Strategy: An Introduction to Game Theory
Prof. Aditya K. Jagannatham
Department of Humanities and Social Sciences
Indian Institute of Technology, Kanpur

Lecture - 42

Hello everyone, welcome to another module in this online course Strategy, An Introduction to Game Theory. So, we are looking at auctions and we have been modeling auction as Bayesian games and we have, so far looked at a first price auction of Bayesian first price auctions, where the valuations are distributed uniformly. We have derived the Nash equilibrium of this first price auction and we also derived the expected revenue of this first price auction. What we are going to look at today is another auction format, an interesting auction format termed as the second price auction.

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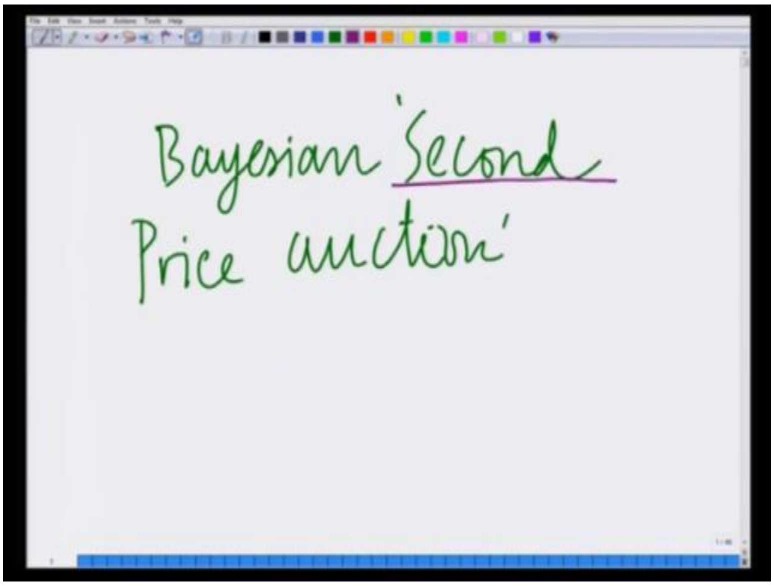

So, we are going to look at Bayesian a second price auction, so what we are going to look at is a second price auction and this is a keyword second. So, this is a second price auction, so we have looked so far at a first price auction, what we are going to look at today is a second price auction. Now, what is a second price auction? Again, let us consider auction with two players that is, let say we have two players or two bidders.

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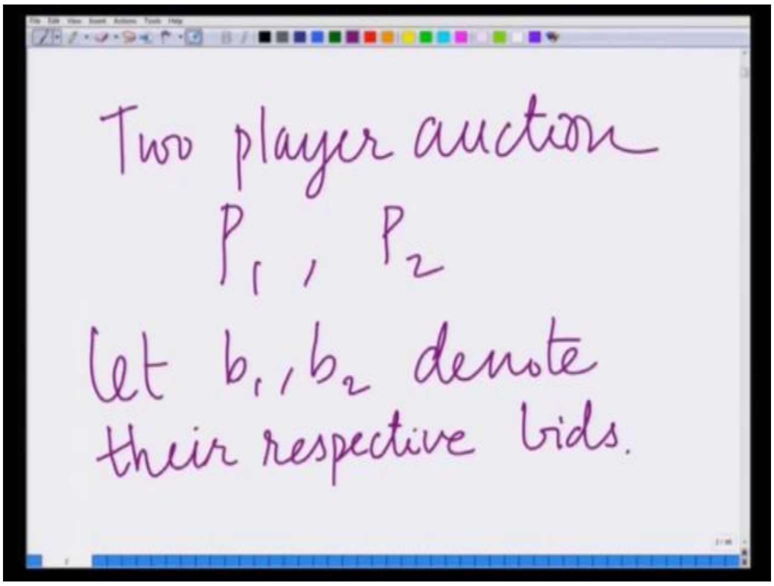

Let us consider a two player auction with the players denoted by P 1 comma P 2, these are the players. And, let the respective bids be denoted by b 1 comma b 2, let b 1 comma b 2 denote in their respective bids, that is we have two players P 1 comma P 2, P 1 is bidding b 1 and P 2 is bidding b 2. And once again, if the player the bid b 1 is higher than or equal to b 2, then player 1 wins the auction, if the bid b 2 is greater than equal to b 1, then the player 2 wins the auction.

