Strategy: An Introduction to Game Theory
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Lecture – 45

Welcome to mooc lectures on Strategy, An Introduction to Game Theory. In this module, I am going to describe a game called Hawk and Dove game.

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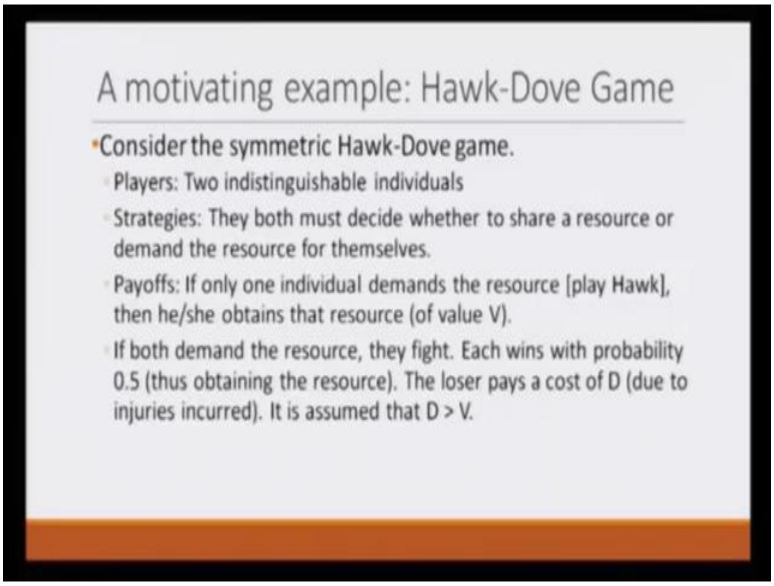

What is hawk and dove game? We are going to consider a symmetric hawk and dove game I will come to the point what does this symmetric mean here. What do we have? Like any game that we have seen in the first week, that if you want to describe a game we can describe using three different things, list of players, strategies for all the players and the payoff corresponding to his strategy profile. This was one way of describing, another also we learnt in the third week that is extensive form game.

But, today we are going to stick to the normal form game and we will see, how useful it is for our next topic that is evolutionary game theory. So, what we have, players two in distinguishable individual, strategies what are the strategy, they both must decide whether to share a resource or demand the resource for themselves, sharing the resource means playing a soft way. So, it is we can call it dove and demanding a resource for oneself is playing the hard way, so we will call it hawk.

What are the payoffs? If only one individual demands the resource, means only one individual is playing hawk and other is playing dove, then the hawk obtains the whole object and let us say the value of whole object is V. If both demand the resource means, both of them are playing hawk, then they will fight and one of them will win with probability half. The loser will pay a cost of D, because fighting will probably, it will also involve some sort of injury and we also assume D is greater than V. So, injury is really, really costly; it cost more than the value of the object for which they are fighting.

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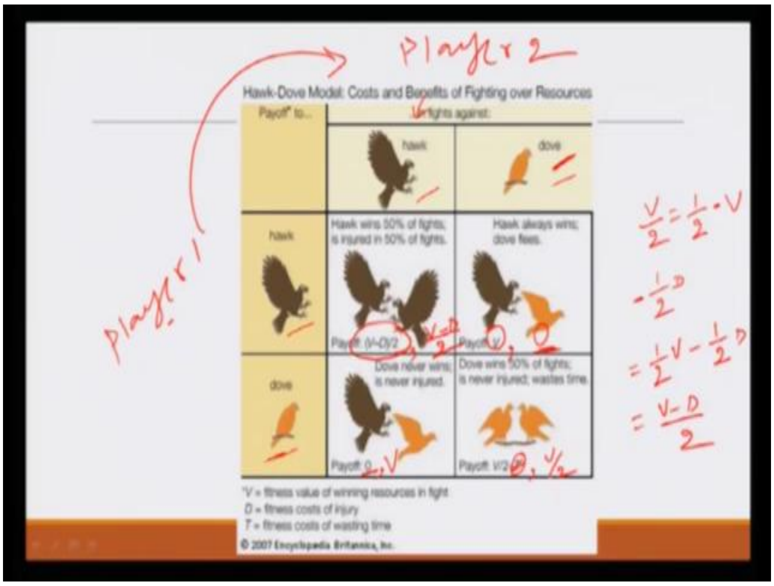

Here, there is a very nice picture from encyclopedia Britannica, here we have hawk, here we have dove, similarly here we have hawk, here we have dove, we can call this row player as player 1, here we have player 2. What do we have? That if player 1 plays hawk and player 2 plays hawk then player 1 gets V minus d by 2 V by 2, because he wins the object with probability half. So, V by 2 is coming half probability multiplied by the value of the object.

