Strategy: An Introduction to Game Theory
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Lecture – 04

Welcome to another module in this online course Strategy, An Introduction to Game Theory. So, in the last module we looked at an interesting example of a game, we looked at the prisoners dilemma and we analyzed the prisoners dilemma, first we look at the best responses. We looked at what is known as the best responses dynamic of the game and from that, we derived a very interesting outcome of the game which is the confess outcome, which we also termed as the Nash equilibrium of the game.

And now to continue to an answer or understanding of this concept, let us try to look at some other games which are similar to this prisoners dilemma game, to see how this game of the prisoners dilemma is applicable in practice. Let us now look at a simple example of two competing firms or two competing vendors or two competing online retail stores, which I am trying to compete by setting either high or low prices. So, and we have already looked at this example, before let us now try to analyze it.

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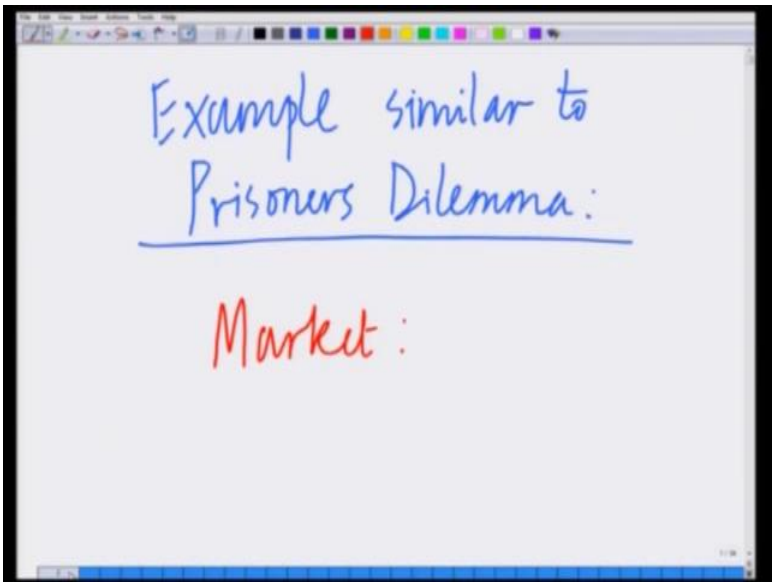

So, let us look at another example similar to prisoners dilemma, let us try to understand a real life application of this prisoners dilemma, but trying to look at a market example. Let us try to look at a simple example of a market, well we have already looked at this game before that there are two competing firms or there are two competing stores or

there are two competing chains of stores for instance.

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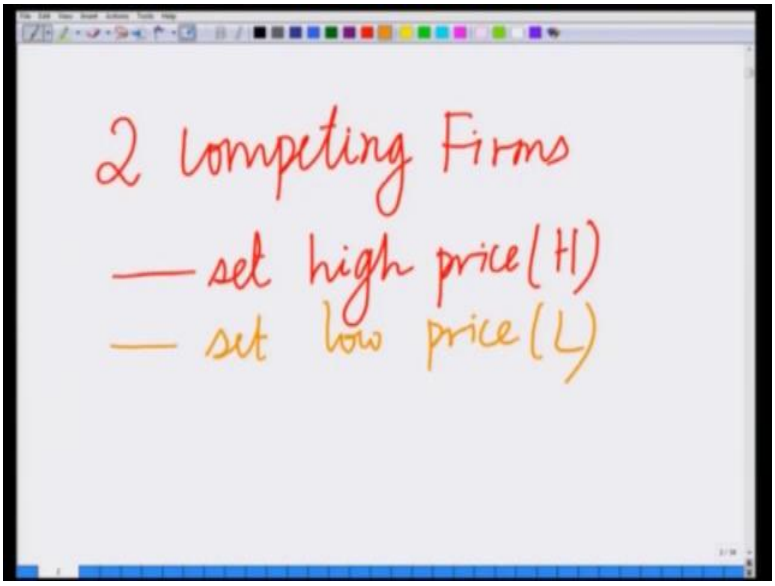

So, there are two competing firms, you have two possible auctions to either set a high price or to set a low price. Both can set a high price H or both can set a low price, both can either set a high price H or both can either set a low price low and now, if both set a high price both get high profits, which are let say both get 500 each.