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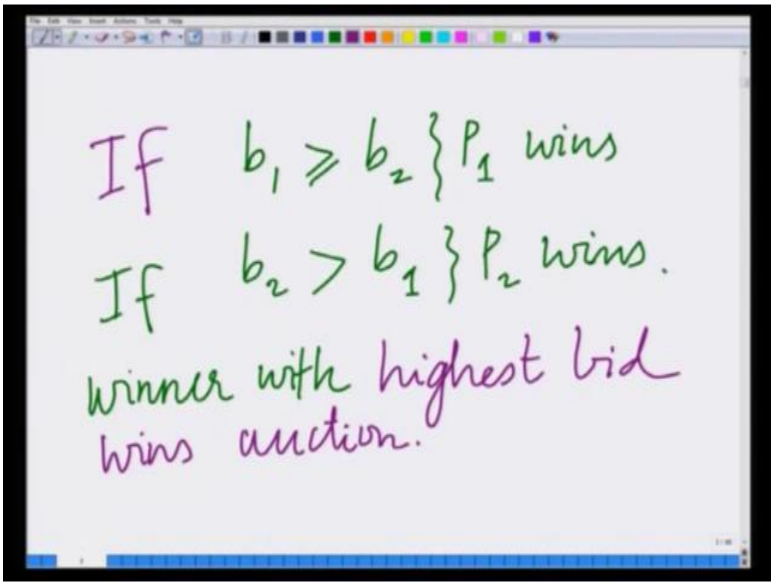

So, the winner, so if again b 1 is greater than or equal to b 2, then player P 1 wins that auction or let me write it a little bit more clearly. If b 1 greater than equal to b 2, then P 1 wins the auction; if on the other hand b 2 greater than b 1, then P 2 wins the auction. So, this part is similar to the first price auction; that is winner win the highest bid wins the auction. So, winner with the highest; however, in the first price auction the winner pays equal to his own bid; however, in the second price auction the winner pays the second highest bid. So, the winner after winning the auction pays the second highest bid, this is the second price auction.

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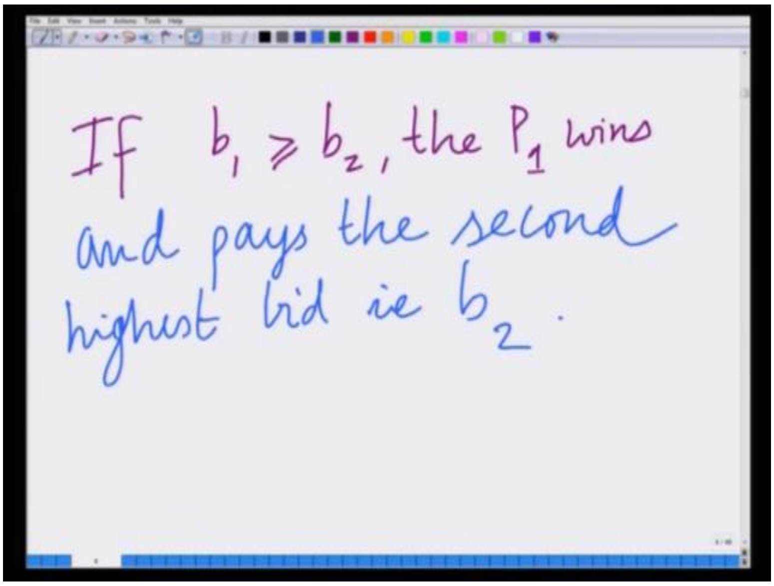

So, if b1 is greater than or equal to b 2, then, P 1 wins and he pays the second highest bid. Now, since there are only two players, we are considering the two player auction, the second highest bid belongs to that of player 2 that is b 2. So, he pays an amount equal to b 2 to get the object and pays P 1 wins and pays the second highest bid that is b 2. So, what we are saying is, if b 1 is greater than or equal to b 2 then P 1; that is player 1 was the highest bids wins and he pays the second highest bid; that is b 2 to get the object.

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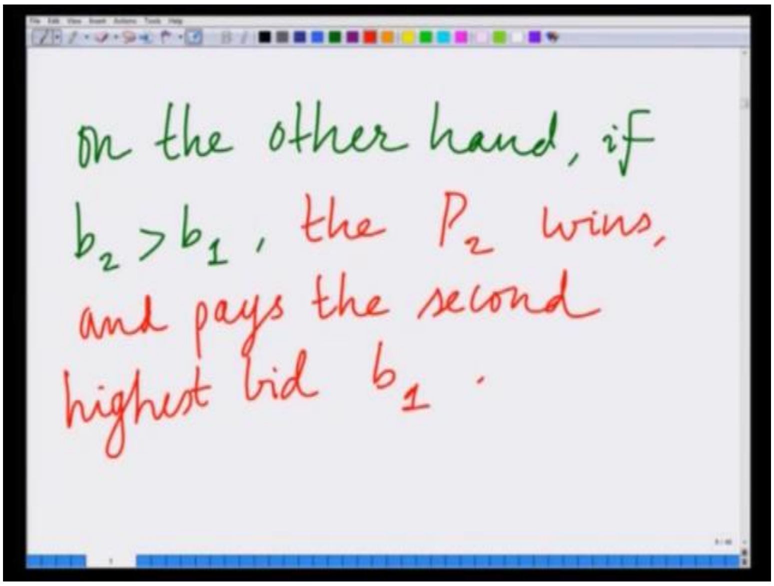

Similarly, if on the other hand, if b 2 is greater than b 1, then P 2 wins the auction and pays the second highest bid that is b 1 and pays equal to the second highest bid that is b 1. So, we are saying that this is second price auction, where the bidder or the player with the highest bid wins the auction and he pays an amount equal to the second highest bid to get the object, therefore this is a second price auction.