But, also we are getting minus D by 2 why, because someone is losing with probability half and losing his costly, because it also involves injury and the cost is D. So, this will be minus, so 1 by 2 V minus 1 by 2 D and the payoff is V minus D by 2. Notice that, in this we have written the payoff of only one player, why the game is symmetric, you bring player 2 here and player 1 here, again you will get the payoff correspondingly.

So, it is symmetric, two players they have the same strategies and they have the same kind of payoff, if you just transpose the matrix you will get the payoff of the other player. We can also write here V minus D by 2 for the other player, if one player is playing hawk and another is playing dove, then what is happening hawk going to win the whole resource and hawk will make V and of course, dove will make 0 we do not need to write 0 here again. Because, here we have saying player 1 is playing dove and player 2 is playing hawk, then player 1 is getting 0 same as here, so it is symmetry in that sense.

And of course, here player 2 will get V and if both of them are playing dove, then they both get V 2 minus T, it is wastage of time, for our example we will take that wastage of time is equal to 0. So, we will get rid of this T and both player will get V by 2 V by 2.

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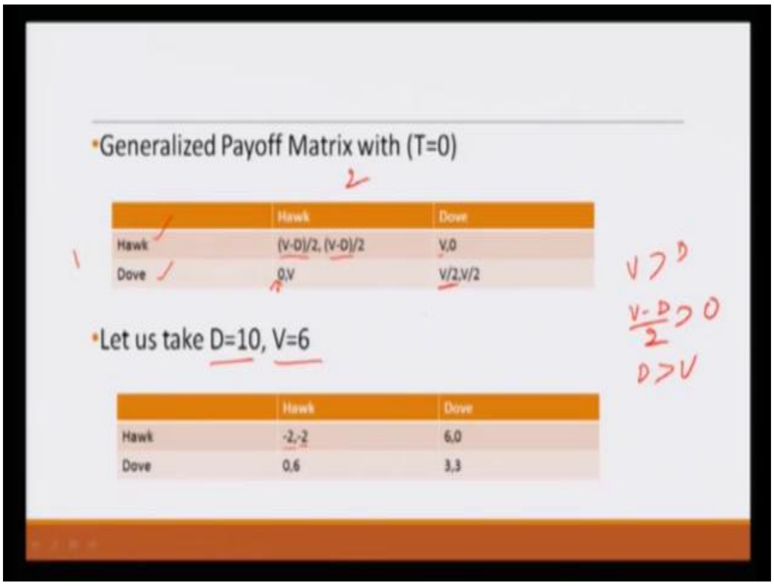

So, here is the generalized payoff matrix, I have written I have taken the dove and hawk out of the picture and I have just written the numbers V minus D by 2 and V minus D by 2 and similarly for all. Let us take a very specific example just for the illustration, what we have is D is equal to 10 and V is equal to 6. Remember, that here in the first slide ((Refer Time: 05:09)) I also talked about that we assume D is greater than V that is why we are taking D as greater than V.

Notice, what happens if D is not greater than V, in other word V is greater than D, in that case what happens, the player 1 and player 2 here, player 1 has a dominant strategy that is hawk. Because, then in that case V minus D by 2, if V is greater than D then V minus D by 2 is greater than 0. So, if player 2 plays hawk player 1 will play hawk, because V minus D by 2 is greater than 0 and if player 1 plays dove, here V is greater than V by 2.

So, player 1 has his dominant strategy it is a symmetric game, player 2 also has the dominant strategy. And if you look at this structure, it is seen as the prisoner's dilemma game. In prisoner's dilemma also what happens, two players, two strategies they both have dominant strategies and the dominant strategy equilibrium gives the Nash equilibrium in which the both players do verse of them one of the options available.

