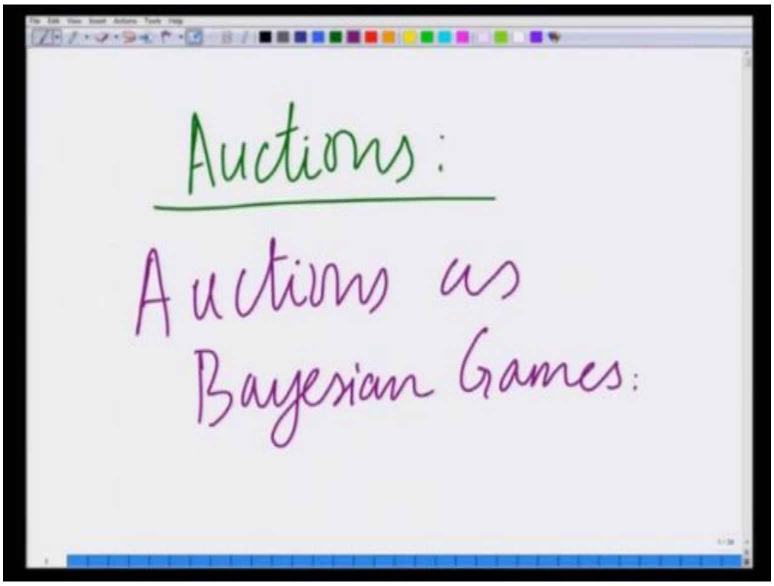
Strategy: An Introduction to Game Theory
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institution of Technology, Kanpur

Lecture – 39

Hello, welcome to another module in this online course Strategy, An Introduction to Game Theory. So, today we are going to start looking at auctions.

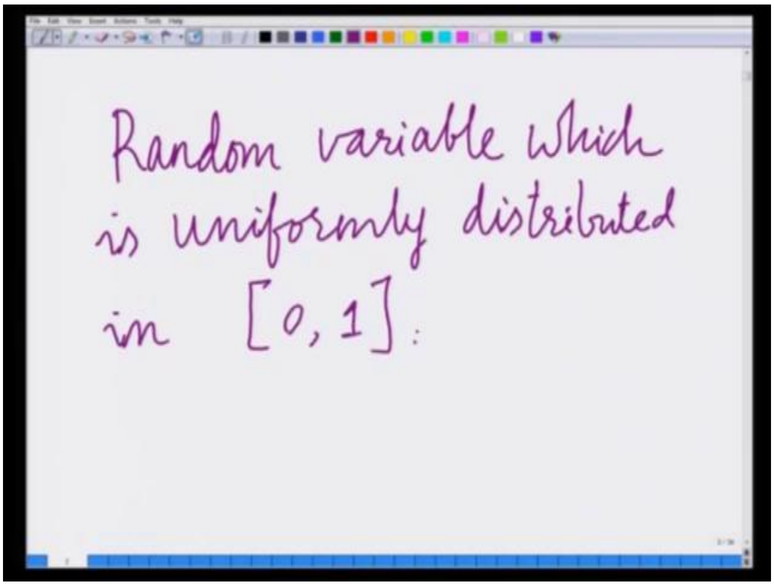
(Refer Slide Time: 00:20)



So, in this module we are going to start looking at auctions, which are another important types of games and we are going to study them using the tools of Bayesian game theory. So, we are going to start modeling auctions as Bayesian games that is games which involve as uncertainty. So, we are going to start looking at auctions as Bayesian games that is, games were uncertainty and throughout our discussions auctions, we are going to focus on probabilities and random variables, etcetera.

