Strategy: An Introduction to Game Theory
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Lecture – 50

Hello, welcome to mooc lecture on Strategy, An Introduction to Game Theory. We are talking about repeated games and what you have seen that, even if prisoners dilemma game is repeated many times players do not achieve any cooperation. Not forever, not even in the earlier stage or later stage, ever they do not achieve any cooperation. So, we are going to see another class of game, where some cooperation is possible.

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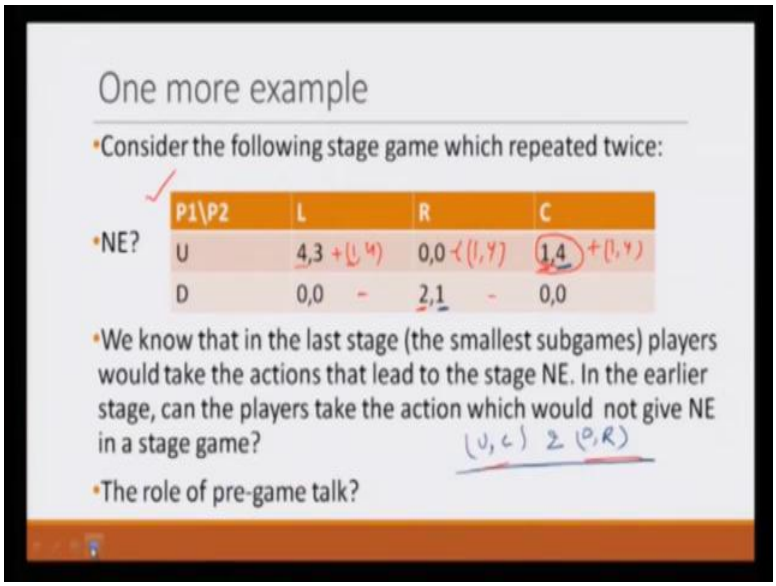

In this sort of game, we will have in all the stage games we will have more than one Nash equilibrium, so let us consider this particular example. We have two players, player 1 and player 2, player 1 has two actions upward and up and down and player 2 has three actions left, right and center. Now, let us find the Nash equilibrium of this game. What do we get as Nash equilibrium of this game? First, let us say player 1 thinks that player 2 is going to play L.

What is the best response of player 1? We will mark it by red bid U, if player 1 thinks player 2 is going to play R, the best response from player 1 is D as 2 is greater than 0. If player 1 thinks that player 2 is going to, player 1 believes that player 2 is going to play C, then the best response from player 1 is to play U. And now, I will mark the best response

of player 2 in blue color, let us talk about player 2's believe. Player 2 thinks that player 1 is going to play U, then the best response from player 2 is to play C.

And if player 2 thinks that player 1 is going to play D, then the best response for player 2 is to play R, L is never the best response for player 2. What do we see here? That we have two Nash equilibrium and these two are U comma C and D comma R, these are the two Nash equilibrium in pure strategies. Now, let us say that this game gets repeated twice that player 1 and player 2 play this game, once they observe the payoff and then they play again, what would happen. Let us see what happens in the game theory.

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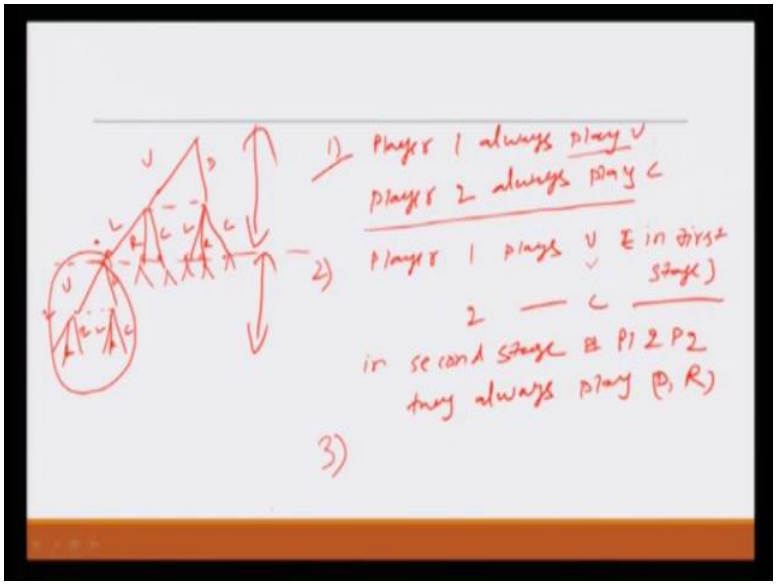

What we have here that player 1 moves first and player 1 can take one of these two action, U or D and then player 2 moves, player 2 does not know which action player 1 has taken. So, left, right and center, left, right and center and then after that again player 1 moves U or D and then, player 2 moves again without knowing the action of player 1 left, right or center left, right or center. The thing is, we will again have to draw here, here, here and for all the possibilities.

