Strategy: An Introduction to Game Theory
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Lecture – 49

Hello and welcome to mooc lectures on Strategy, An Introduction to Game Theory. In this module I am going to introduce a chapter called Repeated Games.

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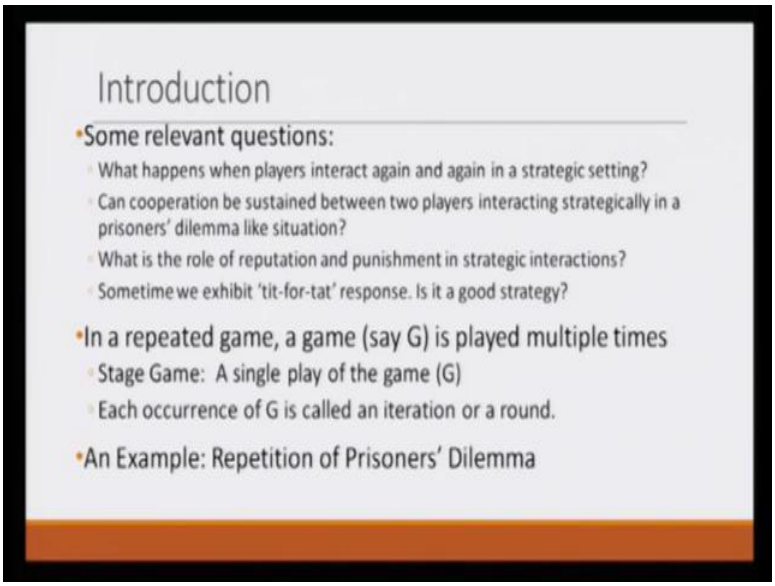

What do we mean by repeated games? Think of a scenario, if you remember when we were talking about games earlier, we were emphasizing that players are interacting with each other one and only once, they are not talking to that player again. Because, what happens if you interact with player again and again then there is a strategy consideration that comes into the picture. So, we wanted to avoid that and that is why we were emphasizing that players interact only once. But, now we are ready to study games in which players interact again and again.

So, the relevant question here are that what happens when player interact again and again in a strategy setting. Most importantly, if you remember the prisoner's dilemma game then the players do not cooperate with each other. So, the relevant question here would be that can cooperation we sustained between two players interacting strategically in a prisoner's dilemma like situation. Again, what is the role of reputation and punishment in any strategic consideration?

Remember, when reputation and punishment does not work when players are interacting only once, their reputation would not play any role. But, if players are interacting again and again, then reputation may play very, very important role and some time in life we heard, the some strategies liked tit for tat, do what other has done to you. Is it a good strategy, we will try to analyze that. So, some definition first that would help us understand it, that in a repeated game let us say again G, G can be anything, it can be prisoner's dilemma game, it can be hawk and dove game, it can be battle of sexes.

What happens a game G gets played multiple time, one after another and after each single play of the game is called stage game and each occurrence of G is called an iteration or a round. One simple example would be that in reputation of prisoner's dilemma game.

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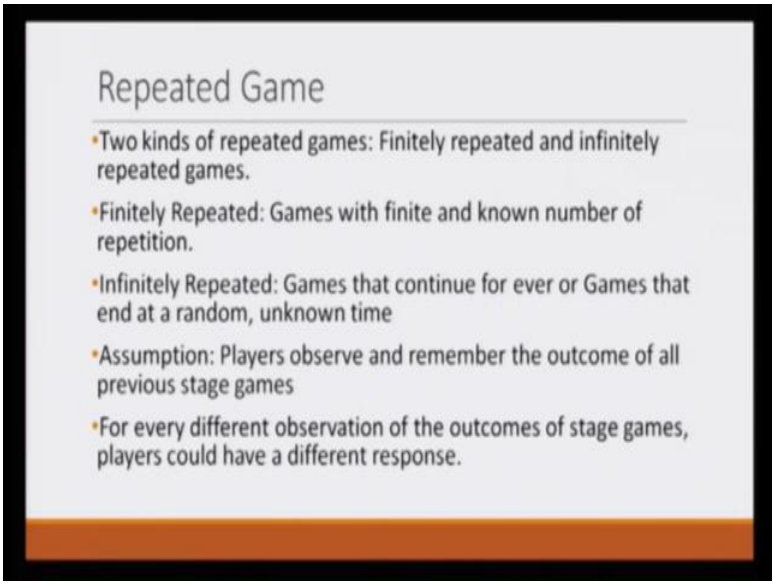

Some more definition we will take about two kind of repeated games, in one type of game we will call finitely repeated game. What happens, that players interact with each other finite number and known number of time. Second is infinitely repeated game, in that this kind of game continuous forever, the end is not known to any one of the players. But, there is also one other possibility we can consider, that game may in this period with some positive probability and continue for the next period with some positive probability, that kind of game will also be studied under infinitely repeated game.

