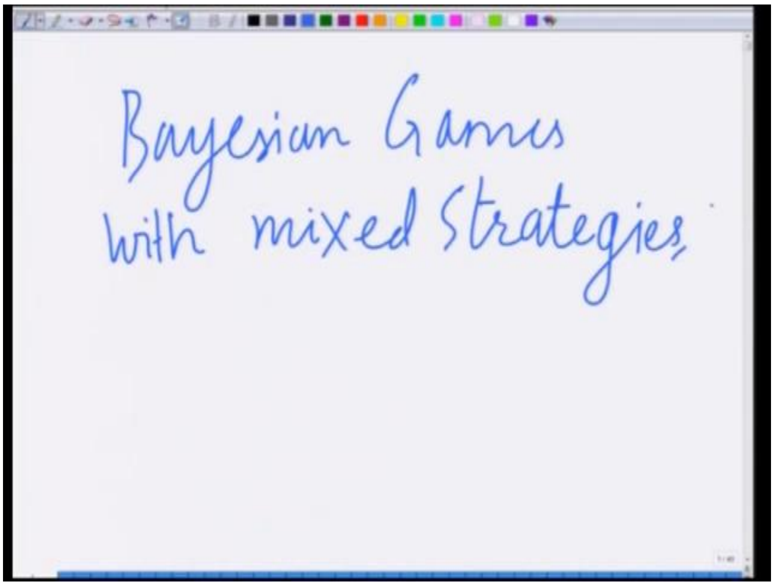
Strategy: An Introduction to Game Theory
Prof. Aditya K Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture -38

Hello, welcome to another module in this online courses Strategy, An Introduction to Game Theory. So, for the past few modules we will be looking at Bayesian games and we have been talking about Bayesian Nash equilibrium. Let us, however we have considered pure Nash equilibrium that is each of the player that is each player of each type is using a pure strategy.

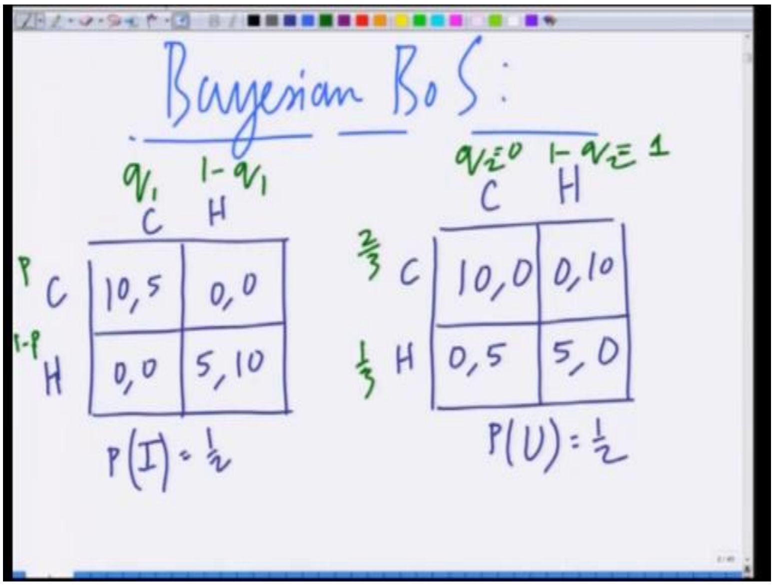
Let us, now extend this to a mixed strategy Bayesian Nash equilibrium that is the Bayesian game, in which players are different types can possibly use mixed strategies rather than pure strategies. So, what we are going to start looking at today is a Bayesian games with mixed strategy or mixed strategy Bayesian Nash equilibrium.

(Refer Slide Time: 00:53)



So, we are going to look at Bayesian games with mixed strategies, where different players or different types can potentially employ the mixed strategy.