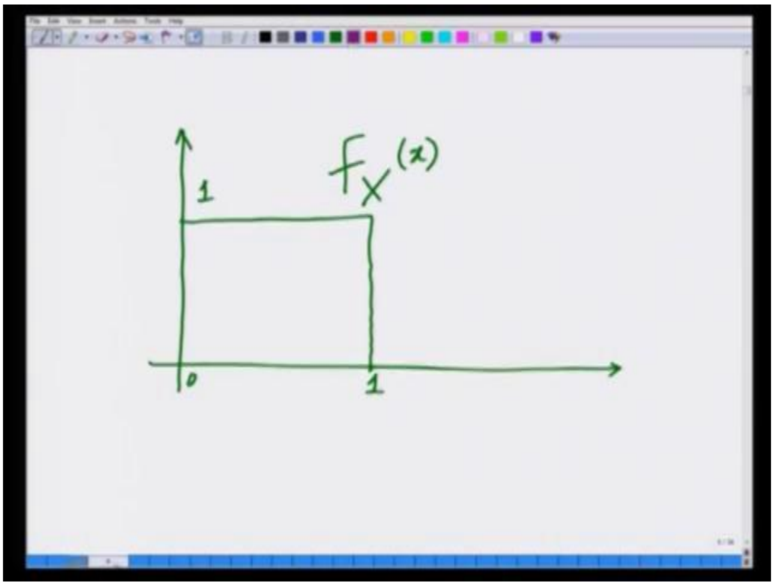
In particular we are always going to use the uniform random variable. Therefore, it is important to understand the properties of the uniformly distributed random variables.

(Refer Slide Time: 01:21)



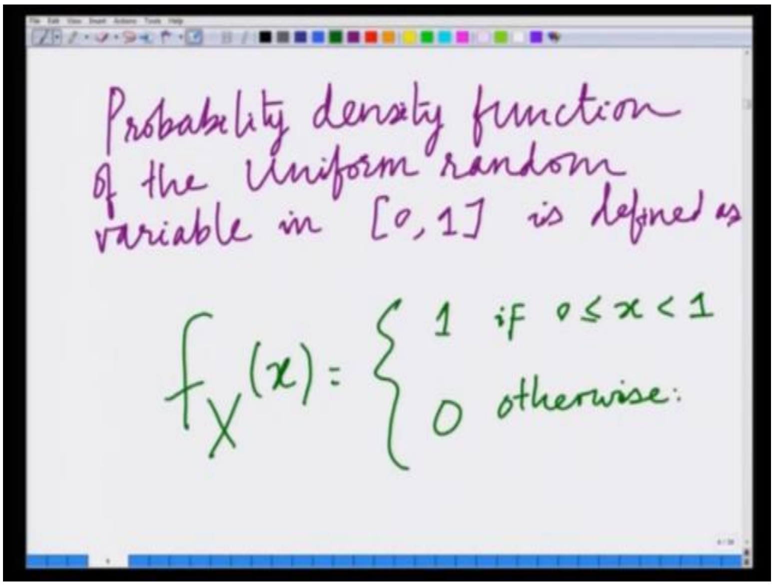
So, throughout our discussion we are going to use the following random variable which is uniformly distributed that is, the random variable which is uniformly in 0 comma 1. So, throughout our discussion on this Bayesian auctions, we are going to consider the random variable which is uniformly distributed in 0 comma 1 and it has a probability density function which is given as follows.

(Refer Slide Time: 02:03)



The probability density function of the random variable, which is distributed uniformly in 0 comma 1 is given as the density f of x. This is the random variable which is uniformly distributed between 0 and 1, it has the density f of x of x which is 1 between 0 and 1 and 0 otherwise.

