Strategy An Introduction to Game Theory
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Lecture - 05

Hello. Welcome to another module on this online course – strategy an introduction to game theory. So, today… Previously, we have looked at several examples of simple games; we have looked at an example of prisoner's dilemma; we have looked at an example of market game, which is also similar to prisoner's dilemma; and, we have look at the important concepts of best response and we have looked at the Nash equilibrium and we have understood to interpret the Nash equilibrium as a self-forcing outcome or a no-regret outcome. What we looked… What we want to look at today is we want to start; we want to look at another concept, which is that of a dominant strategy equilibrium. And, let us try to understand that through the example of a cold war game. Let us look at a cold-war game, which we are going to see, can be modeled similar to a prisoner's dilemma. And, let us try to understand another important concept of a dominant strategy equilibrium.

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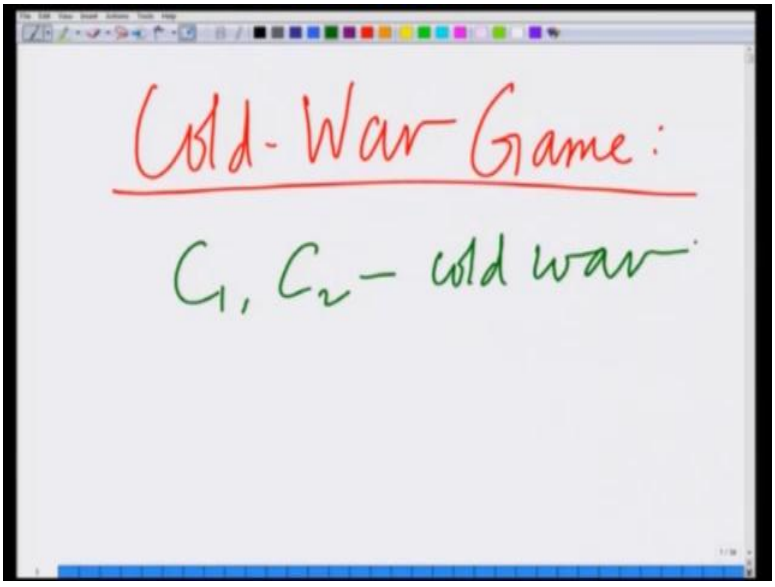

So, let us consider a cold-war game. Let us consider two countries C 1 and C 2, which are involved in a cold war sort of a scenario.

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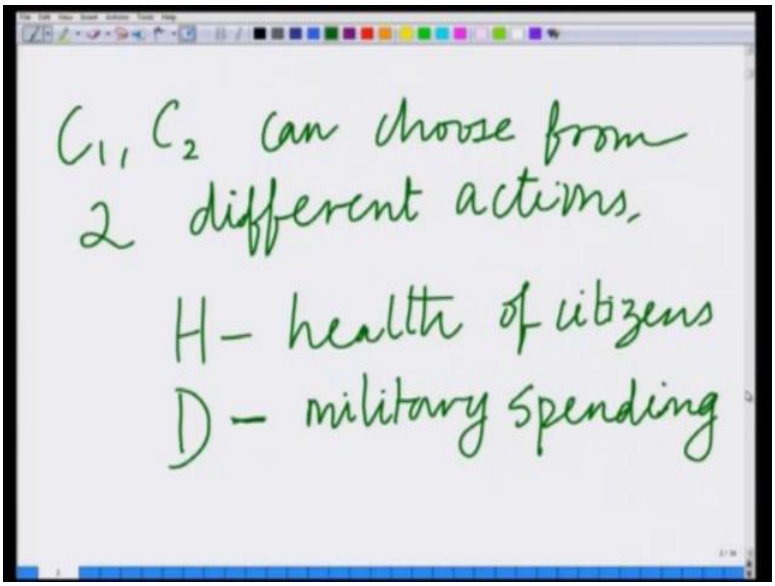

And, both can choose from two different actions: C 1 and C 2. They can choose from two different actions: either C 1; either they can choose to invest or focus their resource on health and development. For instance, C 1 and C 2 can choose to focus on improving the health of its citizens or health of citizens. Or, C 1 and C 2 can choose on investing in defense, that is, to improve their defense preparedness. And therefore, they can get a strategic advantage; all right. They can invest in defense or basically military spending. So, both these countries: C 1 and C 2 have a choice to choose between health and military – military spending. Now, of course, as we see, I think all of you are familiar – more or less familiar now with a concept of a game. So, I do not need to elaborately describe it. I am directly going to formulate this game in terms of its game table.