So, because if they both had played dove and dove, they both would have obtained V by 2 V by 2, but here they end up playing hawk and hawk, in which they both are verse of, but that is the Nash equilibrium, this situation is exactly same as prisoner's dilemma. So, we do not want that we are studying at different game, here we take D greater than V, in that case we do not get prisoner's dilemma why, because very clearly if D is greater than V, then none of the players have any dominant strategy.

Let us look at it, if player 2 is playing hawk, then player 1 is better off by playing dove why, because this 0 is greater than V minus D by 2 and if second player is playing dove, then the first player is better off by playing V by 2. So, in one case hawk is the better response in other case dove is the better response, so player 1 does not have dominant strategy.

So, now, we have a different game, if we take the specialized value if you just plug the value, if V is 6 and D is 10 then V minus D by 2 would be minus 2 that is what we have written here minus 2 minus 2 and similarly we have fill the matrix and we get hawk dove 6 0, because V is 6 and other player is getting 0. Now, can we obtain the Nash equilibrium that you have already done in the first week, but it will also help you refresh the memory. So, let us see let us calculate the Nash equilibrium.

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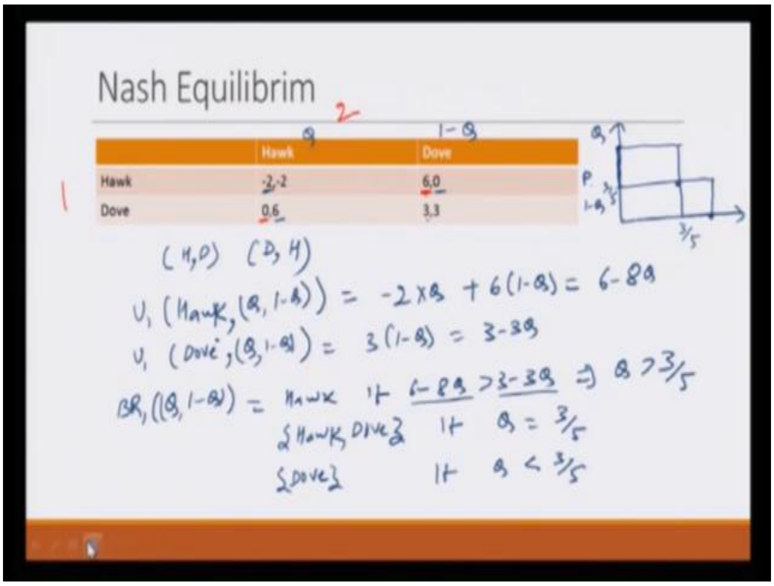

Let us say player 2 plays hawk. What is the best response of player 1? Dove, we already talked about. If player 2 is playing dove what is the best response, player 1 should play hawk. Similarly, if player 1 plays hawk then the best response from player 2 is to play dove and if player 1 is playing dove, then the best response from player 2 is to play hawk. So, there are two Nash equilibrium in pure strategies, hawk I will write as H in short form, so these are the and dove would be D in the short form.

So, H comma D and D comma H these are the two pure strategies Nash equilibrium. Do you have any other Nash equilibrium? You have also learned to obtain mixed strategy Nash equilibrium. So, how can we obtain in the mix strategy Nash equilibrium? We should draw the graph. Let us say player 1 thinks that player 2 is playing hawk with probability Q and dove with probability 1 minus Q.

So, how much hawk will give to player 1? Let us write in the standard way, player 1 is playing hawk and player 2 is playing a mix of Q, 1 minus Q. In this case it is going to be minus 2 multiplied by Q plus 6 multiplied by 1 minus Q and we get 6 minus 8 Q. Similarly, what would be the payoff of player 1? If player 1 plays dove, it is going to be 0 multiplied by Q, 3 multiplied by 1 minus Q, so 3 minus 3 by 3 Q. Clearly, best response of player 1 is, when player 2 is mixing between Q and 1 minus Q, best response of player 1 is to play hawk if 6 minus 8 Q is greater than 3 minus 3 Q.