Also, in this auction similar to the first price auction, each user has or each player has a private valuation. Let us, denote the valuations of player 1 and player 2 by b 1 and b 2 respectively.

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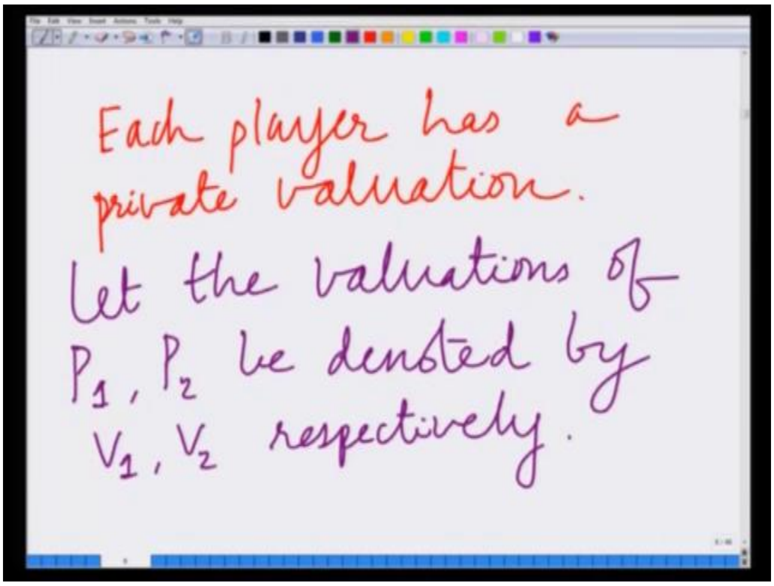

Each player has a private valuation, let us denote these valuations by V 1 and V 2 respectively of players P 1 and P 2. So, let the valuations of P 1 comma P 2 be denoted by V 1 comma V 2 respectively, so let these valuations we denoted by V 1 and V 2, for that similar to the first price auction, we are going to assume that these valuations are independent and are distributed as uniform random variables in the interval 0 to 1.

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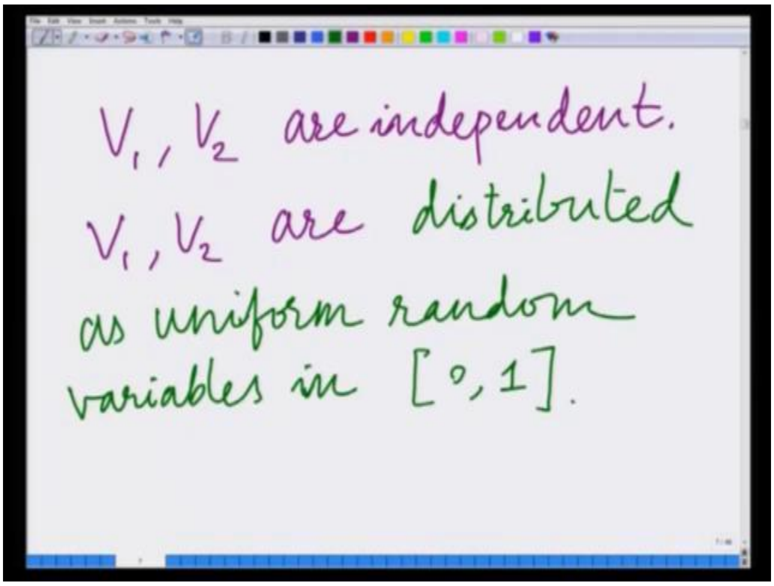

So, we are going to assume that V 1 comma V 2 are independent and V 1 comma V 2 are distributed as uniform random variables in 0 to 1. That is V 1 and V 2 are independent random variables, which are distributed uniformly in the interval 0 to 1.

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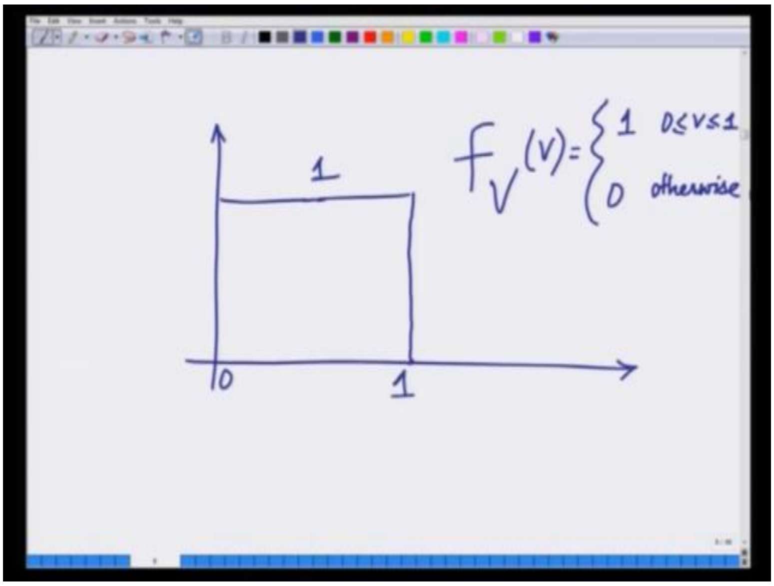

Therefore, the probability density function, you might remember the probability density function of the uniform random variable. That is, if you look at the uniform random variable in the interval 0 to 1, we have the probability density function F of V of v equals 1 if 0 less than equal to v less than equal to 1 and 0 otherwise. So, between the limits, so in the interval 0 to 1 the probability density function is 1 and outside the interval 0 to 1 it is 0 everywhere, so this is the uniform random variable between the interval 0 to 1.

