1 is the earning in the next period, but discounted by delta, it would be delta multiplied by 1. And then, for next to next period it will be delta is square 1, because here is the discounting for one period. Here, is discounting for two period and similarly, delta q multiplied by 1 and so on and the idea is to summit up.

Let us indicate that this is S, this summation is called S. So, if we multiplied this S with delta, what do we get, if we multiplied 1 with delta, we get delta and we will shift 1 plus you will write delta. This delta point delta multiplied 1 multiplied by delta, it is going to be delta square 1 and delta square one multiplied by delta would be delta cube 1 and so on. So, what we can do we can subtract this delta S from S.

Notice, the delta is typically less than 1 and greater than 0; that is what we give the value of discount factor. So, what do we get S minus delta S is basically here, we if do the subtraction all these things will get cancel and what we will get 1. So, S is basically 1 divided by 1 minus delta, but let us say, we have only t periods, then what happen, then here we have 1 plus delta, delta square, delta cube and then, we have delta t minus 1.

In all, you multiply anything by 1, you get the same number and then, again you do the same thing, you take delta S, what you would get, here delta plus delta square plus delta cube with the same logic. Here, you will get delta t minus 1 and when you multiply delta

to the power t minus 1 get delta, you will get delta to the power t and take the subtraction, you will get 1 minus delta multiplied by S.

Here, all these things will get cancel, what would survive, just 1 here and minus delta to the power t here. So, in this case summation would be equal to 1 to minus delta to the power t divided by 1 minus delta. So, now, we can do the calculus for grim trigger strategy.

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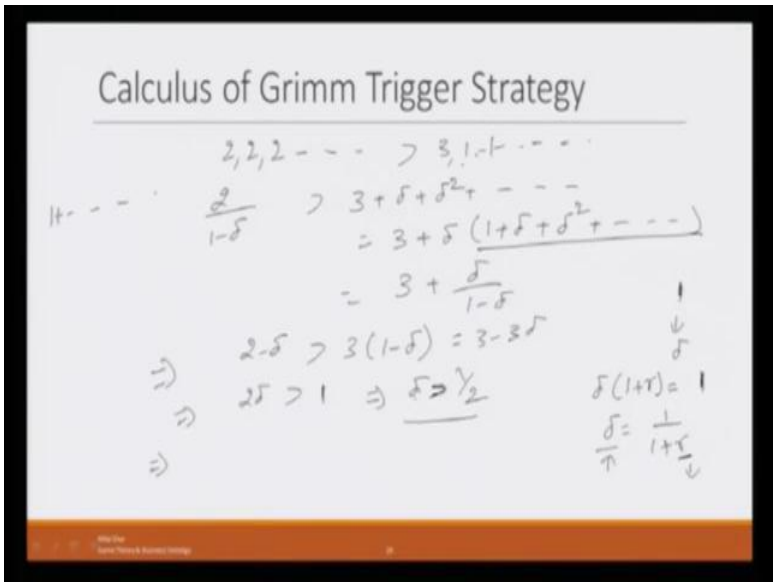

So, person would not like to deviate, if 2 comma 2 comma 2, the stream of the payment of 2 is greater than the stream of payment of 3 comma 1 and then 1 in all the period. And if we take the discounted sum, what do we get, 2 here divided by 1 minus delta y, because if this sum 1, the stream of 1 for infinite period, we get 1 divided by 1 minus delta, this is what we have obtained here.

So, simply we have we can multiply to on all side and here, what we can do, 3 in the first period and then, what we have here, 1 from the second period to the infinite number of period. So, what we can say this is delta, delta square and so on, which is equal to 3, if it take delta common, what do we get 1 plus delta, delta square and so on, this we have already calculated. So, what do we get 3 plus delta divided by 1 minus delta.

So, clear grim trigger strategy works, if 2 divide by 1 minus delta is greater than the entity on the last side. So, if we solve it, what do we get 2 minus delta has to be greater

than 3 multiplied by 1 minus delta. Grimm delta on the left hand side and numbers on the right hand side, what do we get, 2 delta has to be greater than 1 and so delta has to be greater than half, let us check, this is 3 minus 3 delta, yes.