The one the strategy that gives better payoff that is what player 1 would play. So, playing hawk would gives 6 minus 8 Q, playing dove would give 3 minus 3 Q. So, player 1 will play hawk if 6 minus 8 Q is greater than 3 minus 3 Q and that translates into Q is greater than 3 by 5. Similarly, player 1 will play either hawk or dove or any mix of hawk and dove, if Q is equal to 3 by 5 and if Q is less than 3 by 5 it means 3 minus 3 Q is greater than 6 minus 8 Q, then player 1 will play dove, if Q is less than 3 by 5.

And similarly, we can obtain for both the players the player 2 also. So, let us draw for player 1, what would player 1 do, let us say Q is here and now if you mark here p, 1 minus p. If Q is equal to 0 means player 2 is, if Q is equal to 0 then player 2 is playing dove and in that case best response is to play hawk and in that case p has to be equal to 1. So, as long as Q is between 0 to 3 and 5, player 1 will keep on playing the dove.

So, here we go up to let us say it is 3 by 5 and at 3 by 5 any mix will do, best response would be p is equal to 0, p is equal to 1, p equal to any number between 0 and 1. And that case player 1 can also mixed between hawk and dove and he would get the same payoff.

And when p is greater than 3 by 5 then of course, player 1 is better off by playing dove, in that case p has to be equal to 0, this is what would be the best response. Similarly, if you obtain for player 2 what happens, you will get exactly symmetric.

So, let us to try, you can try on your own, you will get something of this sort, this is what you will get. So, there are three equilibrium one, at this point, one at this point and one at this point, this we have already obtained. What is this? This is hawk comma dove, this is dove comma hawk and this is the mixed hawk one and this is what we get. As the Nash equilibrium, there are three equilibriums that we obtained, you should also obtain the value of this, I have already written 3 by 5 this would be 3 by 5.

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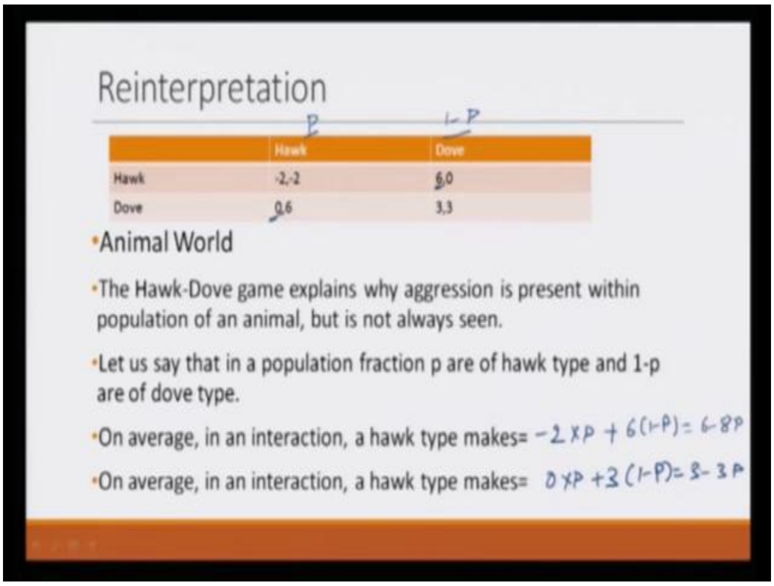

Now, let us try to reinterpret this game slightly differently, what happens game remains the same. But, rather then two rational player playing this game, what happens that this game is being played in the animal world between two animals and it will help us explain why aggregation is present with in population of an animal, but is not always see, it is not always seen in all the cases, some time we see the an animal is aggressive, some time we see animal of the same as space is not aggressive.

We cannot say that these animals are deciding this being aggressive based on some rational, some reasoning, some ration that we cannot say. So, here how can we explain, we say that a population of an animal is made of two types, one is hawk type, another is dove type. Hawk types are the aggressive type and dove types are kind of they are reclusive, there are withdrawn type or you can say they are piece loving type.

What happens now? Let us say that in the population we have p proportion of hawk and 1 minus p proportion of dove. So, let us say when an hawk type of animal interact randomly with an animal of the same is space is, then what happens what would be the hawk type animals payoff, it would again we have already calculated, it would be minus 2 multiplied by p plus 6 multiplied by 1 minus p that we had already calculated it comes out to be 6 minus 8 p.