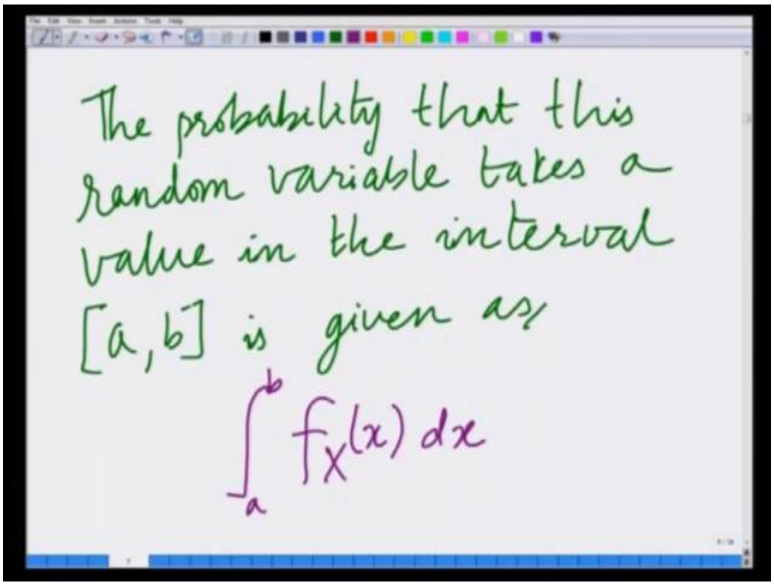
(Refer Slide Time: 02:37)



So, the probability density of the uniform random variable in the probability density function of the uniform random variable in 0 comma 1 is defined as f of x, 0 less than or equal to x less than 1 and it is 0 everywhere else. So, the uniform random variable between 0 and 1 has a probability density function f of x of x which is equal to 1 if the random variable x lies between 0 and 1 and it is 0 elsewhere. And this ((Refer Time: 03:45)) we have already plotted we are showed here a plot of the probability density function of the random of the uniform random variable between 0 and 1.

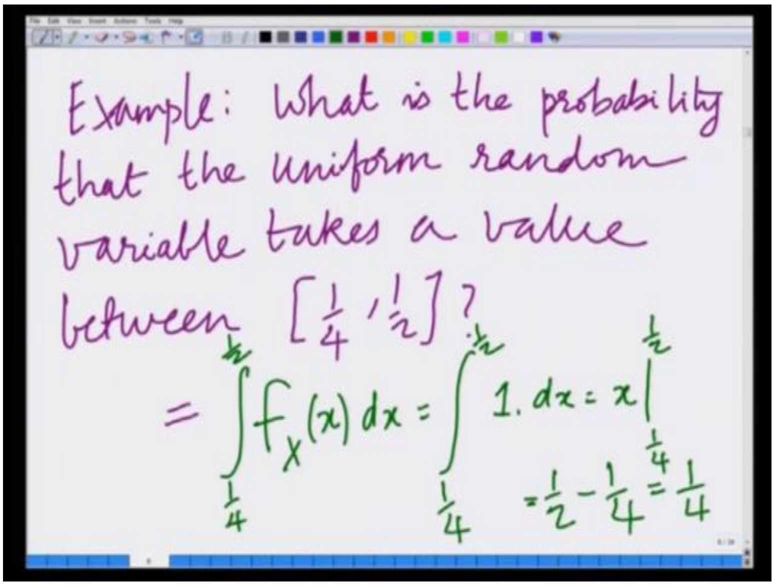
So, and this random variable which is uniformly distributed with in 0 and 1 can take any value between 0 and 1 with uniform probability, this can take any value between 0 and 1.

(Refer Slide Time: 04:18)



Further, the probability that it lies in an interval a comma b is given as the probability that this random variable takes a value in the interval a comma b is given as integral a to b f of x of x d x, that is the probability that the random variable x lies in the interval a to b is given by integral a to b f of x d x. For instance, we can ask the question what is the probability that this random variable, the uniform random variable between 0 and 1 lies in the interval 1 by 4 half.