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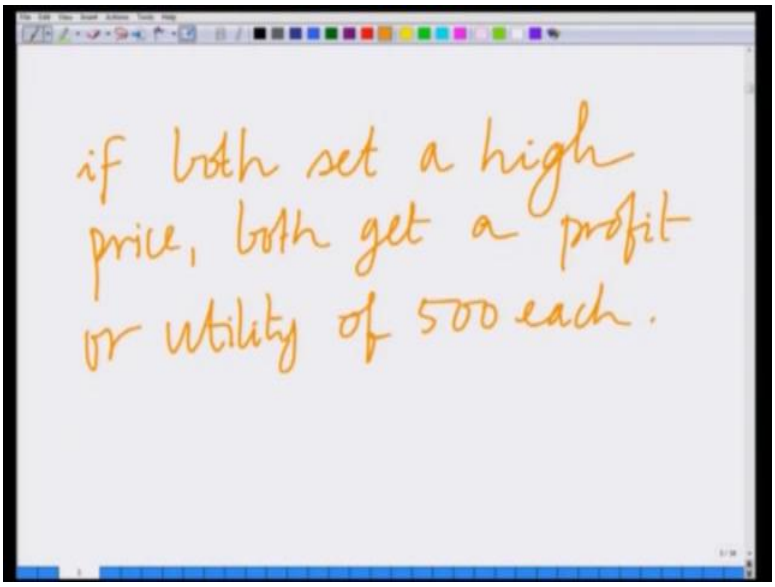

So, if both set a high price, both get a profit or utility of 500 each.

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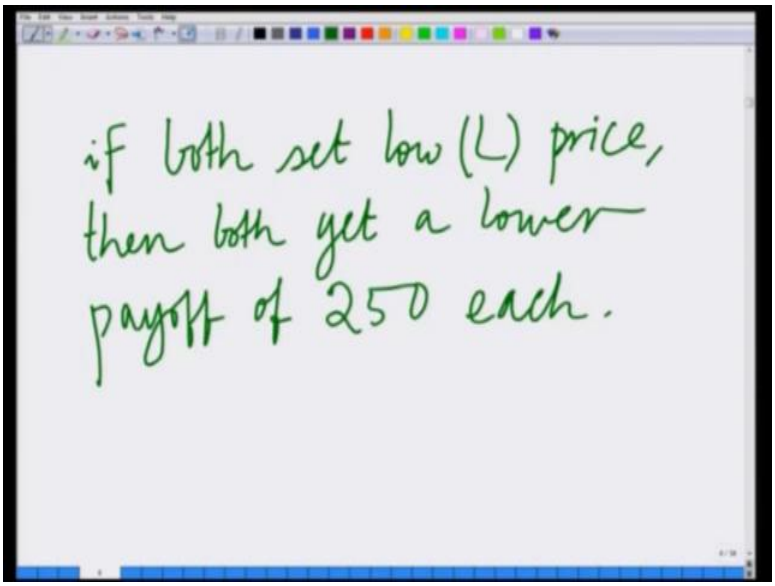

However, if both choose to play if both set a low price, that is if both choose the auction L then both get lower profit over a lower payoff of 250 each, then both get a lower payoff of 250. So, if both set a high price both get 500 a payoff of 500 each, both set a low price they get a payoff of 50 each. However, something interesting happens, when one sets a high price while the other sets a low price, another shop or the firm which sets a low price for it is good, attracts all the market therefore has captured the entire market.

And therefore, he is able to manage a profit a higher profit of 750, while the firm which sets the high price does not capture any market. Because, they are going to ((Refer Time: 04:40)) other firm which have set a low price therefore, since he does not capture any market share, since he does not have any customers, it is going to end up with 0 let us say.

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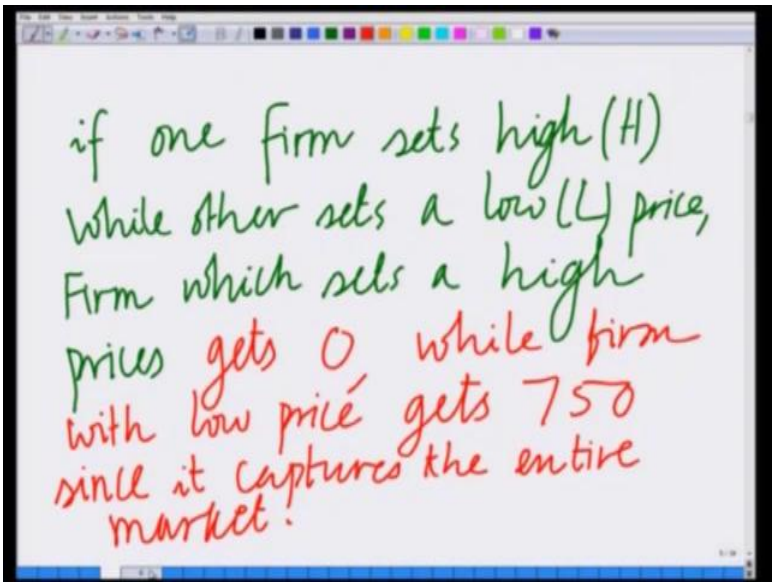