So, we have to again start using the backward induction to solve the game, in any last stage players are playing basically this game ((Refer Time: 03:45)) that we have given in the table. And in the last stage players would always play either U comma C or D comma R, any of these two are we have given in the table and in the last stage, players would always play either U comma C or D comma R, any of these two are possible.

So, let us say here they are playing U comma C and now we are coming back to here, in the original game here we write the payoff whatever we get the payoff here, we add it up like we did earlier in the case of prisoners dilemma game. So, now there are several possibilities, one possibility is that player 1 always play U and player 2 always play C. What happens in the second stage? They are always playing U and C, so basically when you came to the first stage ((Refer Time: 04:57)) U comma C, you are getting 1 comma 4.

So, when you are using the backward induction you are solving this game again in the first stage, for all the stages you will add 1 comma 4, 1 comma 4, 1 comma 4. Why you will add this? Because, this is what players would be getting in the second stage. Remember, the game is of common knowledge, players are rational, they know the whole the structure of the game and they account for all these things.

So, when we add this 1 comma 4 in all the boxes then 1 comma 4 would not impact anything and again the stage Nash equilibrium would be U comma C or D comma R. So, one possibilities that player 1 always play U and player 2 always play C, here I am saying that player 1 always play U in the first stage as well as in the second stage, second possibility. I am not going to list all the possibility, but some I just want to identify that player 1 plays U in the first stage and player 2 plays C in the first stage and in second stage I want this to be different then the first one.

So, in the second stage because we have to think about, what is the other Nash equilibrium D comma R. In second stage, player 1 let us say p 1 and p 2 they always play D comma R. So, again what I am saying in the second stage that starting after this point, here you have first stage, here you have second stage in the second stage they always play D comma R. So, they both get 2 comma 1 and in the first stage in here, here at all these six location we will have to add 2 comma 1.

And when we are adding the same number, they can either play U comma C or D comma R, here I am hypothesizing they play U comma C. Similar to one we can have third that they always play D comma R, but these all things are not very interesting in the sense that players are playing a strategy which would give them stage Nash equilibrium, that Nash equilibrium that they get in the stage game.

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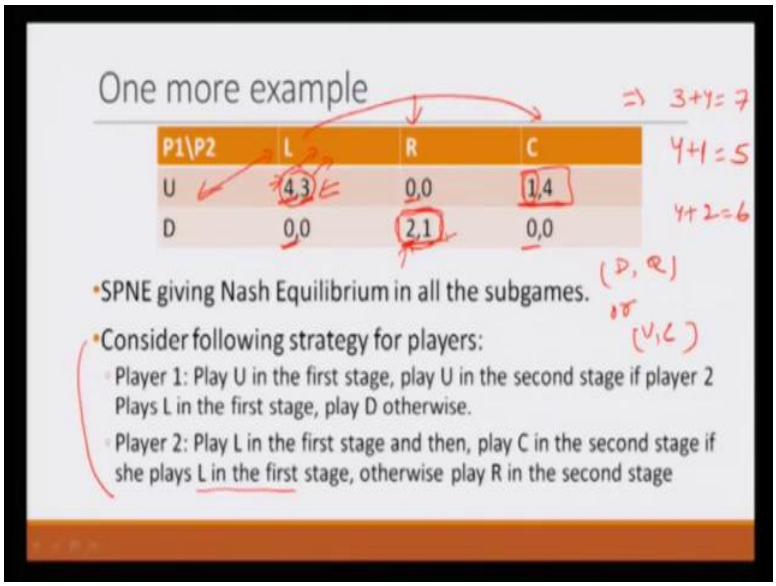

The key term is can we get any sort of cooperation, because notice if you look at L, U comma L, U comma L gives 4 to player 1 and 3 to player 2, where here 2 comma 1 they both are towards off. So, can there be something, can there be a mechanism, can there be a plan which would make them play U comma L in the first stage. Notice that ((Refer Time: 08:13)), U comma L is not one of the Nash equilibrium, there are only two Nash equilibrium U comma C or D comma R.

So, let us consider one possibility that player 1 play U in the first stage and play U in the second stage, if player 2 play L in the first stage. So, notice what I am saying ((Refer Time: 08:36)), because what would happen, here you will have sub games starting for player 1 and player 2 they will be playing. So, what I am saying, if player 1 will play U in the beginning and if player 2 is playing L in the beginning he will again play U.

But, notice if here a sub game player 1 plays U, but here player 2 is playing R, so a sub game starting after U comma R, player 2 would not play U. So, this is what I say that player 1 plays U in the first stage and play U in the second stage, only if player 2 play L in the first stage or otherwise if player 2 does not play L in the first stage, then play D otherwise. Similarly, let us look at the player 2. What is there for player 2? That player 2 play L in the first stage as promised and then, play C in the second stage, if C plays L in the first stage; otherwise, play R in the second stage.