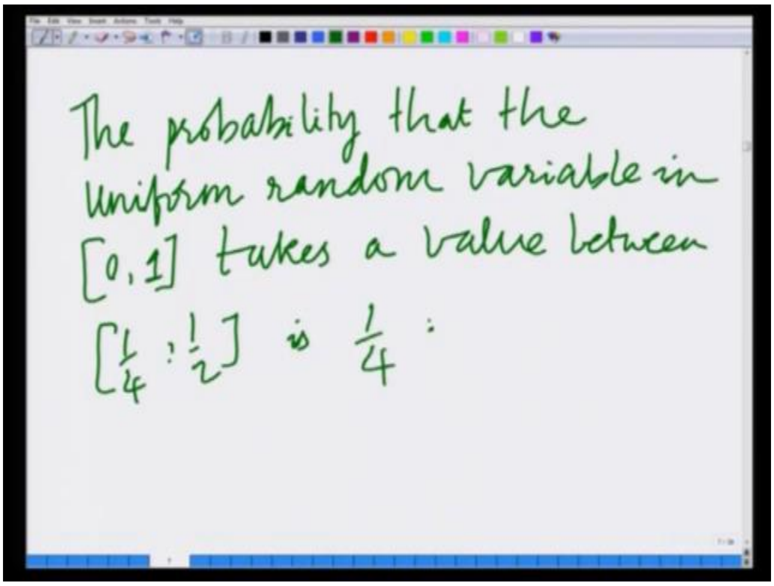
(Refer Slide Time: 05:28)



So, let us take a simple example, let us consider an example. What is the probability that the uniform random variable takes a value between 1 by 4 comma half that is what is the probability that the uniform random variable between 0 and 1 lies in the interval 1 4th to half. And this probability as we can see is given by this probability is equal to the integral between 1 by 4th and half of f of x x d x, which is equal to the integral between 1 by 4 to half remember between 1 4th and half f of x is given equal to 1.

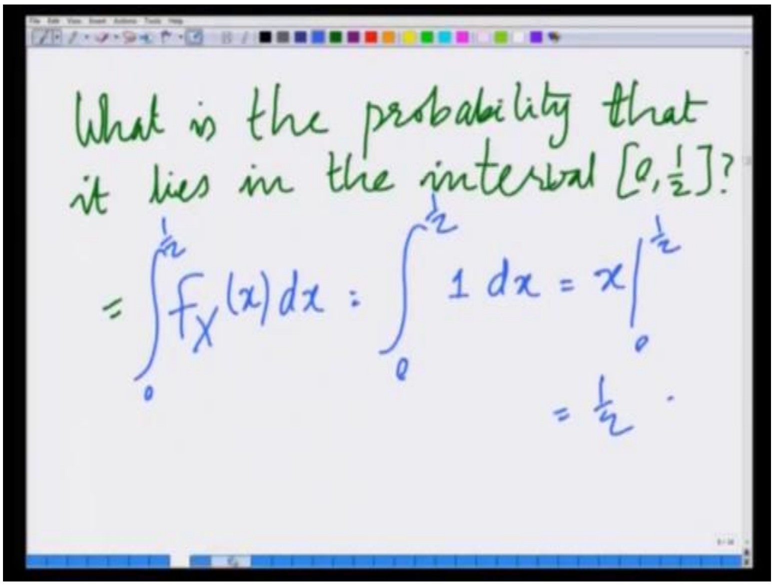
So, therefore, this is the integral 1 by 4 to half 1 times d x, which is equal to x between the evaluated between the limits 1 4th to half which is equal to half minus 1 4^{th}, which is equal to 1 4th. Therefore, the probability that this uniform random variable between 0 and 1 takes a value in the interval 1 4th to half is given by 1 4th.

(Refer Slide Time: 07:12)



Therefore to summarize the probability that the uniform random variable in 0 comma 1 takes a value between 1 4th comma half is 1 4th. So, that is the probability that it takes a value between 1 4th comma half is I 4th. So, that is the probability that it takes a value in the interval 1 4th. What is the probability, again similarly just to repeat a similar example, what is the probability that it takes a value in the interval 0 to half?