We have to make certain assumptions, what are those assumption that player observe and remember the outcome of all previous stages. So, it is a game of perfect recall, they

arable to recall what they have gone through in the past. For every observation of the outcome of a stage games, player could have a different response. In other word, we can think of a strategy contingent upon previous responses, previous outcome of previous stages.

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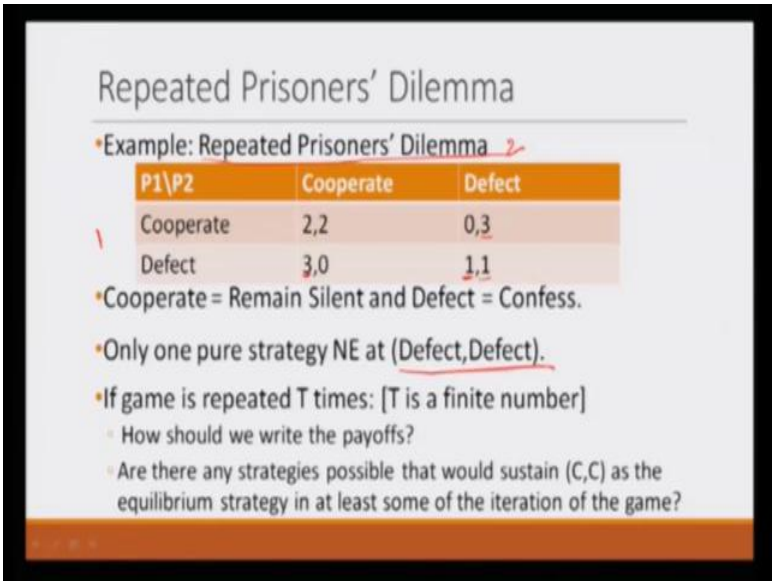

So, let us take the prisoner's dilemma game, you remember this game, either you have done exactly the same game or slightly a modified version of this game. But, just easier what happens there are two prisoner's, they can cooperate or they can defect, corporation means they remains silent and defection means that they confess, there is only one Nash equilibrium let us check, if prisoner's one thinks player prisoner's 2 is going to cooperate then the best response for prisoner 1 is 2 defect and if prisoner 1 think, prisoner 2 is going to defect then again the best response is defect.

So, we clearly see that prisoner 1 has a dominant strategy that is to defect and similarly prisoner 2 has a dominant strategy defect, because defect is always the best response no matter what prisoner 1 is doing. So, we get basically at dominant strategy equilibrium which is all dominant strategy equilibrium or Nash equilibrium. So, we get a Nash equilibrium that is defect and defect. Now, this you have done in the first week, so what we are going to add here is that now if this game is repeated T times and first we will discuss the scenario in which T is finite.

The first thing that we have to worry about that how should we write the payoffs, so we will revise some mechanism how to write payoff probably simple edition or discounted

edition, we will talk about it. But, more important question would be are there any strategies possible that would sustain cooperation, remember in one if prisoner's dilemma game is played only once, the both players would defect, they would not cooperate.

So, here question is are there any strategies possible that would sustain cooperation as the equilibrium strategy at least in some of the iteration of the game, let us say if not towards the end, then how about in the beginning, so we will discuss that.

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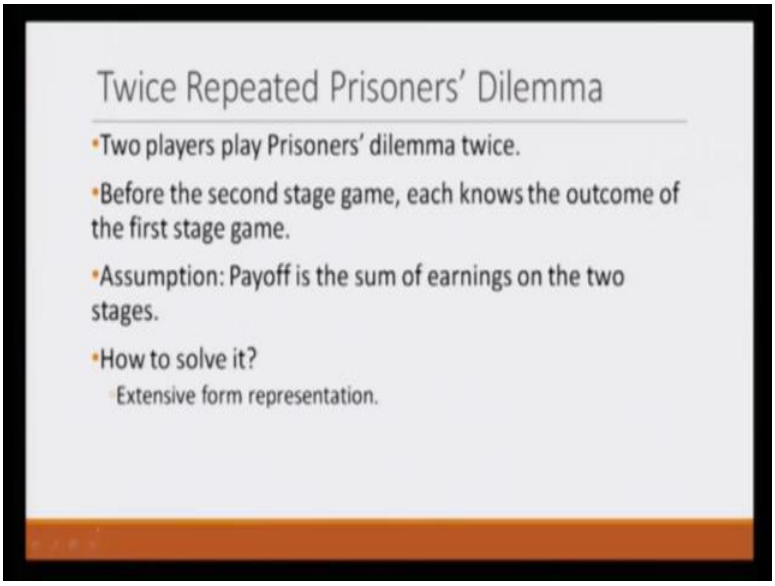

So, first let us take the most simple example that this prisoner's dilemma game is played only twice, before the second stage game each know, each of the players they know the outcome of the first stage game. So, they know what happens in the first stage game, what is the assumption, we were worried about how to get the payoff, here we will make the assumption that payoff is the sum of earning in the two stages, we will simply add the payoff from these two stages. How to solve such game?