So, we are saying both the valuations V 1 and V 2 are independent and they are distributed as uniform random variables in the interval 0 to 1. Now, let us again analyze this game in terms of it is Nash equilibrium, we are going to demonstrate that the Nash equilibrium of this game.

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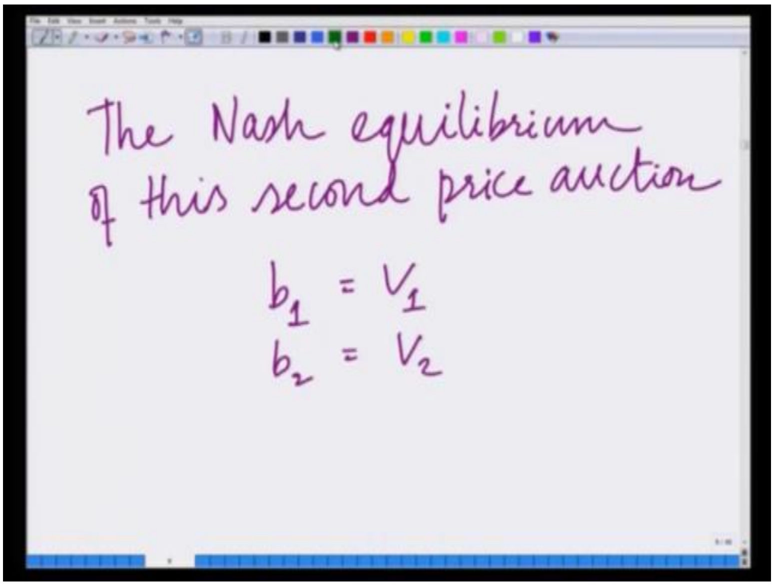

So, the Nash equilibrium of this second price auction is given as b 1 equals V 1 and b 2 equals V 2, that is what we are saying is that each player bidding his true valuation. That is b 1 equals V 1 and b 2 equals V 2 is the Nash equilibrium of this Bayesian second price auction. To analyze this, to verify that this is indeed the Nash equilibrium, let us assume that b 2 is bidding, the player 2 is bidding his true valuation; that is b 2 equals V 2 and let us try to derive the best response of player 1.

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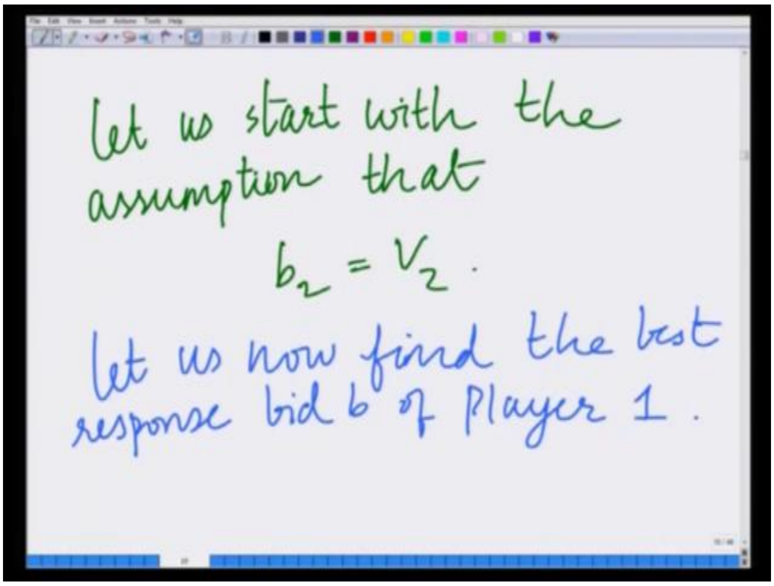

So, let us start with the assumption to demonstrate that this is indeed the Nash equilibrium. Let us start with the assumption that b 2 equals V 2 that is player 2 is bidding his true valuation b 2 equals V 2. What is, let us find the best response of player 1, let us find the best response bid b of player 1. Let us, now find the best response bid b of player 1, for this let us consider two scenarios, let us assume first, that the valuation V 1 is greater than or equal to V 2.