So, if one firm or store sets high H, while other sets a low L price; firm which sets a high prices gets 0, while firm with low price gets 750. Since, it is able to capture the entire market away from the other firm, since it captures the entire market share. So, that is what we are saying. So, we are saying that we are trying to model a simple market place, competition between two shops or two retails stores or two change stores which are trying to sell the same good by trying to, but and they have two auctions to choose from either both, either they can set a high price or they can set a low price.

If they both set a high price and they get higher profits of 500 each, if they set, both set a low price both get a lower profit of 250 each. But, if one sets a high price and the other sets a low price, the once which sets a low price captures the entire market. So, it is able to get profit of 750, while the one which sets higher price does not capture any of the market and because it does not capture any of the market, it has a net payoff for utility of 0. So, now let us again try to formulate, I have just said the game table is a convenient way to set of comprehensively summarize this game, comprehensively denote the payoffs of this game and analyze this game.

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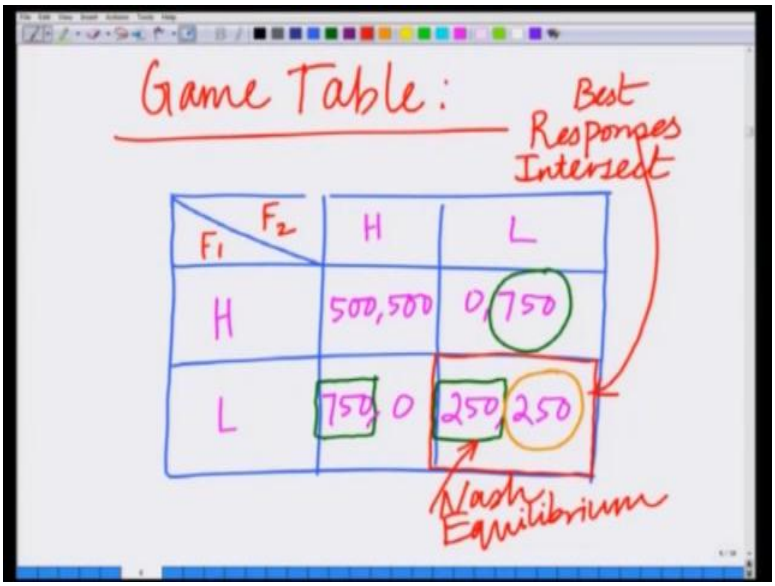

So, let us try to draw the game table for this game, for the simple market game, for the simple market competition between two firms or two online retail stores. And therefore, what we said again is that this game table it is going to have, well we are going to have a row player and we are going to have a column player. And let us denote the row player by firm 1, the column player by firm 2, firm 1 that is the row player can either choose high price or a low price, firm 2 can either choose a high price or low price.

So, as we said the auctions of the row player are listed along the rows, auctions of the column player are listed along the columns. And if both firm 1 and firm 2 choose high price, then as we said before they are going to get a payoff of 500 each, but on the other hand if both choose a lower price, a lower both choose to set a lower price then both get a payoff of 250 each. But, a firm 1 chooses a high price, firm 2 chooses a low price then firm 1 get 0, firm 2 gets a payoff of 750.

On the other hand if firm 1 chooses a low price, firm 2 chooses a high price then firm 1 gets a payoff of 750, firm 2 gets a payoff of 0. So, this is the game table which comprehensively summarizes the game scenario over this strategic interaction between these two firms in the market. Now, again remember the way to analyze the game is to look at the best response dynamic. So, let us start by looking at the best responses that each of these firms can play.

For instance, now again to find the best response we have to restrict one of the firms to one of it is auctions. So, let say firm 2 chooses the auction high that is, it chooses to set

high price, then choosing high yields 500 for firm 1, while choosing a low yields 750. So, naturally low is the best response of firm 1 therefore, firm 1 chooses low if firm 2 sets high price.