(Refer Slide Time: 08:07)

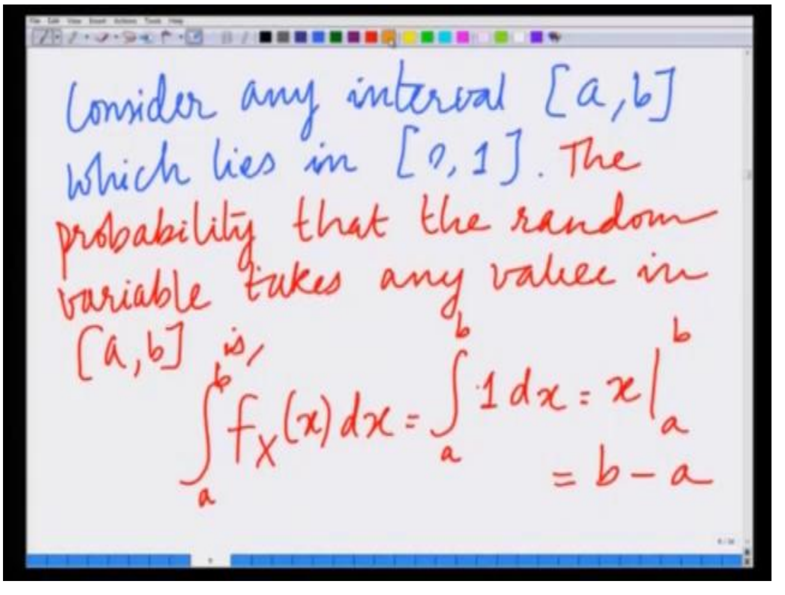


What is the probability it lies in the interval 0 to half this is again given as integral 0 to half f of x x d x which is integral 0 to half 1 d x which is x evaluated between the limits 0 to half which is equal to half. Therefore, we see that the probability it lies in the interval 0 to half is half and in fact, if you look at this probability density function ((Refer Time:

09:02)), you can said that it is uniformly distributed in 0 to half.

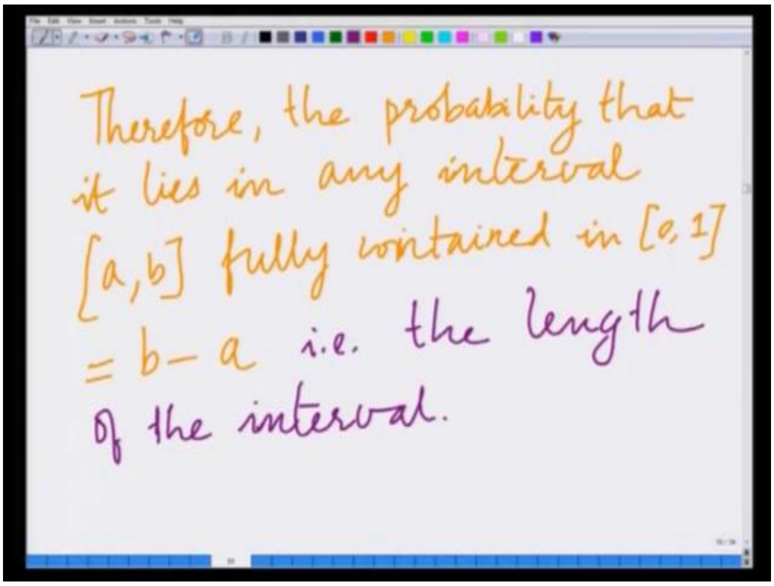
Therefore, the probability that it lies in 0 to half is identical to the probability that it lies in half to 1 and both this probabilities therefore, is equal to are equal to half. Since, the sum of the probabilities must sum to 1 therefore, the probability that it takes any value in the interval, in the first half of the interval that is 0 to half is equal to half. Further to generalize this notion you can see that the probability consider in interval a comma b which lies entirely in 0 comma 1.