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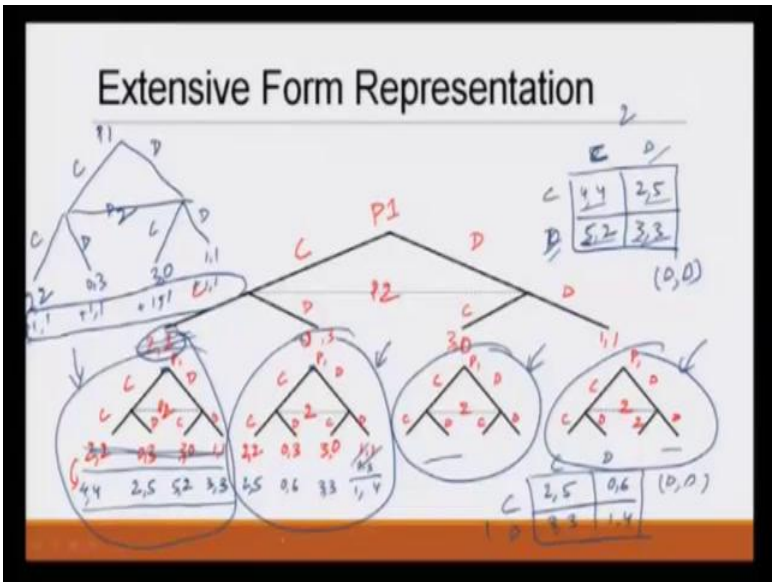

Let us look at the, we know something called extensive form game, so we can think of this as extensive form game, here you have prisoner 1, prisoner 1 plays either cooperate or defect, here is prisoner 2, prisoner 2 does not know whether prisoner 1 has played cooperate or defect. So, prisoner 2 can again decide cooperate and defect. What happens here the payoff become known to the all the player. So, I am going to write the payoffs just, so here is 2 comma 2 in case of cooperate and defect, when player 1 cooperates player 2 defect, then player 2 gets 3 and player 1 gets 1 3 comma 1 and if both defect, they both get 1 comma 1 let me check the payoffs, here we have 0 0.

Now, player 1 sees this, here again player 1 get to move and then player 1 can again decide C D, C D and here player 2 is decided. Similarly, we have to fill for all the possibilities player 1 decides C or D, then player 2 decides without observing the action of player 1. So, let me just finish writing this and this is the game, this is the way game is played. Now, we have to write again the payoff and in the second stage, payoff would be again same numbers we have to write every where 2 comma 2, 0 comma 3, 3 comma 0, 1 comma 1 and so on for the all other possibilities.

Now, what we have to think that of course, the outcome is known to the players, but we can say that after this they realize the total payoff and how much is the total payoff, let me write it in the blue color. Because, for all these four outcomes it happens after both the players are played C comma C in the first stage. So, C comma C, playing C comma C would give both of them 2 comma 2. So, we have to simply add 2 comma 2 in all the payoffs. So, it will be 4 comma 4, it will be 2 comma 5, again 5 comma 2 and it would

be 3 comma 3.

So, in other word we do not need to write this 2 comma 2 here and we do not need to write this payoffs here, we will simply write these are the payoffs and simply one more let me do it, here we have to add 0 comma 3 in all. So, if we add here 0 comma 3 we will get 4 and here we will get 1. So, we do not have to write all these things, similarly here we add, here we will get 3 comma 3, here 0 comma 6 and here 2 comma 5 and so on we have to do everywhere.

Notice, if we write this game, because here player 1 after player 1 and player 2 they have participated in a stage 1 and now they are playing the stage 2, stage 1's payoff have already them known to them that 2 comma 2 they are going to get. So, basically now they are playing C comma C, D comma D the payoff would be here. So, here C comma D here payoff would be 4 comma 4, C comma D 2 comma 5, 5 comma 2 and 3 comma 3 this is for this particular stage game.

What would happen, again we see that for both the players D is dominant strategy, let us see let us thing that player 1 things that player 2 are prisoner 2 is going to play C, what is the best response play D because D gets 5 and C gets 4. Similarly, player 1 things that player 2 is going to play D then what is the best response of player 1 again play D, because D gives 3 and C gives 2, so D is clearly the dominant strategy.