It may sound funny that because player 2 is thinking about himself, when player 1 is playing L in the first stage, then why we are basing his second stage move on his first stage move, he knows what he is going to play. But, this we had already discussed in the extensive form game when we were talking about strategy. Basically, what they are saying in this, forget about this description that may sound really complicated sometime, what basically player 1 let us say this is what player 1 is saying to player 2.

That player 2 let us cooperate on U comma L, I know this is not Nash equilibrium. So, this is if it was one stage game, then it would not happen in forcible, why because player 2 would like to deviate from L to C. But, here this is the proposition from player 1, player 1 is asking to player 2 that play L in the first stage and I am going to play U this will benefit me, this will give me 4. If you do that in the next period, we are going to coordinate on U comma C which is a Nash equilibrium, which is self in forcible we are going to coordinate on U comma C.

But, if you do not help me in the first period, then I am going to punish you and of course, I will punish you by playing D and then, you will be forced to play R and we both will be worse off in the second stage. At least you will be worse off, because not player 1 will be worse off player 1 will be worse off in U comma C, so this is the plane that player 1 is happen. So, why this plane may work let us see in the second stage players would play the strategy, the action that would give them Nash stage equilibrium.

So, they can coordinate either on D comma R or U comma C in the second stage in all these are the only two possibilities, because only these two are self inferable. So, in second stage there is no problem, let us say in the first stage one can think that player may have incentive to deviate. So, let say player 2 deviates player 2 does not follow this plan, what happens let us say player 2 follows the plane how much player 2 earns 3 in the first stage and 4 in the second stage.

So, total earning is 3 comma 4 and that is 7, let us say player 2 cheat player 2 does not follow this plane and player 2 cheat play when player 2 cheats what would he do, he would play C instead of L and by playing C player 2 would get 4, but because this will be observed by player 1 at the end of first period. So, player 1 is going to play D to punish player 2 and when player 1 is going to play D then they will reach to this outcome and in which player 2 would make 1 and that is 5.

So, player 2 does not have any incentive to deviate, similarly can we thing of whether player 1 has an incentive to deviate or not, let us say if according to this plan player 1 makes 4 here in the first stage and 2 in the second stage, so 4 plus 2 player 1 makes 6. What if we deviate in the second stage? There is no point of deviating, because as player 2 is playing R, if he deviates you would get 0. So, no point of deviating, so second stage we cannot deviate, how about first stage if in first stage if he deviates he gets less than 4 that is 0 or 1.

So, he has no incentive to deviate, so here we get a Nash equilibrium which is sub game perfect and in which in the earlier stage players do not play the action which would lead to the stage Nash equilibrium. But, it helps them the reputation helps them sustain some cooperation in the form of that in first period they in the playing U comma L.

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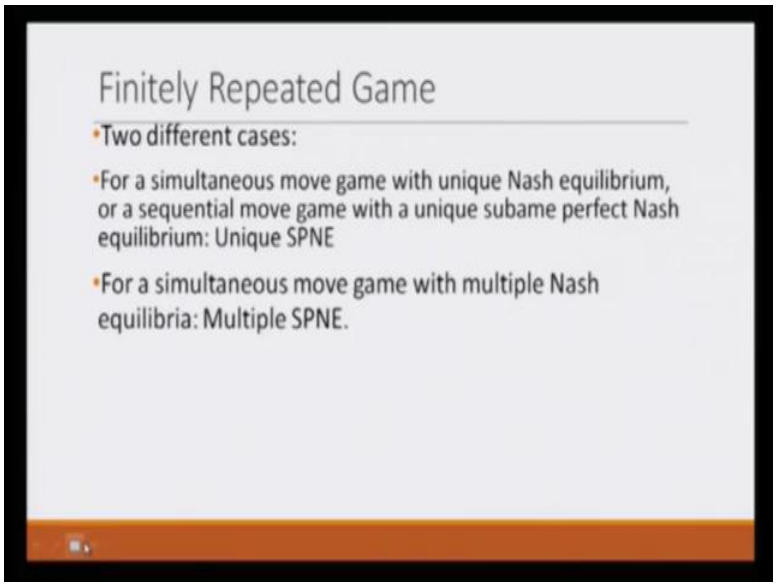

So, let us conclude this finitely repeated game we discussed two different cases for a simultaneous move game with a unique Nash equilibrium or a sequential move game with the unique sub game perfect Nash equilibrium, what do we get when we repeat such games we get unique sub game perfect Nash equilibrium, there is only one like when we repeat prisoners dilemma we all players always defect. Why? Because, prisoners dilemma as unique Nash equilibrium.

What for a simultaneous move game with multiple Nash equilibrium? We may get multiple sub game perfect Nash equilibrium, like the example we just of course, one

possibilities that they clay the actions that will lead to stage Nash equilibrium. But, reputation may also sustain preparation in the sense that there exists some sub game perfect Nash equilibrium in which players take the action which would not lead to stage Nash equilibrium in earlier stages, like we had in the first stage.

Thank you.