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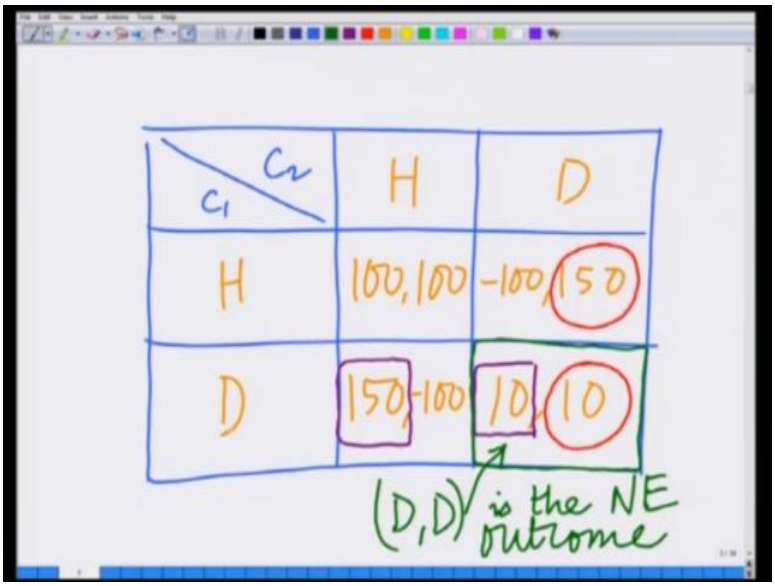

So, now I can formulate this game in terms of this cold-war game in terms of the game table; that is, if I look at it between countries, of course, I am going to represent country C 1 on the rows; that is, my row player is C 1, my column player is C 2. And both of them of two options either to invest in the health or defense. C 1 also has option to either invest in health or defense. When both of them invest in the health of their citizens, both of them have an outcome of 100; that is, since their citizens or the health of their citizens is improving both of them; let us say we can quantify it as both of them receiving utility of a 100. And, if both of them invest in defense, of course, because there is not enough resources to focus on the health of their citizens, the health of their citizens can suffer. But, because they are focusing on their military preparedness, let us say both of them receive a payoff of 10 comma 10.

However, on the other hand, if one of them focuses on the health of its citizens, well other focuses on defense. So, it is a country, which focuses on defense, has a strategic advantage over the other country, which can often be perceived to be important. The one which focuses on health gets a payoff of minus 100. The one which focuses on defense gets a payoff of 150.

Similarly, if C 1 focuses on defense, while C 2 focuses on health; since C 1 has a strategic advantage, it gets a payoff of 150; while C 2, which has lost the strategic advantage, gets a payoff of minus 100. So, we have the cold-war scenario between two countries, which can be modeled by the simple game C 1 versus C 2. And, each of these countries have an option to either choose between health and defense. If both of them

choose to focus on health, then they get a payoff of 100 each; if both of them choose to focus on defense, then they get a payoff 10 each. While if one chooses to focus on defense, while the other chooses to focus on health; then, the one which has focused on defense; for instance, if C 1 chooses defense, C 2 chooses health; C 1 gets a payoff of 150, since which is it has a strategic advantage over C 2; while C 2 which has lost the strategic advantage gets a payoff of minus 100. Similarly, if C 1 chooses to focus on health, while C 2 chooses to focus on defense; then, C 2 since it has gained a strategic advantage over C 1, it gets a payoff of 150; while C 1 which is focusing on health gets a payoff of minus 100.

Again we can try to analyze this game in terms of its best responses. If C 2 chooses health, then the best response of C 1 is to choose defense, because defense gives it a payoff of 150, while health gives it a payoff of only 100. So, C 1 chooses to go with defense. Similarly, if C 2 chooses defense, then choosing health gives C 1 minus 100; while choosing defense, gives it a payoff of 10. Therefore, C 1 again chooses to go with defense. Again similarly, we can mark the best responses of C 2. If C 1 chooses to go with health, then the best response of C 2 is to go with defense, because defense gives it a payoff of 150 in comparison to health, which gives it a payoff of 100. And similarly, if C 1 goes with defense, the best response of C 2 is to again go with defense. And therefore, we can see the outcome, where the best responses intersect. This is the outcome – D comma D is the outcome, where the best responses are intersecting. As you can see, the best response to defense of country 1 is defense for country 2. The best response of defense of country 2 is defense for country 1.

And, since the best responses are intersecting, this is the Nash equilibrium D comma D, that is, both focusing on defense; that is, military preparedness is the Nash equilibrium – is the Nash equilibrium outcome of this game; where, both the countries focus on defense preparedness is indeed the Nash equilibrium outcome of this cold-war game, which sort of explains the kind of outcomes that we expect to see frequently in this cold war scenarios, where both the countries involved in a cold war are diverting a significant amount of resources towards military preparedness or they are basically towards defense spending. So, D comma D is the Nash equilibrium. And, this is another – yet another game of – similar to prisoner's dilemma; because if we can see the Nash equilibrium outcome, there is one Nash equilibrium; and, the Nash equilibrium outcome as we discussed in the last example is not a Pareto optimal outcome, because if both the

countries choose H comma H, both of them can simultaneously improve their payoff by choosing H comma H; because country one can improve its payoff from 10 to 100, country 2 can improve its payoff from 10 to 100. Therefore, by choosing H comma H, both the countries can simultaneously improve their payoff. Therefore, D comma D is not Pareto optimal outcome.