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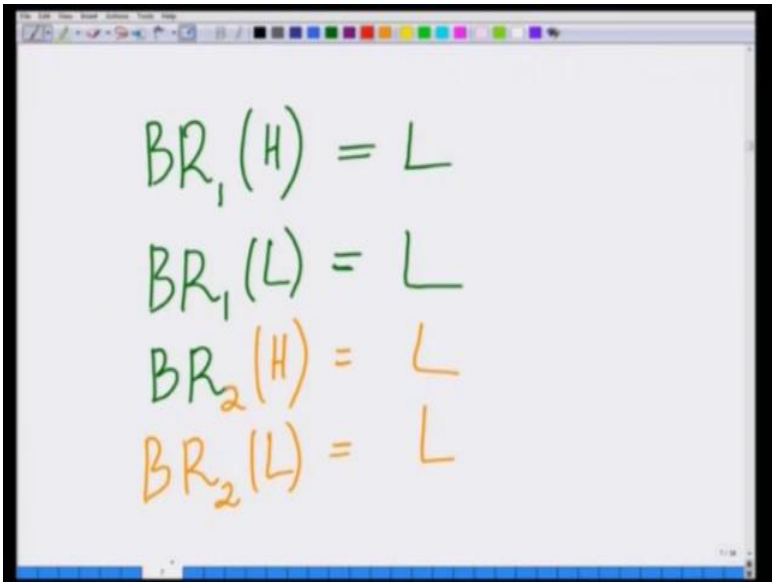

And remember the way we mentioned it is by saying, the best response of firm 1 for the auction H of firm 2 is to choose low ((Refer Time: 10:04)). Similarly, on the other hand if firm 2 chooses to set a low price then we are restricted to the second column, setting a high price gives 0 to firm 1 while setting a low price yields 250. So, firm 1 is better off setting a low price of 250, so setting a low price and getting 250 and therefore, the best response of firm 1 is to choose ((Refer Time: 10:29)).

And just we can also write this as best response of firm 1 to auction low of firm 2 is to choose low ((Refer Time: 10:41)). So, firm 1 if firm 2 chooses the auction H that is it sets high price best response of firm 1 is low, if firm 2 chooses to set a low price best response of firm 1 is again low. Now, let us look at the best response of firm 2, if firm 1 chooses to set high price that is it is restricting the game to the first row. Then, the best response of firm 2 is again to set a low price, because low yields it is 750, whereas high yields at 500. So, firm 2 chooses low which gives it a payoff of 750.

So, best response of firm 2 to the auction H of firm 1 is low ((Refer Time: 11:28)) and again if firm 1 chooses to set a low price, then low for firm 2 yields 250 while high yields it 0 therefore, firm 2 chooses to set a low price which gives it 250. Therefore, best response of firm 2 to low of firm 1 is again low and that completes the best response

dynamic for this game ((Refer Time: 11:59)).

And now once again if you go back you can look at this table, there is one box or one outcome which is special, where the best responses are intersecting and that is the L L box, if you can look at this carefully again you can see that basically there is a box where the best responses are intersecting, this is the box where the best responses intersect. Therefore, this is the Nash equilibrium of this game, we said what is the Nash equilibrium.

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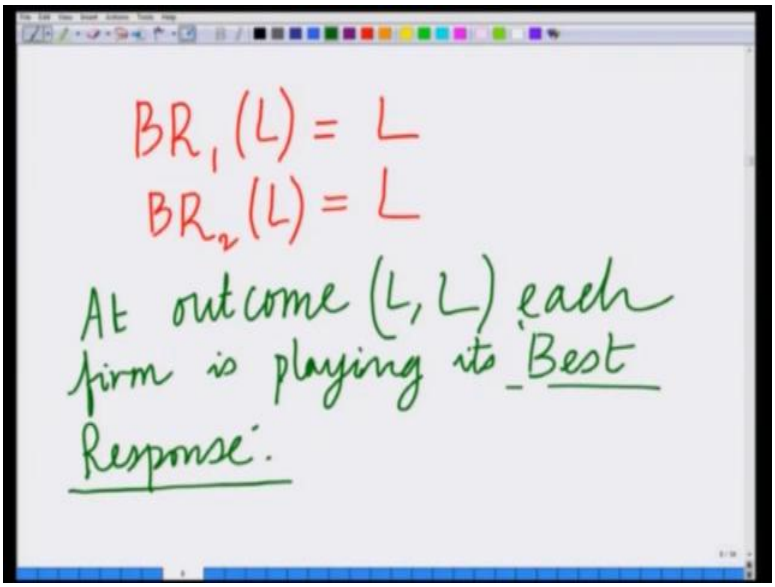

If you look at the L L outcome, the best responses intersect that is the best response of firm 1 to L of firm 2 equals L and the best response of firm 2 to L of firm 1 is L. So, at outcome L comma L, each firm or each online retail store or each shop is playing it is best response.