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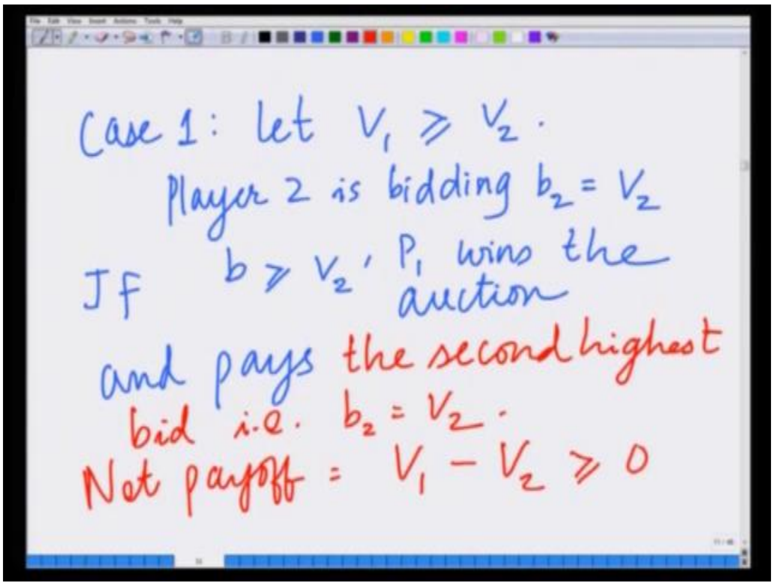

Let us, consider the first case, case 1 let V 1 v greater than or equal to V 2. Now, if V 1 is greater than or equal to; that is the valuation of player 1 is greater than equal to V 2, let us try to find the best response bid b of player 1. Now, if b remember player 2 is bidding b 2 equals V 2, now therefore, if player 1 bids any b greater than or equal to b 2 that is any v greater than equal to V 2 he wins the auction. So, if b greater than equal to V 2, then he wins he wins player 1 wins the auction.

Then, player 1 wins the auction and this payoff is and is payoff is remember once he wins the auction his paying the second highest bid, but the second highest bid is V 2. So, is payoff is valuation V 1 minus the amount he pays that is V 2, so he wins the auction and pays the second highest bid; that is b 2 equals V 2, so he is paying an amount equal to V 2. Therefore, is payoff is his is pay of his is, basically the valuation minus the amount his paying. So, net payoff equals valuation V 1 minus among his pay V 2, which is greater than or equal to 0, because V 1 is greater than or equal to greater than or equal to V 2.

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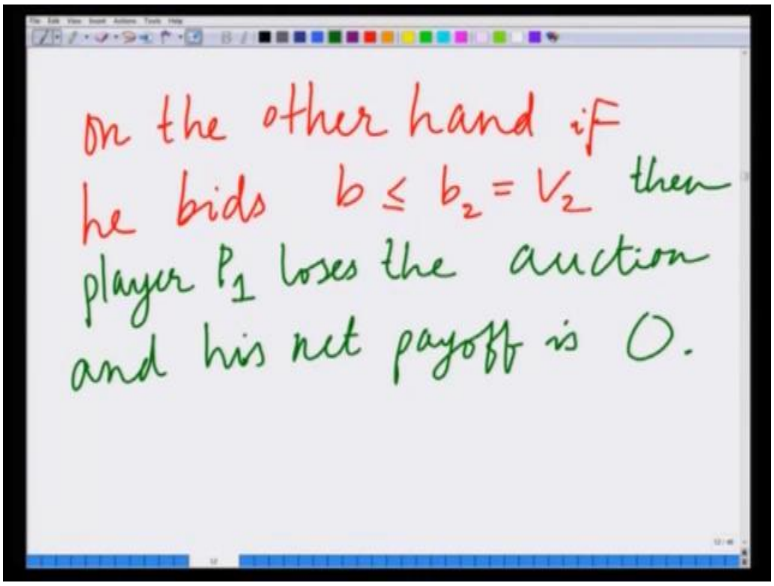

On the other hand, if he bids b less than equal to b 2 equals V 2, then he loses the auction, then player 1; that is player P 1 loses and his net payoff is 0. Since, if he loses the auction is net payoff is 0. Therefore we have two cases, one if the bid b is greater than or equal to b 2 equals V 2 wins the auction and his payoff is well is valuation V 1 minus V 2, which is greater than or equal to 0 on the other hand, if b is less than V 2. That is b is less than b 2 equals V 2 which is the bid of player 2.

Then, he loses the auction and if he loses the auction there is not pay anything he does not get anything, therefore is net payoff is 0. And therefore, you can clearly see if he bids any bid b greater than equal to V 2 his payoff is V 1 minus V 2, which is positive.

Therefore, any bid b greater than equal to V 2 is a best response in particular V 1, which is greater than equal to V 2 is a best response bid.