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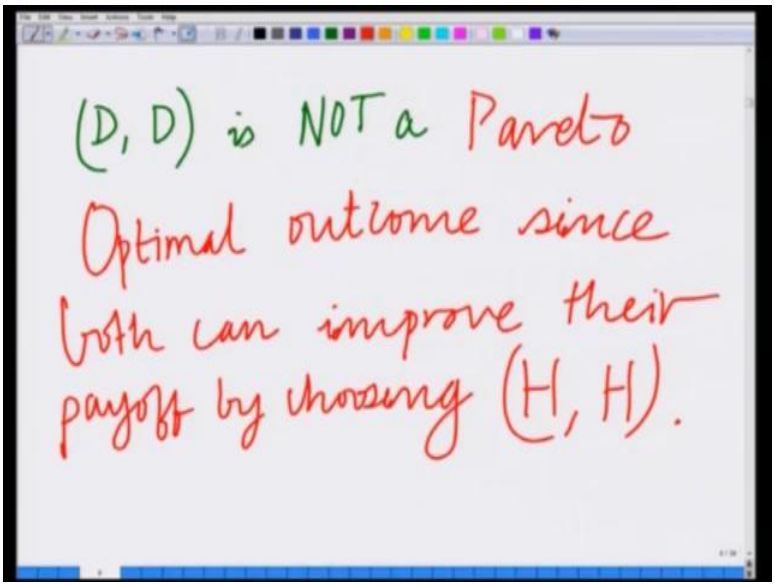

D comma D is not a Pareto optimal outcome since both can improve their payoff by choosing H comma H. So, in this cold-war example, we are basically seeing that, the Nash equilibrium is D comma D, which is the equilibrium outcome, which is the intersection of the best responses. And, again this D comma D outcome is not a Pareto optimal outcome because there is another outcome, which is H comma H, using which or in which basically both the players can simultaneously improve their payoff. Therefore, D comma D, which is the Nash equilibrium outcome is not a Pareto optimal outcome. And, this game is therefore, similar to the prisoner's dilemma.

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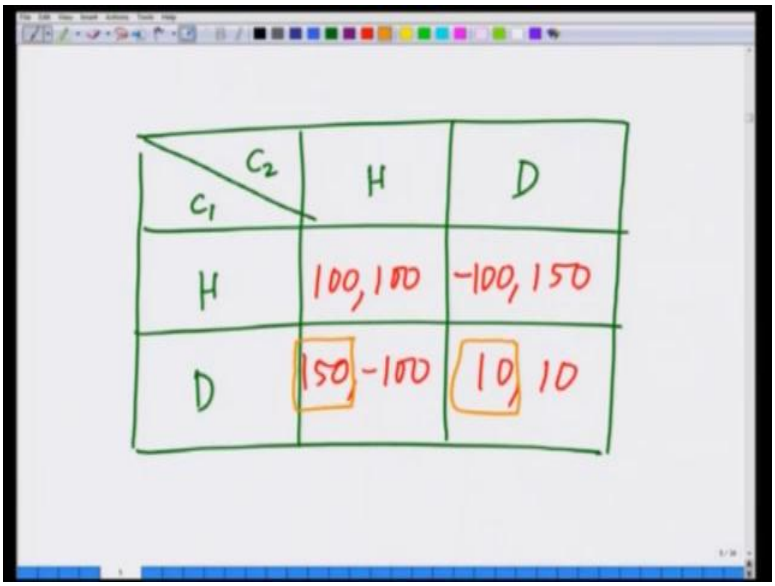

We want to introduce now another concept. Again let me redraw this game table for this cold-war game. Again if I redraw the game table for this cold-war game, again I have two countries: C 1, C 2, which can invest in health or defense health or defense and the payoffs are as follows. 100 comma 100 minus 100 comma 150, 150 comma minus 100 and 10 comma 10. Let us try to again look at the best response dynamic. If C 2 or country 2 chooses to invest in health, then the best response of C 1 is to choose defense. That is what we had seen. And similarly, if C 2 chooses to invest in defense, the best response of C 1 is to again choose defense. Therefore, you can see that, irrespective of the action chosen by C 2, the best response of C 1 is always to choose defense irrespective of the action of C 2. Such a strategy, such an action, which is always the best response irrespective of the actions of the other players, is known as a dominant strategy. Therefore, D is a dominant strategy play for C 1.