Similarly, D is dominant strategy for player 2 and they will end a playing D comma D that happens here they play D comma D, how about here. Similarly, we can draw the table here and we write C comma D, C comma D and what are the payoffs 2 comma 5, 0 comma 6, 3 comma 3, 1 comma 4. Again we will see the D is the best response for player 1 no matter what player 2 is doing and these the best response for player 2 no matter what player 1 is doing. So, they both would play D, so D comma D is again a dominant strategy equilibrium that gives the Nash equilibrium.

And similarly you can check in this game and in this game, you should notice basically if you remember from the extensive form game what are these, this is one sub games starting here, there is another sub game starting here, this is another sub game, this is another sub game, these are the four sub games in the second stage and what is happening everywhere here we have added 2 comma 2. So, we subs tract 2 comma 2 does not matter that is not impacting the decision, because 2 comma 2 happens in both the boxes.

So, when we subtract that 2 comma 2 we get the original prisoner's dilemma game back. So, we know what happens in the prisoner's dilemma game that players all there are dominant strategy that is D and D comma D happens. So, in this sub game players will play D comma D, in this sub game players would play D comma D and in this sub game player would play D comma D and in this sub game player would play D comma D.

So, if you do the backward induction what do we get, let me draw here, here we have player 2, here we have player 1 player 1 can confess or defect player confess or cooperate and do not cooperate or defect and we already know the payoff here 2 comma 2, here we have 0 comma 3, 3 comma 0 and then we have 1 comma 1. But, we know that after that player again interact and in all possible interaction they inter playing D comma D. So, in all the cases they earn 1 comma 1. So, we add here 1 comma 1, here 1 comma 1, here again 1 comma 1 and here again 1 comma 1.

Since, we are adding the same number everywhere again it would not impact, it would remain a prisoner's dilemma game in which both the players are dominant strategies again D. So, players would play D, so what we have observed that if prisoner's dilemma game is repeated twice in stage 1 players play D in stage 2 they always play D. So, we do not get any cooperation, now what we have learn that if we repeat a prisoner's dilemma twice nothing changes, players always defect in all the stages.

So, you may be think the whole idea that we started with that if we have this reputation then can we sustain some short of cooperation between them it since for part two for repeating this game twice we do not get any cooperation, so let us go one step for that.

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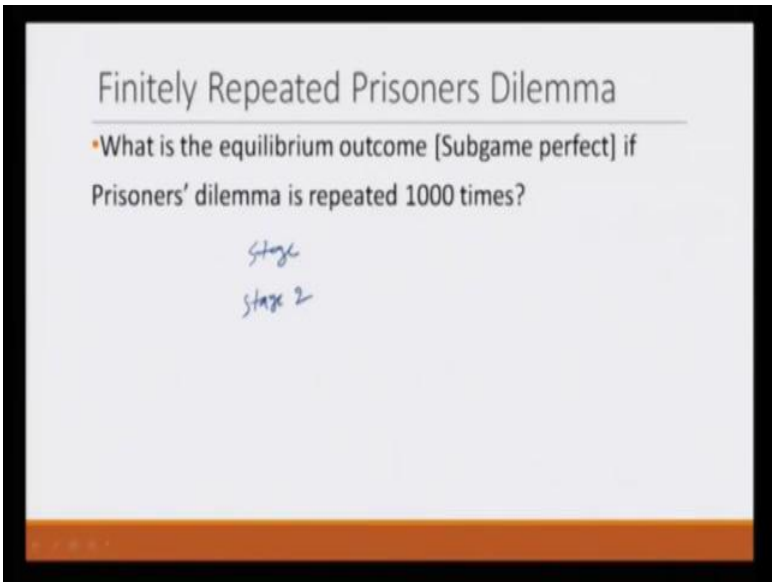

Let us say that they play the prisoner's dilemma 100 time, 1000 times, 10,000 times a finite number of times. So, there will be stage 1, they play stage 2 again the play and so on the game would see just with two stages ((Refer Time: 15:31)), we have 1, 2, 3, 4 and one whole five sub games. So, it will become fairly large game very fast, but we know in the last stage they will be playing the prisoner's dilemma to in all the possibilities what is going to happen, in the last stage because after that they know they are not going to play they are not going to strategically interact.

So, they are better are by defecting, defect is the dominant strategy for both the players in the last stage. Now, we can take the same logic that we had earlier used, if let us say there are 1000 rounds what we figured out that in the last round player would always defect. So, second last round last, but one round what happens, when the last round player would know in when they are playing the second last stage they know that in the last stage they both are going to defect, so nothing changes.

So, in the second last round also they would defect and if you take this reasoning back come back to the first stage their also players would again defect. So, it does not matter how many times we repeat the prisoner's dilemma game, the only thing that we have to keep in mind that we are repeating this prisoner's dilemma game for finite number of times, what would happen players would always defect and we do not get any cooperation. Let us take that result is very sad that you know reputation does not help, it does not guarantee any cooperation, let us take one different game.