(Refer Slide Time: 09:39)



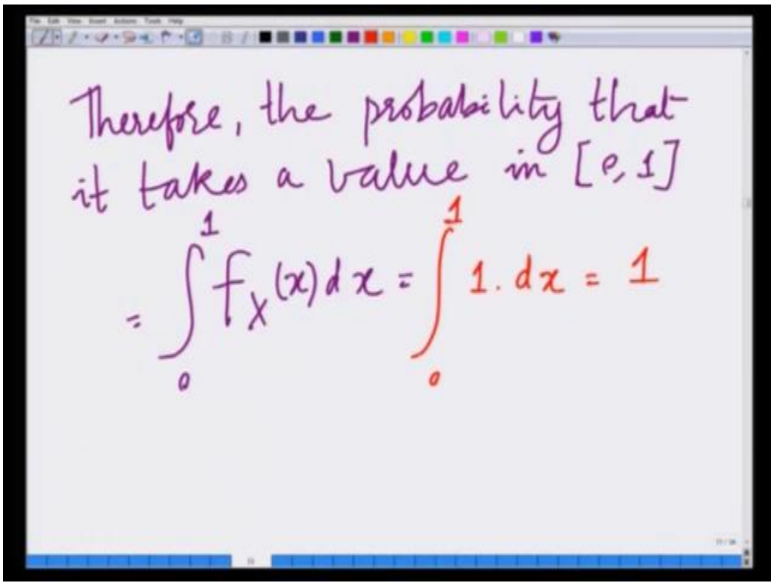
Consider any interval a comma b which lies in 0 comma 1, the probability that the random variable takes the value any value between in this interval a comma b. The probability that the random variable takes any value in this interval a comma b is integral a comma b f of x x d x can since the interval a comma b is contained in the interval 0 comma 1 when the probability density is 1, this is equal to integral a to b of 1 d x which is equal to x evaluated between the limits a comma b which is equal to b minus a. Therefore, the probability that it lies in any interval a comma b fully contained in 0 comma 1 is equal to b minus a that is the...

(Refer Slide Time: 11:14)



So, you consider this uniform random variable which is distributed uniformly between 0 and 1 and you consider any interval a comma b which is fully contained in 0 comma 1 in the probability that it lies in this interval a comma b is equal to b minus a, which is the length of the interval. Therefore, the probability is equal to essential basically the length of the interval, hence it naturally follows that if consider the entire interval 0 to 1 then the probability that it takes a value in 0 to 1 is basically 1 minus 0 which is 1.

(Refer Slide Time: 12:41)

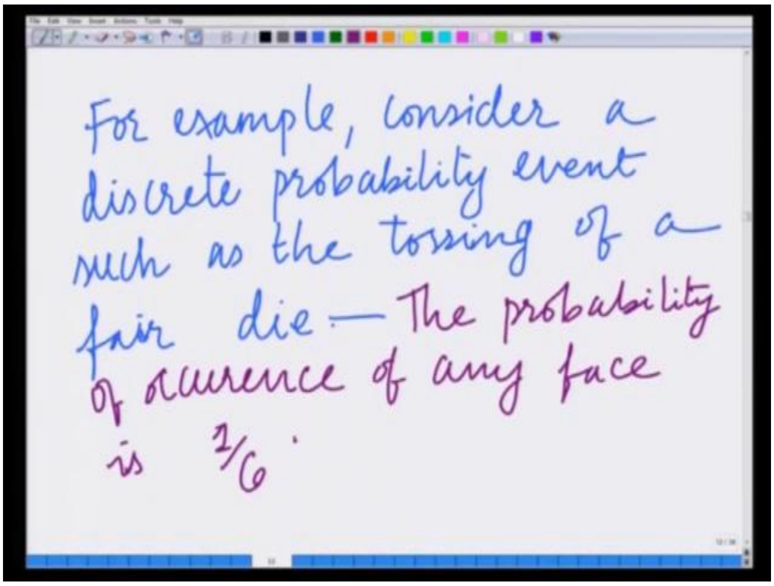


Therefore, the probability that it takes a value in 0 comma 1 equals 0 comma 1 f x x d x. But, in the interval 0 comma 1 the probability density function is unity. So, therefore, it is 0 comma 1 1 d x equals 1, which means the probability which probability 1 it takes a value in the interval 0 comma 1 and this can be expected because the random the probability density function is non zero only on the interval 0 comma 1, this probability density function is non zero only in the interval 0 comma 1.