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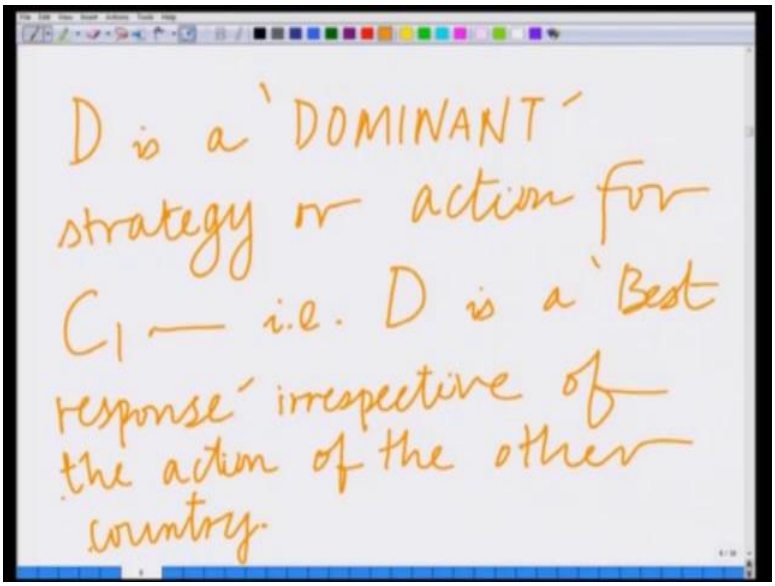

D is a dominant or dominant action for C 1; that is, it is better to choose D, that is, D is always a best response for C 1 irrespective of the action of the other player; that is, that is to say D is a best response irrespective of the action of the action of the other country or the other player. Such an action is known as a dominant action. What is a…

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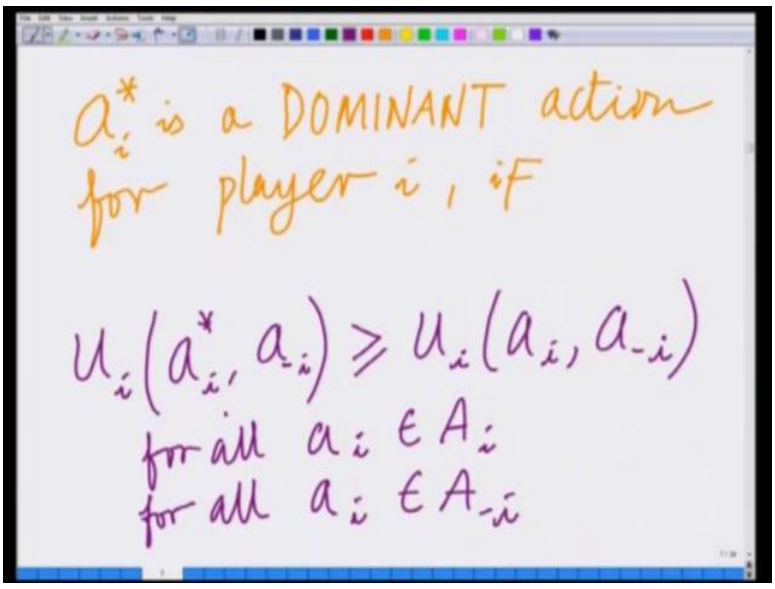

Mathematically, if we have to define a dominant action again using our notation, a i star is a dominant action for player I, if u i a i star comma a minus i is greater than or equal to u i of a i comma a minus i for all a i element of A i; that is, for all actions a i; that is, a i star is the best action comparison to all other actions A i for player i. And, this holds true irrespective of the action for all a minus I – element of A minus i; that is, this holds for

all actions, that is, irrespective of the action a minus i of all the other players. Remember – we use a minus i to denote the actions of other players.

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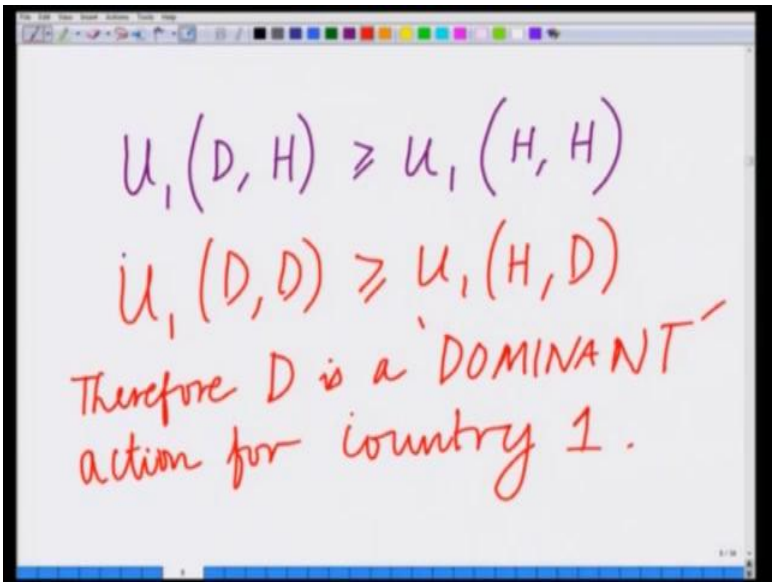