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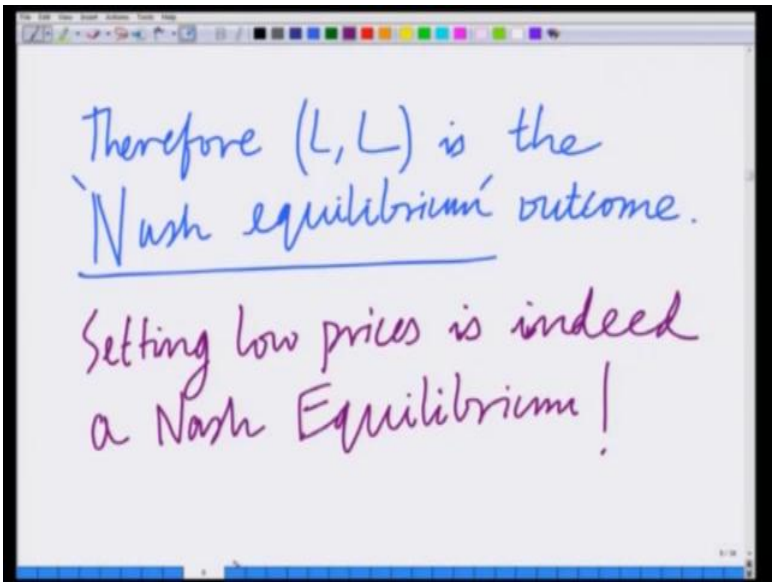

So, that is where the best responses intersect and therefore, L comma L is the Nash and therefore, L comma L is the Nash equilibrium outcome. What is the Nash equilibrium outcome? Remember, we said the Nash equilibrium outcome is where the best responses intersect. So, when both of them firm 1 and firm 2 are playing the best response each is playing the best response to the other, firm 1 choice of L is it is best response to L of firm 2 and similarly L of firm 2 is the best response to L of firm 1 and since the best responses are intersecting therefore, this is a Nash equilibrium.

And this simple example involving two players in the market, shows as why different competitors gravitate towards low prices in a market place. This simple example shows as that setting low prices is indeed such setting low prices is indeed the Nash equilibrium in a market competition game. So, this simple example shows as that low prices is indeed a Nash equilibrium and that is the reason why when you have different firms or different shops competing the tent to gravitate towards lower prices and we they tent gravitated towards low prices that is better for the consumer.

Because, you have these goods at low prices and that is how a free market essentially. So, this sort of ((Refer Time: 16:04)) basic rudimentary example shows as how a free market words by illustrating how in a Nash equilibrium, this different competing firms or these different competing shops are trying to set low prices to attract to capture the market share to capture as much of the market share as possible and thereby increase are there profit, this is similar to the kind of competition that is the now going on between different online retails, stores trying to capture as much market share as possible by

setting no prices for similar goods. And we also said Nash equilibrium is indeed that are some as several other properties of the Nash equilibrium.

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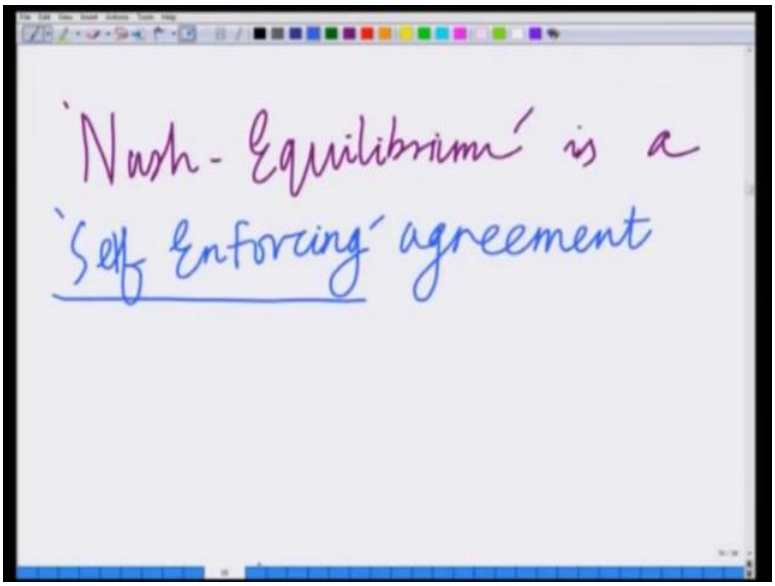