So, bring 2 on the right hand side, so 3 minus 2 is 1 and 3 delta minus delta is half. Notice, this is also what we are saying that, 1 rupees tomorrow and 1 rupee in the next state has the value of only delta. In other word, if I keep delta in the bank then, I earn some interest and let us say, it is a simple interest. So, delta multiplied by 1 plus r should give 1.

So, it means, we can established a relation between discount factor and interested delta is equal to 1 divided by 1 plus r. So, higher the delta lower would be the interest rate. So, grim trigger strategy sustain on it own, when delta is at least as large as half, higher than half would sustain on it own.

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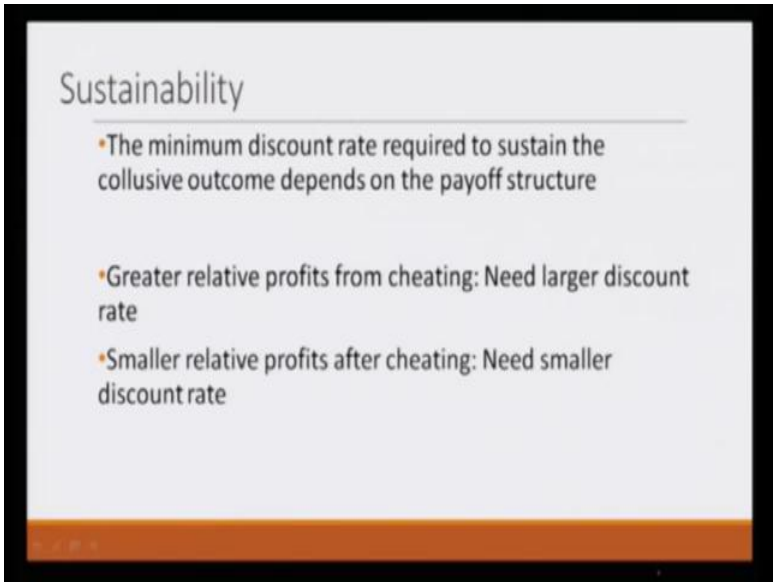

The thing is the minimum discount rate require to sustain the collusive outcome depends on the payoff of structure, what is the payoff of structure 2 comma 2, 3 comma 0, 0 comma 3 and 1 comma 1 that we obtain. It depends the value of delta depends on the difference the game that one can make from cheating. So, greater relating profit from cheating, it needs larger discount rate and smaller relating profit after cheating need smaller discount rate. So, in this particular case, we have seen that cooperation would

sustain, players would cooperates forever under the thread of grim trigger strategy, if delta is greater than half.

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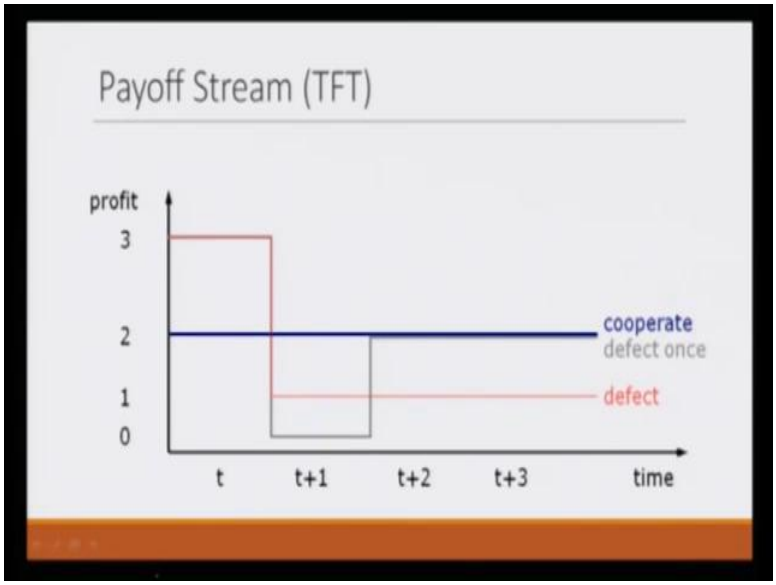

Now, let us look at the tit for tat, what happens in the tit for tat, a player cheats, player gets 3 in this period, but in the next period, other player would mimic. The defect in that period he would get 1 and again, they will be back to 2, because your punishment is only for one period.