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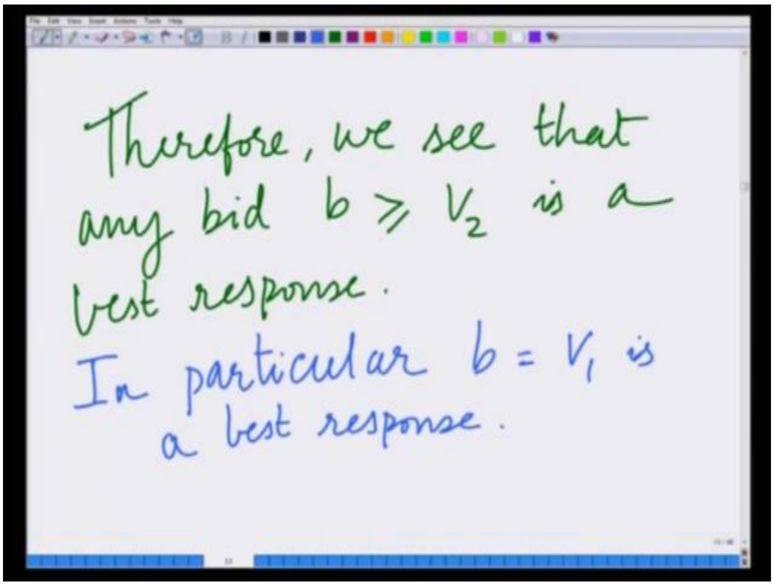

So, we see any bid, therefore we see that any bid b greater than equal to V 2 is a best response in particular b is equal to V 1 is a best response. So, what we are seen is that, if the valuation V 1 of player 1 is greater than or equal to V 2 the valuation of player 2 and player 2 is bidding b 2 equals V 2, then b equals V 1 or b 1 equals V 1 is a best response bid of player 1. So, we have consider the game successfully analyze the case when V 1 is greater than or equal to V 2.

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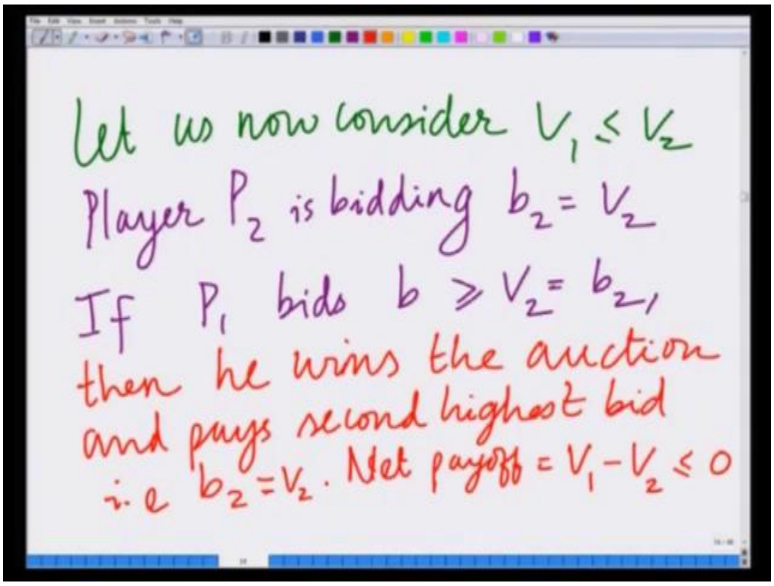

Let us, now consider the other case, where V 1 is less than or equal to V 2. And player P 2 is bidding b 2 equals V 2. Now, if player 1 bids b greater than equal to V 1, if P 1 bids any b greater than or equal to b 2 equals V 2, then he wins the auction and he pays the second highest bid, which is b 2. And therefore, net that is b 2 equals V 2 therefore, net payoff is his valuation V 1 minus the amount pay, which is V 2, which is less than equal to 0.

Because, V 1 is less than equal to V 2, if he bids any amount b greater than or equal to V 2. He wins the auction and he pays the second highest bid, the second highest bid is b 2 equals b 2. So, he pays an amount equal to V 2 and what is this net payoff is V 1 minus V 2, which is less than or equal to 0, because b 1 is less than or equal to V 2.

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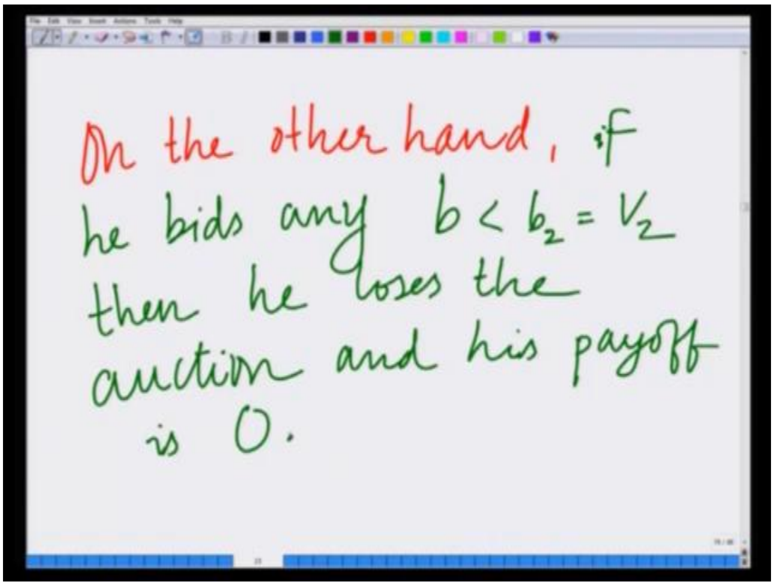

On the other hand, if he bids any b, which is less than b 2 equals V 2, then he loses the auction, and his payoff and his net payoff is 0. So, if b less than b 2 that if is bids any bid b, which is less than b 2 equal to V 2, which is the bid of the second player; that is player 2 he loses the auction and his net payoff is 0. So, bids any b that is greater than or equal to b 2.