For instance, we also said that the Nash equilibrium is a self enforcing is we also said that the Nash equilibrium is a self enforcing agreement. What does it mean to say that the Nash equilibrium is a self enforcing agreement, for instance let us go back to again, let us look L comma L, if both of them are let us redraw the game again to basically sort of...

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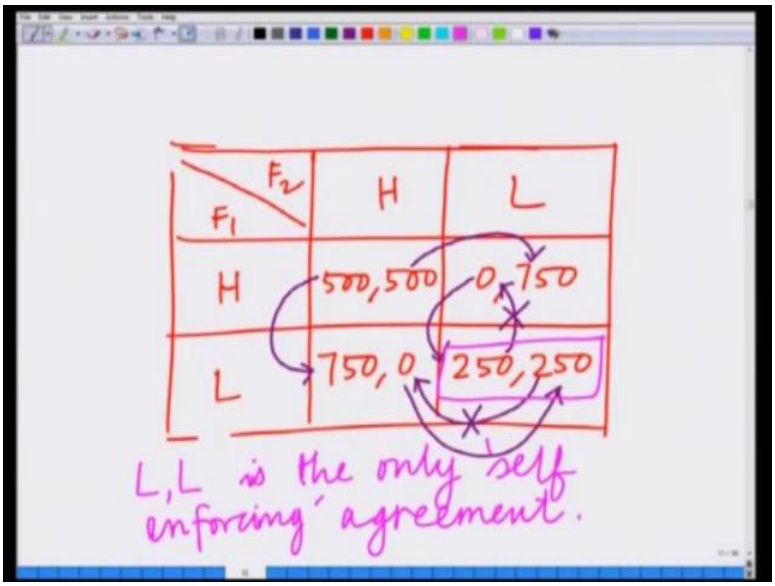

So, as that I can clearly illustrate the concept of a self enforcing agreement, we have two

firms firm 1 and firm 2 both can set either high or low prices and high comma high gives 500 comma 500 high comma low gives 0 comma 750, low comma high as 750 comma 0, low comma low gives 250 comma 250. Now, you can see if both of them let say agree prior to the game that they are going to set high price are they going to strict to that agreement well no.

Because, firm 1 can deviate from setting a high price to the low price captured the entire market, thereby increasing which market share at profit to 750. Similarly, firm 2 has an incentive to deviate and increase it is profit from 500 to 750. Similarly, if they both agree on H comma L you can see that basically firm 1 has an instinctive to deviate to set a low price. Similarly, if both of them agree on L coma H you can see that in this scenario firm 2 has an instinctive to deviate.

Only in the outcome if they agree on the outcome L comma L no one has an instinctive to deviate, because if firm 1 deviates it is going to decrease it is payoff from 250 to 0. So, that is not going to be possible and similarly if firm 2 deviates from L to H it is going to decrease it is profit firm 250 to 0, so that is not going to be possible. So, you can clearly see L comma L is the only self enforcing agreement and that is important, because in an market place firms f 1 and f 2 cannot collude to artificially set high prices that is illegal that is basically termed as a collusion and that is there are several loss which prevent collusion are known as anti collusion loss.

Therefore, in the absence of such anti collusion laws they cannot have a formal agreement aspects high prices at informally also they cannot set high prices, because as you seen that is not a self enforcing agreement. Therefore, the only possible outcome is that they set both set low prices, which is the self enforcing agreement, which is the Nash equilibrium. And therefore, all this competing different competing shops are firms in the market place tend to gravitated towards lower prices, which is good from the consumer and that is over free market economy or free market system works.

And therefore, and this is the simple example of a competition price are competition. So, this is simple example of a price war also turned as the price war between two companies or between two firms. We are going to later develop and define this competition between two firms to make this example slightly more sophisticated and slightly more practical and more closer to real life scenario of competition between two firms and also let us observe one more, let us observe another important point in this game.