Speaking in terms of our cold-war game, you can see that, u 1 of D comma… that is, if country 1 is choosing country 2 is choosing health, u 1 of D comma H is greater than or equal to u 1 of H comma H; that is, if country 2 is choosing health, country D is the best response or D yields higher payoff for country 1 in comparison to H. Similarly, on the other hand, if country 2 is choosing defense, again defense is country 2 is choosing defense, again defense yields a higher payoff in comparison to – in comparison to – in comparison to health or in comparison to health; that is, irrespective of country 2 choosing H or D, the strategy D or the action D always yields a higher payoff in comparison to the action H. Such an action is known as a dominant action. Therefore, D is a dominant action for country 1 or player 1. Therefore, D is a dominant – D is a dominant action for country 1. D is a dominant action, that is, its yields; it is always the best response irrespective of the action of the other player.

Similarly, if you look now at country 2; again, if country 1 chooses health, the best response of country 2 is to choose defense, that is, D. And, if country… And, if country 1 chooses D, the best response of country 2 is to choose D. So, again you can see irrespective of the action of country 1, the best response of country 2 is to always choose D. Therefore, D is also a dominant…

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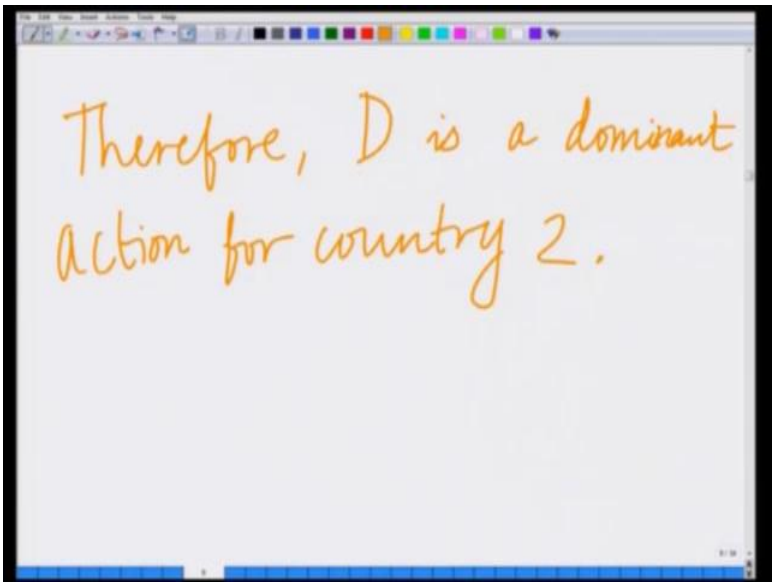

In fact, the game is symmetric. Therefore, D… therefore, D or choosing defense is a dominant action for country 2. So, then you can see there is a dominant action for country 1; there is a dominant action for country 2. And, the importance of a dominant action is naturally… Since the dominant action is a best response irrespective of what the other player is doing, the players always choose their dominant action. In any equilibrium, the players choose their dominant actions, because the dominant action is a best response irrespective of the action of the other player. And therefore, when the players of the game have a dominant action, these dominant actions – the combination of these dominant actions forms the Nash equilibrium of the game; that is, the dom… And, this is also known as a dominant action equilibrium or a dominant strat… or a dominant strategy equilibrium of the other game.

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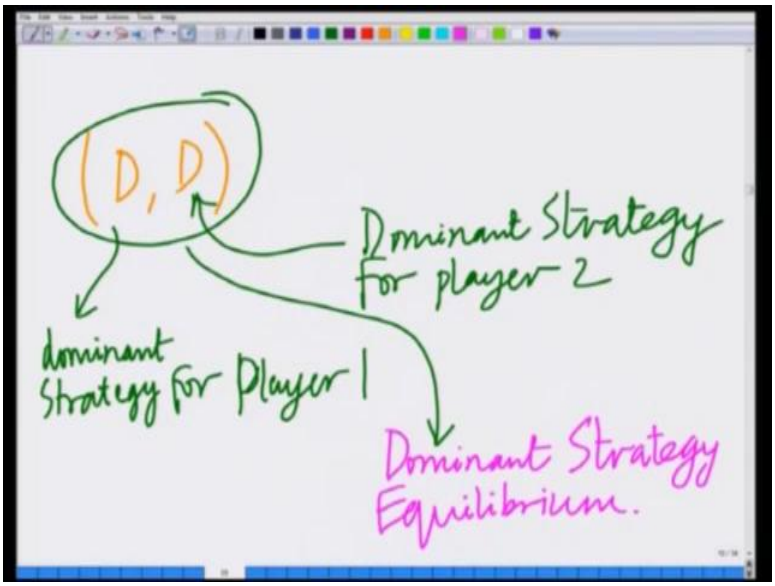

So, if you look at D comma D, that is, both D comma D; where, D is dominant strategy for player 1 and D is dominant strategy for player 2. And therefore, this is also known as the special kind of a Nash equilibrium, which is basically formed from a combination of dominant strategies. This is also known as a dominant strategy… This is also known as… This is also known as a dominant strategy equilibrium.