And let us say if animal is dove type and randomly it interacts with an animal of the same space is what would be it is payoff, it would be 0 multiplied by p why, because there is probability p that it would be interacting with the hawk type of animal, because p is the proportion of hawk types in the whole population. So, 0 multiplied by p and the probability is that the other animal would be of dove type is 1 minus p, in that case the payoff to the animal this animal dove type of animal would be 3, but with probability it would be 3 multiplied by 1 minus p. So, 0 multiplied by p plus 3 multiplied by 1 minus p, so we are saying 3 minus 3 p.

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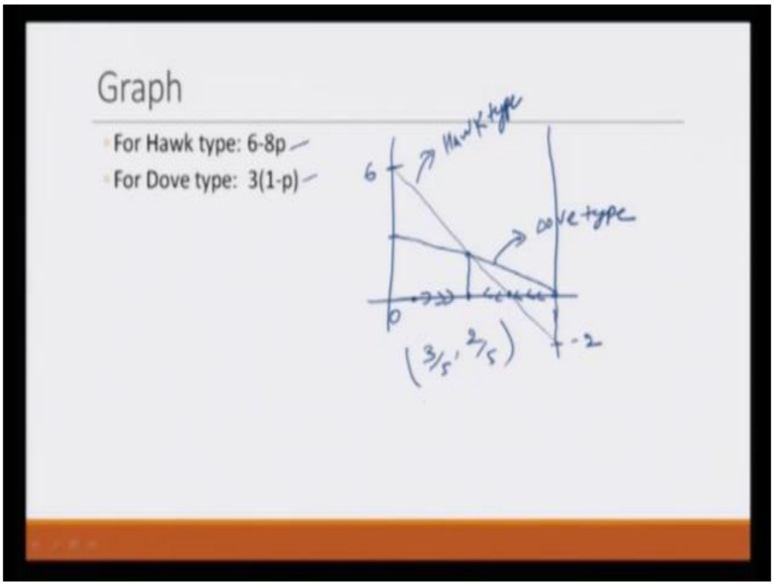

Now, let us see I have already written for hawk type 6 minus 8 p and for the dove type 3 multiplied by 1 minus p and p where is between 0 to 1. So, let us draw the graph how payoff would change of a hawk type based on the proportion of hawks in the whole population. So, it is going to be starting from 6 when p is equal to 0, 8 multiplied 0 would be 0, 6 minus 0 would be 6. So, starting with 6 and when the whole population is made of hawk type, then how much is the payoff 8 multiplied by 1 because 1 represents the whole population and 6 minus 8, so minus 2 here would be let us say minus 2.

And for a line it is a straight line, so two points are good enough we draw like this and this is the payoff of hawk type of as the function of p. And similarly the payoff of dove type is starting when p is equal to 0, it is equal to 3 when p is equal to 1, it is equal to 0. So, it is starts at a much lower level, but rate of decreases much less and this is for dove type. So, let us extend because now when we are talking about animals they are not rationally deciding anything.

Whether they are hawk type or their dove type that is genetically encoded it is not a consist this is in the have taken to become hawk type or dove type, it is guided by jeans. So, what happens if a population is entirely made of hawk type, it means we are here and what happens if we introduce at dove type in the population. An average dove type would do much better and we will formalize these notions in the next module.

But, doing much better means that it will have better fitness, it will able to do well, it will have batter a productive success and so it is proportion in the population would increase, it means if doves population for proportion is increasing then hawk proportion is decreasing p would decrease and you would moving this particular manner. Ultimately when you reach to this point, there is no longer any advantage in being dove type.

Similarly, if you start here where p is fairly low in that case what happens, hawk type does much better than the dove type. In that case of course, hawk would have better fitness, they would do well, they would have better in productive success and there proportion in the population would increase and it move in this direction. So, this is the point where equilibrium would happen or with these payoffs we do not know the payoffs exactly for a particular animal population.

But, let these are the made up number in with these made up number we say that we see that 3 by 5 of the whole population is hawk type and 2 by 5 or of dove type. This depends of course, the exact value that we have obtained depends on what is the value of V and A. The idea of this fitness and reproductive success it belongs to evolutionary biology and that is we are going to apply game theory now. So, in the next module I am going to talk little bit about evolutionary biology and then we will talk about evolutionary game theory.

Thank you.