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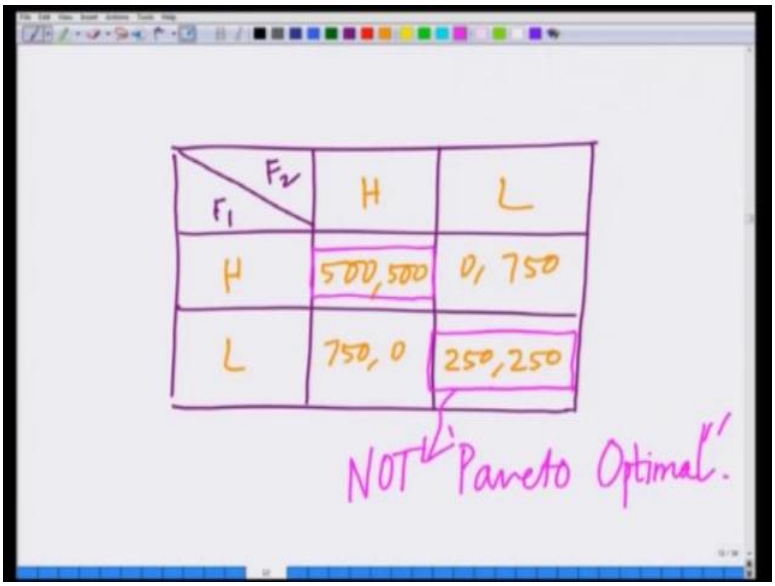

The other important aspect of this game is that let me redraw the game again. So, as to illustrate this clearly that we have two firms F 1, F 2 and both either high low, high low and you can see the Nash equilibrium outcome is L comma L when each of them is getting 250 at each. Now, in this Nash equilibrium outcome compare to there is another outcome, where both of them can simultaneously improve their profit or payoff although it is not an equilibrium outcome.

If you look H at H comma H there is an outcome H another outcome H cunningly H comma H in which both of these competing shops or both of these competing firms can simultaneously increase their payoff. Therefore, we say that such an outcome compare to in comparison in relays such an outcome is a not a Pareto optimal outcome. So, this outcome this L comma L is not Pareto optimal. What is the meaning of Pareto optimality? Given any outcome, if there is no other outcome such that both can simultaneously improve their profit or utility or payoff such an outcome is a Pareto optimal outcome.

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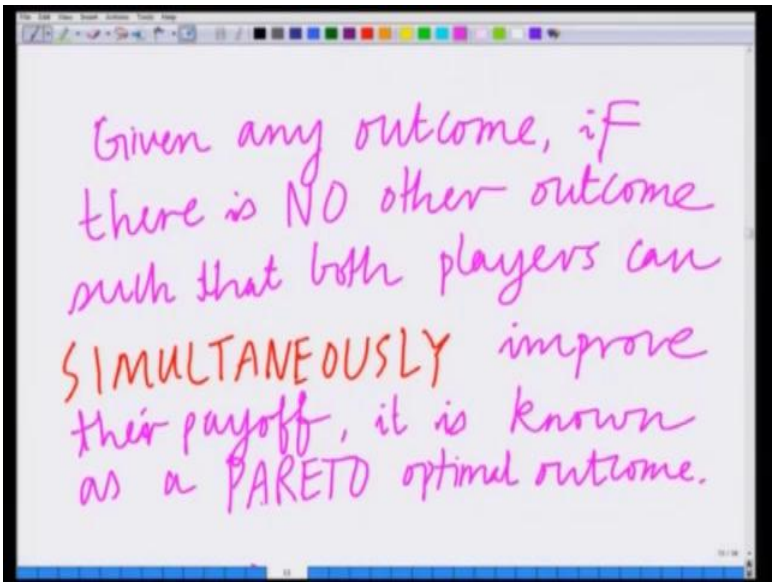

Given any outcome, if there is no other outcome such that both players or such that all players both players can simultaneously improve their payoff if there is no other outcome such that both players can simultaneously improve their payoff it is known as a Pareto optimal outcome ((Refer Time: 24:56)) it is not a Pareto optimal outcome, let us go back to the game. Let us for instance ((Refer Time: 25:07)), let us look at the game you can see that if both of them choose H comma H both of them get a payoff of 500 comma 500 they cannot choose a different outcome were both can increase the payoff.

For instance, if they choose H comma H firm 1 is decreasing it is payoff from 500 to 0, while if they choose L comma H firm 2 is decreasing which payoff from 500 to 0. So, there is a outcome H comma H is such that there is no other outcome, where both can simultaneously increase their payoff therefore, outcome H comma H is Pareto optimal. Similarly, you can argue that outcome L comma H and outcome H comma L are also Pareto optimal outcomes try to reason this out for yourself.