Then, he gets a negative payoff if the bids any bid b, which is less than V 2, then he gets a payoff 0; obviously, bidding any bid b, which is less than b 2 is best response in particular, since V 1 is less than equal to V 2 bidding his valuation V 1 is best of source.

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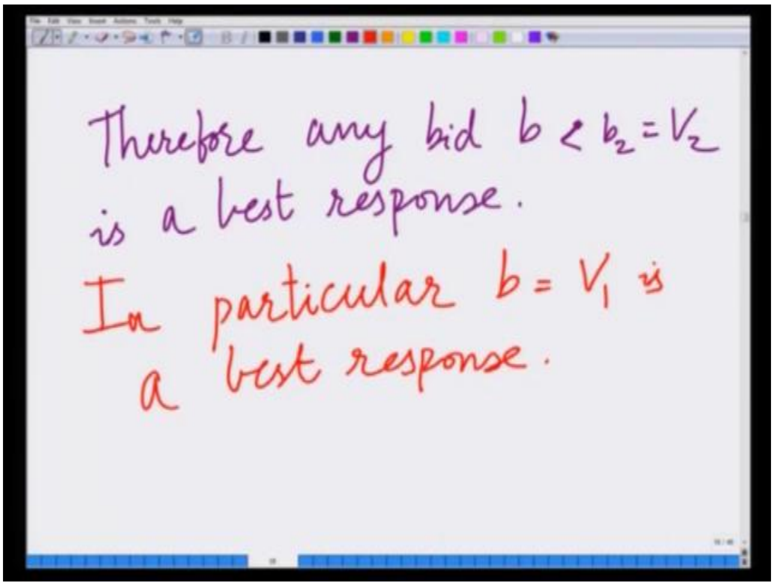

So, a bidding any bid b, therefore any bid b, which is less than b 2 equals V 2 is a best response in particular b equals V 1; that is bidding is two valuation V 1, which is less than V 2 is a best response in particular bidding b equals V 1 is a bidding b equals V 1 is a best response. So, we have consider two cases we have consider a case, where either V 1 is greater than or equal to V 2 or V 1 is less than V 2.

And, in both the cases we have shown that bidding b equals V 1 that is the bidding b equals V 1 is the best response for player 1 that is bidding is two valuation b 1 equals V 1 is the best response for player 1.

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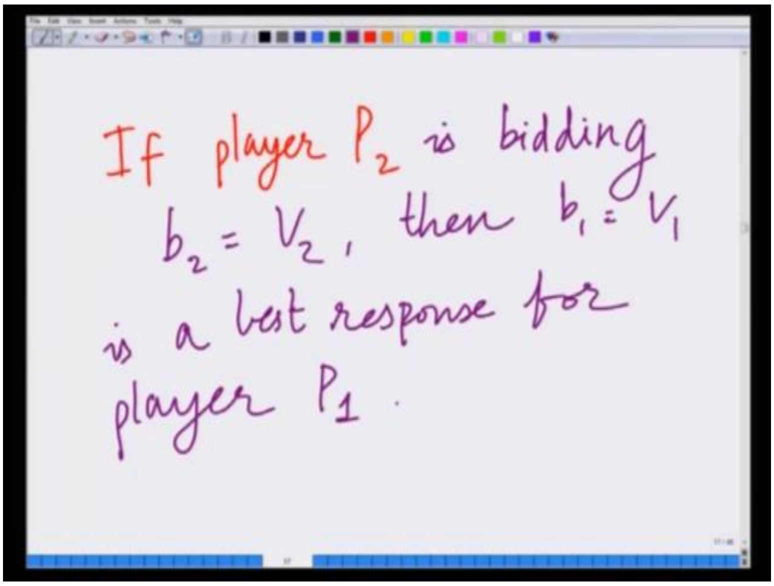

And, therefore we can conclude that basically that, if player P 2 is bidding b 2 equals V 2, then b 1 equals V 1 is a best response for b 1 equals V 1 is a best response for player p 1.