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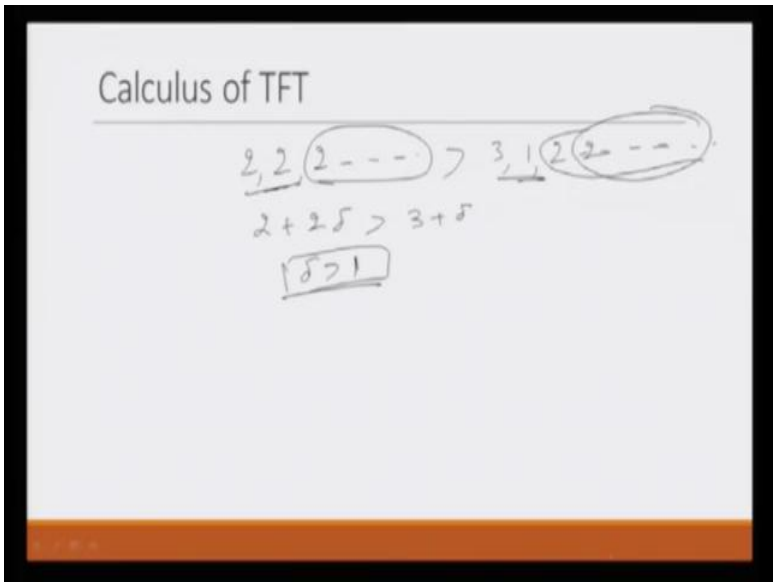

So, here cooperation would sustain, if the stream of 2 comma 2 is greater than the stream of 3 comma 1 and then, 2 comma 2 forever. So, what do we need to say that 2 comma 2 delta, because we can be ignore this is stream is same as this is stream here. So, what we need to compare this 2 with these 2. So, 2 plus 2 delta should be greater than 3 plus delta and what do we get here, delta has to be greater than 1.

So, this is not possible, it is because of payoff, typically what happens, it is difficult to sustain cooperation and that tit for tat, there is a very small chance, it depending on the payoff in this particular case, because of the payoff structure, tit for tat would not sustain cooperation ever. Players would like to cheat all the time, but we saw that even the these payoff, grimm trigger strategy would sustain cooperation.

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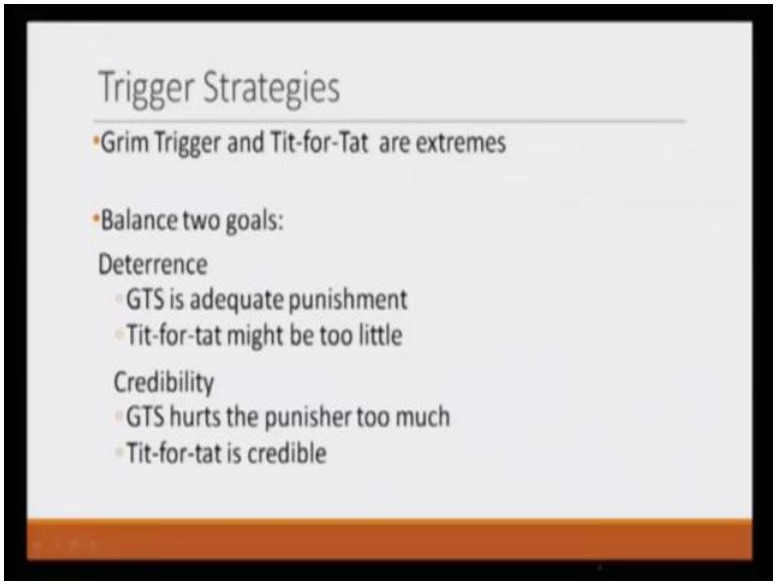

So, what we have learn that there are 2 goals, what happens in grim trigger strategy, when you punish your opponent, you are punishing yourself also forever. So, that is little less credible that to punish other you would punish yourself. Tit for tat is more credible, because you are saying I would punish you for only one period, but in grim trigger is strategy, it would deter other strong deviating, it would deter others from cheating.

But, in tit for tat, it does not have enough deterrence. So, basically the idea is to balance these 2 goals, deterrence versus credibility. But, we have seen that there exists possibility in infinitely repeated game that cooperation would sustain forever.

Thank you.