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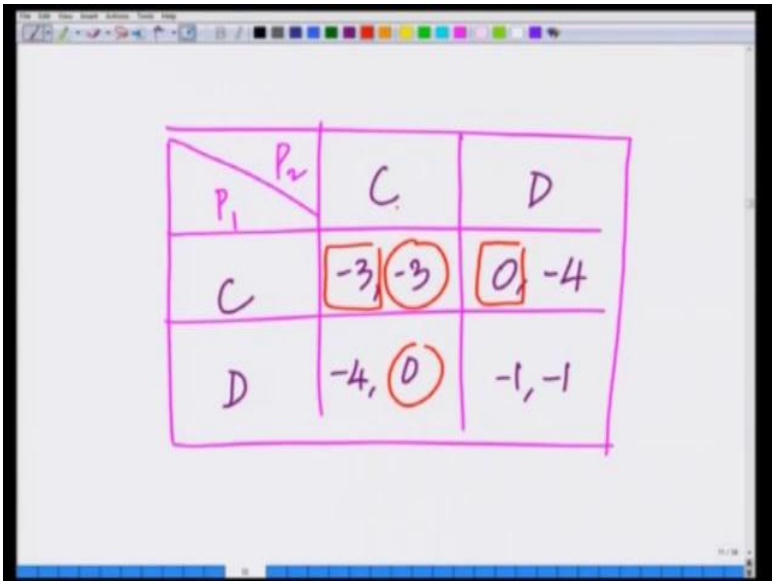

In fact, let us go back to our example of the prisoner's dilemma. Let us go back to our earlier example of prisoner's dilemma and look at the Nash equilibrium of this game again. So, we have two prisoners: P 1, P 2; either they can confess or deny confess or deny payoffs are as follows. And, you can see once again that, if P 1 chooses P 2 chooses

to confess, best response of P 1 is to confess. If P 2 chooses to deny, the best response of P 1 is to confess. So, irrespective of P 2 either confessing or denying, best response of P 1 is to always confess. So, C is a dominant strategy for P 1. Similarly, if P 1 chooses to confess, best response of P 2 is to confess. If P 1 chooses to deny, best response of P 1 is to again confess. So, irrespective of P 1 choosing confess or deny, best response of P 2 is to always confess.

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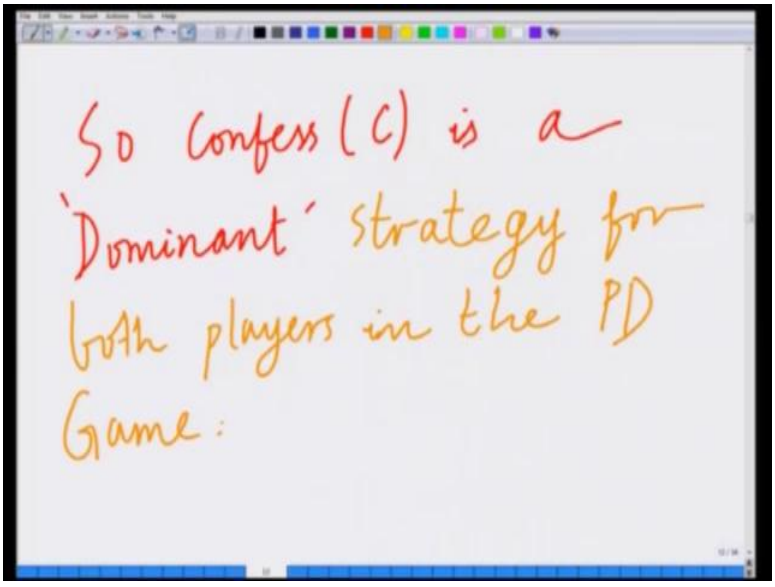

So, confess is… So, confess… So, confess of C is a dominant – is a dominant strategy for both players in the PD game.