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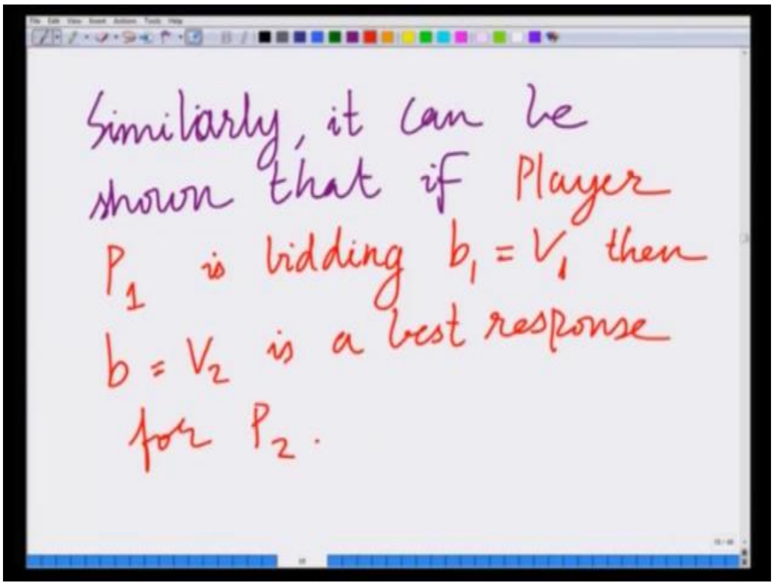

Similarly, we can repeated from the perspective of player 2 that is if player P 1 is, similarly it can be shown it can be argued that similarly it can be shown that if player P 1 is bidding b 1 equals V 1. Then, b equals V 2 is a best response for player 2 that is p 2, therefore b 1 equals V 1 constitute the best response to b 2 equals V 2 b 2 equals V 2. And, similarly when b 1 is equal to V 1 b 2 equals V 2 is a best response of player 2 therefore, b 1 equals V 1 and b 2 equals V 2 at the best responses of each of the players.

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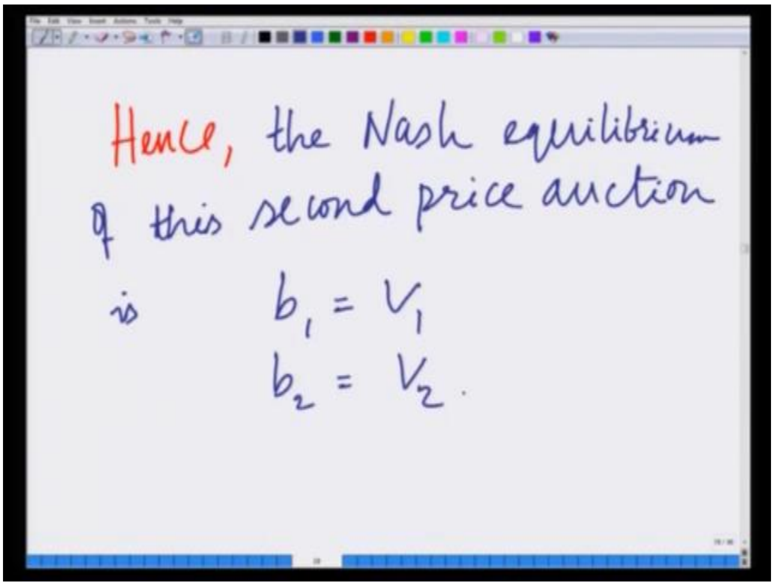

Hence, the Nash equilibrium of the second price auction is b 1 equals V 1 and b 2 equals V 2. Hence and what is important here is to realize that each player is bidding his true valuation; that is player 1 is bidding is two valuation V 1 and player 2 is bidding is two valuation V 2, therefore each player bidding his true valuation is the Nash equilibrium of the second price auction.

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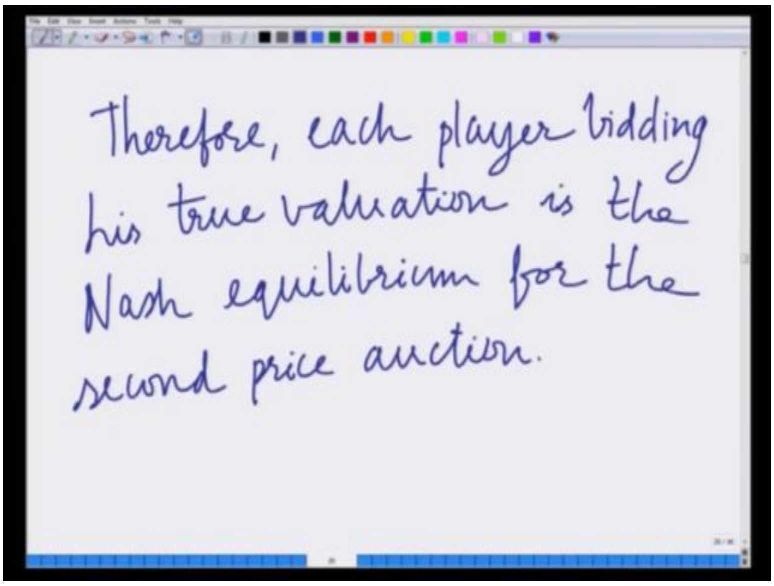

Therefore, we have show that, therefore, each player bidding his true valuation is the Nash equilibrium for the second price auction. To summarize b 1 equals V 1 and V 2 equals V 2 concludes the Nash equilibrium for this Bayesian second price auction and in the next module; we are going to derive the expected revenue to the auction here for this Bayesian second price auction.

Thank you.