(Refer Slide Time: 01:23)



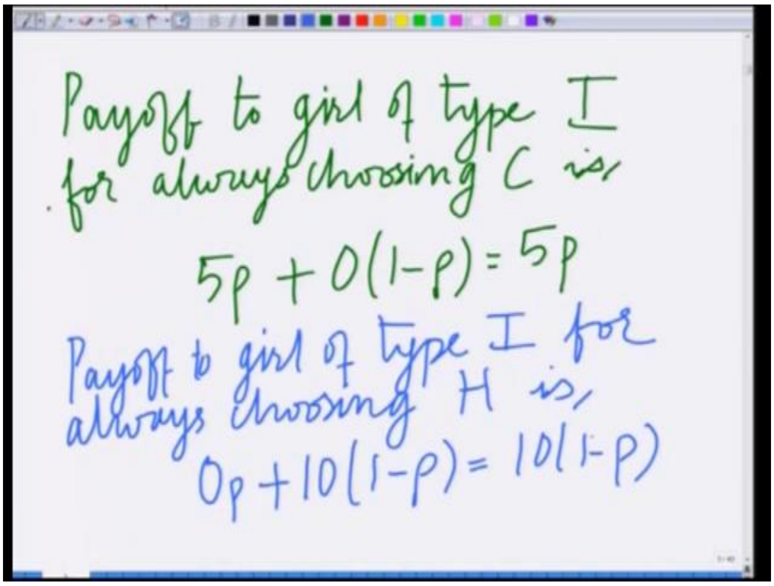
So, let us go back to our Bayesian battle of sexes game, I hope everyone remembers the Bayesian, the game tables for the Bayesian battle of sexes. So, there is a boy and girl can be of two types, as player 2 the girl can either be interested and the probability of interested equals half and the probability of the girl being uninterested is half and the different actions are C H C H C H C H. And, the payoffs corresponding to the girl of type I are 10 comma 5, 0 comma 0, 0 comma 0, 5 comma 10 and the payoffs corresponding to girl of type U are 10 comma 0, 0 comma 10, 0 comma 5, 5 comma 0.

And, this is the Bayesian battle of sexes, in which there is boy of a single type, you can player 2 the girl player is of two types that is she can be of type interested, which occurs with probability I equals half, at that corresponds and the game table corresponding to that is given on the left and she can be of type U that is uninterested, the probability corresponding to that is half and the game table corresponds to that is given at the right.

Let us, now consider a mixed strategy for the boy and the girl player of each type. So, let say the boy employs mixture probability p and 1 minus p, the girl is employing a mixed strategy q and so the girl of type I is employing the mixture q 1, 1 minus q 1, girl of type U is employing the mixture q 2 and 1 minus q 2. So, the boy is mixing with probabilities p and 1 minus p, which means the boy is choosing C and H with probabilities p and 1 minus p respectively.

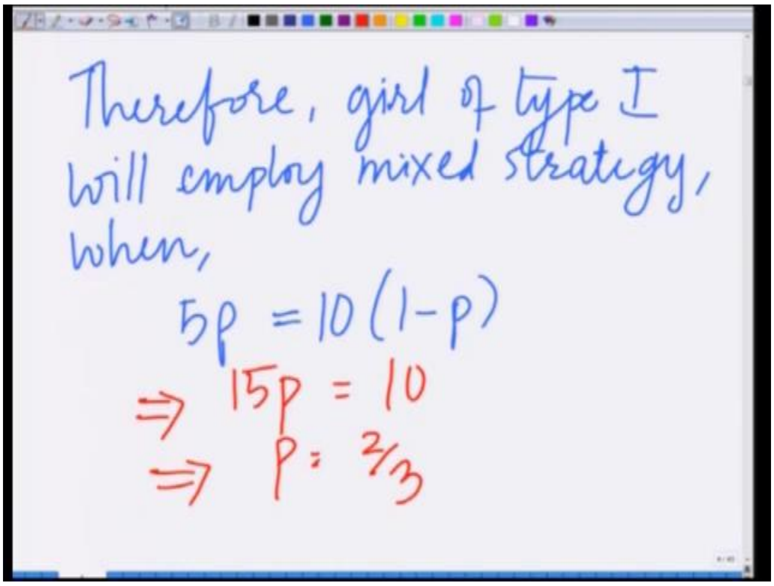
Girl of type I is choosing C and H with probabilities q 1 and 1 minus q 1 respectively and girl of type U is choosing C and H with probabilities q 2 and 1 minus q 2 respectively. Now, let us look at the payoff of girl of type I from choosing C, well the payoff of girl of type I to choosing C is 5 times, when if she choose always choosing C is probability p she gets a payoff of 5, with probability 1 minus p she gets a payoff of 0.