However, if you look at L comma L that is where both the firms are a sitting a low price, this is not a Pareto optimal outcome. Because, there is another outcome namely H comma H with both of them can simultaneously increase their payoff. For instance, if they choose H comma H instant of L comma L then firm 1 can increase it is payoff from 250 to 500 firm 2 can increase it is payoff from 250 to 500 therefore, there is another outcome through which both of them can simultaneously increase their payoff L comma L it is not Pareto optimal outcome therefore, if I have to summarize in order to summarize this thing.

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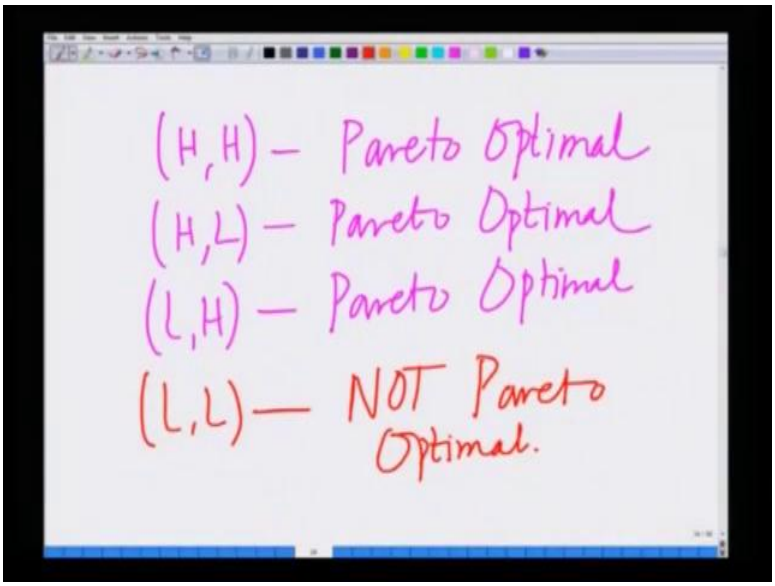

If you look at H comma H, H comma H is a Pareto optimal, H comma L is also Pareto optimal, L comma H is also Pareto optimal. The Nash equilibrium outcome L comma L is unfortunately not Pareto optimal, since there is another outcome through which both of them can simultaneously increase their payoff therefore, L comma L which is the Nash equilibrium outcome is the only outcome in the game which is not Pareto optimal.

In fact, this is characteristic of the games such as the characteristic of a prisoners dilemma game that is the Nash equilibrium of this game which is L comma L. And in fact, you can go back to the prisoners dilemma game and you can look at the Nash equilibrium outcome which is the confess outcome. And you can again argue that while all the other outcomes denied denied, confess denied, denied confess are Pareto optimal the only outcome that is confess confess which is the Nash equilibrium outcome is not Pareto optimal, so this is the characteristic of the prisoners dilemma game.

So, this is not a Pareto optimal outcome, why is at not Pareto optimal again, because if you look at it in this market example both of them can simultaneously increase from L comma L by choosing H comma H both of them can simultaneously increase their payoff from 250 to 500 for both of them. Of course, as we show earlier it is not a self enforcing agreement, you although H comma H is a Pareto efficient or a Pareto optimal outcome as we it seen earlier it is not a Nash equilibrium, because from H comma H both the players have an incentive to deviate.

So, it is very interesting how in these strategic games, because of these competing

behavior, because of the sense of competition between these different agents or these different firms or these different job they end up ultimately in outcomes which are not necessarily good for either of them. That does not mean that it is a bad outcome for instance, when such as we have seen in this example with these firms are competing the prices tend to be low which good for the consumers ultimately.

However, it just means for both of them it is not a Pareto optimal or a Pareto efficient outcome. So, that is an important concept in that we will learn today which is Pareto optimality, think about that and Pareto optimality is also another important to way to understand and analyze different games that is after Nash equilibrium one can also ask the question, if the equilibrium outcome of these games is a Pareto optimal outcome. And sometimes it can be a Pareto optimal outcome, although we have not seen an example.

We are going to see some examples of such games in the feature, but we are saying that in the prisoners dilemma and game such is the market example, which are similar to the prisoners dilemma, the net equilibrium outcome and the Nash equilibrium outcome is not a Pareto optimal outcome. So, let us end this module here thank you very much and will continue in the next module later.

Thank you.