Therefore, it has to take a value it is restricted to values between 0 and 1 and hence the probability that it takes a value any value in the interval 0 to 1 is 1 that is it does not take any value outside of this interval 0 comma 1 and this can also be seen by integrating the probability density between the limits 0 comma 1 and therefore, which is the total probability that is the area under the probability density function, which integrates to which as that the total probability of this random variable is basically 1 which is also the area under the probability density function the area under the probability density function has to integrate to.

So, this is the very specific probability density function which corresponds to that of the random variable which is uniform between 0 and 1. And this is also to speak a sort of a broad sense, this is also the continuous random variable analog of a discrete probability mass function or discrete are discrete probability random a discrete random variable which takes values or which takes values with equal probability. For instance, if you consider of a toss of a fair die each face can occur with the probability 1 by 6 that is the probability of occurrence of each face that is when you toss a fair die.

(Refer Slide Time: 15:02)

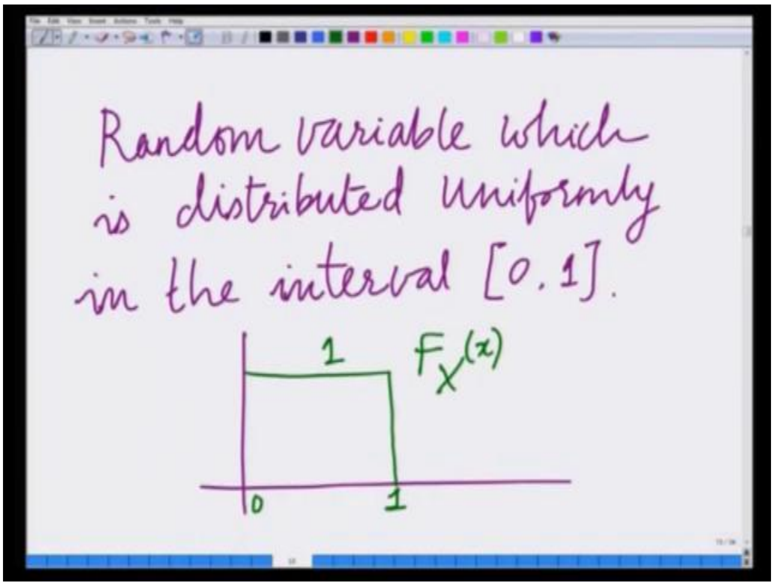


For example, consider a discrete probability event such as the tossing of a fair die then the fair die as six faces each face can occur with equal probability therefore, the probability of each faces 1 by 6. The probability of occurrence of any face is can you toss a fair die basically 1 by 6. Therefore, we say that all the faces are equip probable that is all the faces of the die or equip probable let that probabilities equal to 1 by 6 for each face and the analog of that for a continuous random variable that is I can random variable which can take any value, any real number between 0 and 1 which is a continuous random variable.

Therefore, it can take an infinite number of or any of the infinite number of real numbers between 0 and 1, the analog of that if for continuous random variable is uniform distribution between a certain interval. And therefore, since this random variable is uniformly distributed in 0 comma 1 we say that this random variable can take values uniformly in the interval 0 to 1.

So, the random variable which is uniformly distributed in the interval 0 to 1 can take values uniformly in this interval 0 to 1. So, this is the concept of a uniform random variable that is we are considering a uniform random variable which is distributed in 0 comma 1.

(Refer Slide Time: 17:21)



That is random variable which is distributed uniformly in the interval 0 comma 1 and this is given by the probability density that we have already drawn which is of height 1 in the interval 0 to 1 and 0 everywhere else. And this it is important to understand this random variable and the key properties of the random variable that we have mentioned previously.

Because, this is the random variable and we are going to use this random variable and properties of that, this random variable in the subsequent analyzes of auctions where we model auctions as Bayesian games. So, I would be let us stop this module here and continue in the next module.

Thank you very much.