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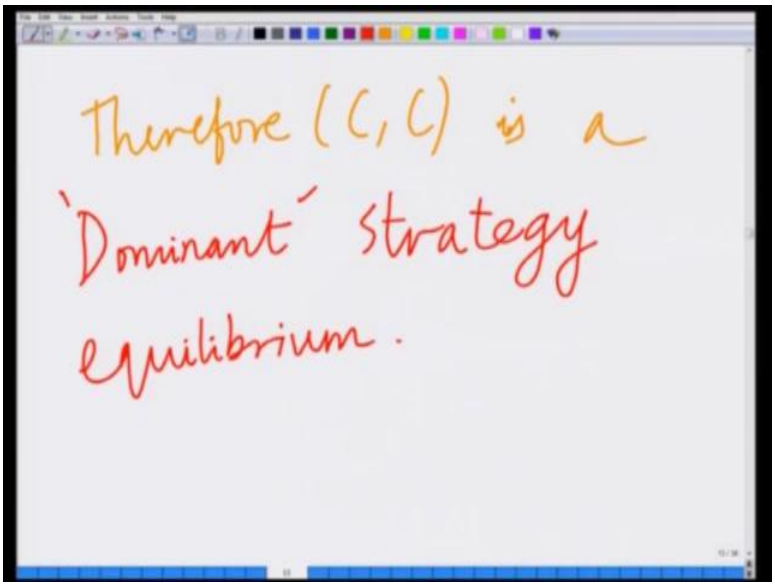

Therefore, C comma C is a dominant strategy. The Nash equilibrium C comma C is a dominant strategy. It is a dominant strategy equilibrium. And, it is not necessary for all Nash equilibria; or, whenever there is a Nash equilibrium for it to be a dominant strategy equilibrium. But, in this case, it happens that, the C comma C, which is basically composed of two dominant strategies, where C is a dominant strategy or a dominant action for each player, such a Nash equilibrium in which each player is playing as dominant strategy or dominant action is also known as a dominant strategy equilibrium. And, several examples of such dominant strategy equilibria can be found in real life. Let us take a simple example. Let us take a simple example of multiple conversations happening in allowed party.

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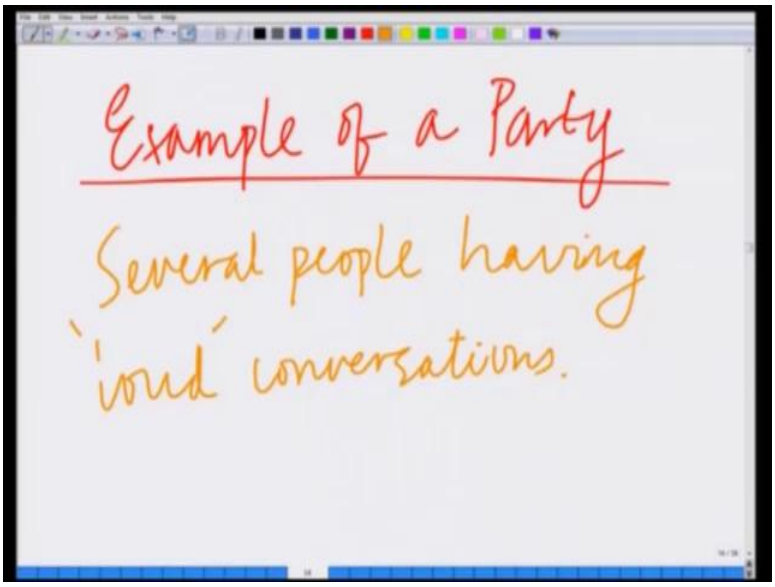

Let us take a simple real life example; let us not… I mean we are not going to look at it… Let us try to reason it out a sort of intuitively. Let us look at an example, where there are several loud conversations taking part, for instance, at a concert or a party. So, let us take an example – example of a party, where there are several loud conversations. And, each person has two options: either to shout at the top of his noise, that is, to speak loudly, that is, to shout or person can speak softly.

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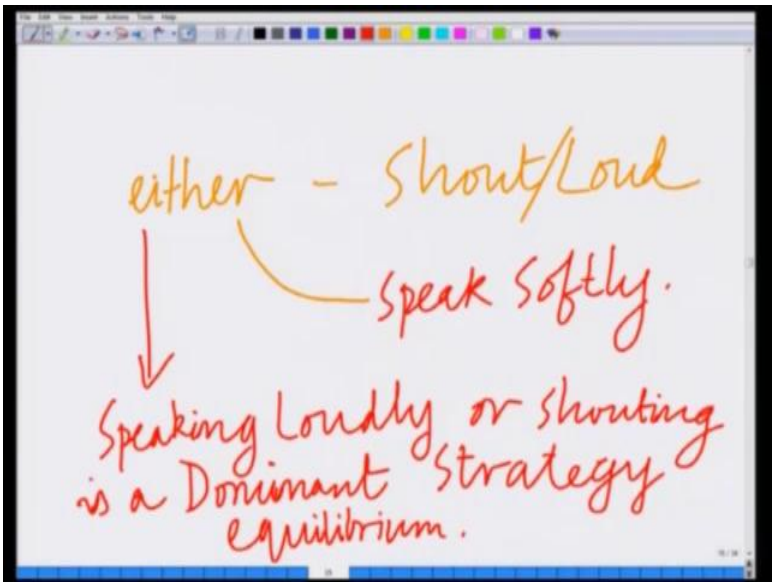

So, each person can be either loud, either shout or be loud or they can be soft – they can speak softly. And, you can easily guess what is the dominant strategy or the dominant action equilibrium in this scenario. If everyone is speaking in a soft voice, then the best response – then your best response is to shout, because shouting will get your voice… because shouting will make your voice be easily heard or shouting makes your voice easily heard to the person whom you are talking to. And, even if all the other persons, all the other people in the party or in the venue are shouting, that seems even more important for you to shout to get your voice across. So, irrespective… So, it is clear that, irrespective of what the other people are doing, one of your best response is always to shout or always to speak loudly, so that your voice is heard clearly – so that your voice is heard clearly by the person with whom you are having a conversation. So, speaking loudly or shouting is a dominant action or a dominant strategy frequently at such events.

And therefore, what you find is everyone choosing is dominant strategy, which is to shout or which is to have a loud conversation. And, that is what you find frequently happening. And, this is a very practical or this is a very practical example. For instance, if you go to a venue, which is where you have a function or where you have a party, you will frequently find a large number of people having loud conversations, because that is indeed the dominant strategy equilibrium. So, speaking shoutly tends to speaking loudly or shouting – speaking loudly or shouting; speaking loudly or shouting is a dominant strategy equilibrium. This is a dominant strategy equilibrium. Similarly, you can look at it – look at another example of where people are trying to… And, this is a more common

example, where a large number of people are attending a concert. And, frequently, you tend to find that, all the people eventually even though there are chairs in the concert, all the people eventually end up standing. So, at a concert…

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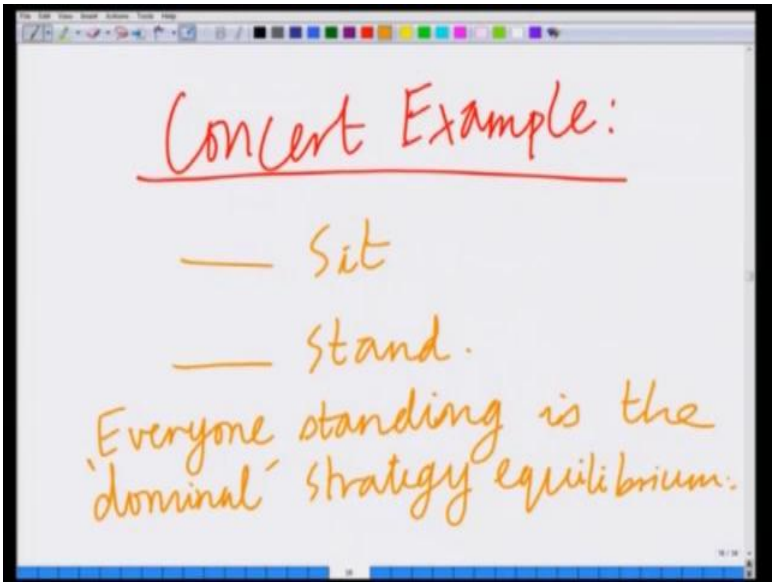

So, example of a concert; let us take a concert example, where we have… It is an open air concert, where there are lot of chairs that people can actually sit and enjoy the concert. So, all people have two actions: either they can sit or they can stand. And now, let us try to analyze this in terms of a dominant strategy. If everyone is sitting, then it is your best response is to stand, so that you get a better view or a clear view of the concert or the performance. And, even if it is everyone is standing, it is all the more important for one to stand, so that he gets a better view of the performance. So, irrespective of what the other people are doing, your best response – the best response of people often tends to be to stand up. Even if others are sitting or standing, irrespective of what the other people are doing, that is what the other people are sitting or standing, the best response tends to be to stand, so that you can get a better view; and therefore, enjoy the concert performance, enjoy… get the best… get the most out of the concert.

And therefore, frequently what you find in such real life scenarios is everyone standing up, everyone playing their dominant strategy, which is to stand up. And, frequently you find that, in such when you see everyone is standing up to get the best possible view of the concert and maximize their enjoyment. So, what they are trying to do is they are trying to play the dominant strategy. If that is what this is. This is a very simple example. These are simple examples of dominant strategy equilibrium; we do not need to really

model it or putting down numbers and drawing game tables, but intuitively, we can understand that all these scenarios are very similar to a prisoner's dilemma, where there are obviously other Pareto optimal outcomes. For instance, in this scenario, where everyone can sit and probably enjoy the concert in a more relaxed fashion; but eventually it ends up, where everyone is standing to get a better view, because that ends up being the Nash equilibrium of this kind of a scenario. Or, that ends up… That is indeed the dominant action equilibrium or the dominant strategy equilibrium. So, stand or everyone standing… So, everyone standing is the… So, where everyone is standing ultimately ends up being the dominant strategy equilibrium. So, this kind of… So, these are examples, where of different games, which have a dominant strategy example. And, these are similar to the prisoner's dilemma.

Thank you. Thank you very much. And, we will continue in the next module.