(Refer Slide Time: 04:23)



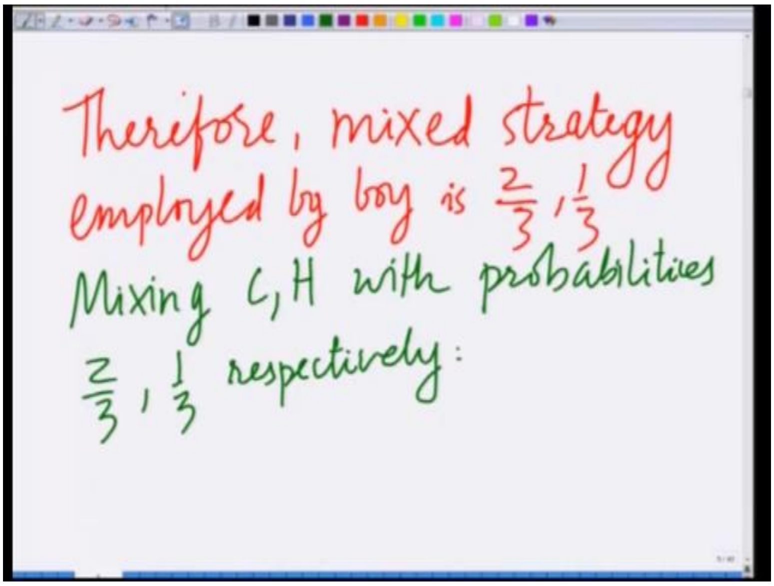
So, payoff to girl of type I for always choosing C is 5 times p plus 0 times 1 minus p equals 5 times p. Similarly, payoff to girl of type I for always choosing H is well, that is if she is always choosing H she is getting 0 with probability p and 10 with probability 1 minus p, so that is 0 times p plus 10 times 1 minus p equals 10 times 1 minus p. And therefore, girl of type I that is girl that is player 2 who is interested will choose a mixed strategy, that is she will randomly mix C and H only if these two payoffs are equal, which means 5 p equals 10 times 1 minus p.

(Refer Slide Time: 05:55)



Therefore, will employ a mixed strategy when only when payoff from C and H are equal, which means 5 times p equals 10 times 1 minus p, which basically implies 15 p equals 10, which implies p equals 2 by 3 which implies p equals 2 by 3. Therefore, the mixed strategy employed by the girl of type I is 2 by 3 comma 1 by 3, therefore or therefore, the mixed strategy this is the mixed strategy employed by the boy, we are looking at the probability p.

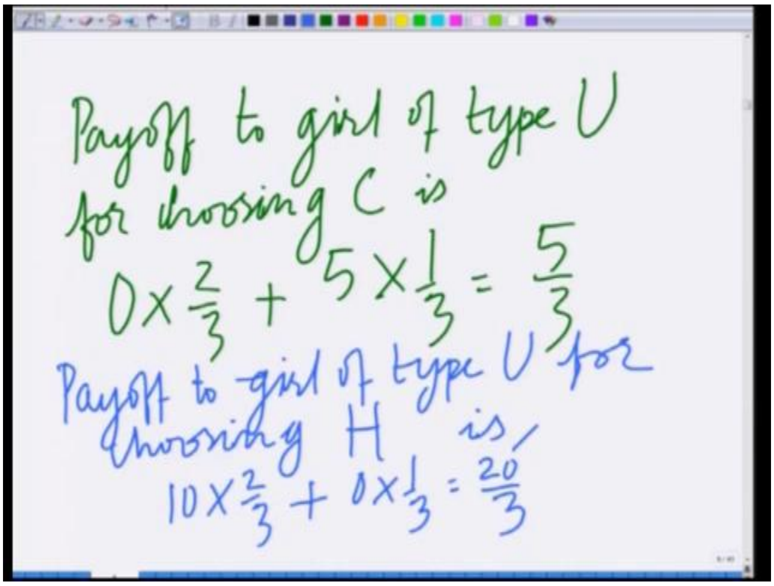
(Refer Slide Time: 07:01)



Therefore, the mixed strategy employed by the boy is 2 by 3 comma 1 by 3, that is mixing C comma H with probabilities 2 by 3 comma 1 by 3 respectively. So, we have derived that p equals 2 by 3 1 minus p equals 1 by 3 therefore, the mixed strategy employed by the boy is 2 by 3 comma 1 by 3. Now, if the boy is mixing that is player 1 is mixing with probabilities 2 by 3 1 by 3, let us look at the best responses of girl of type U.

Now, if p equals 2 by 3, so p equals 2 by 3 1 minus p equals 1 by 3 therefore, payoff to girl of type U from choosing C is 5 times 0 times 2 by 3 plus 5 times 1 by 3.

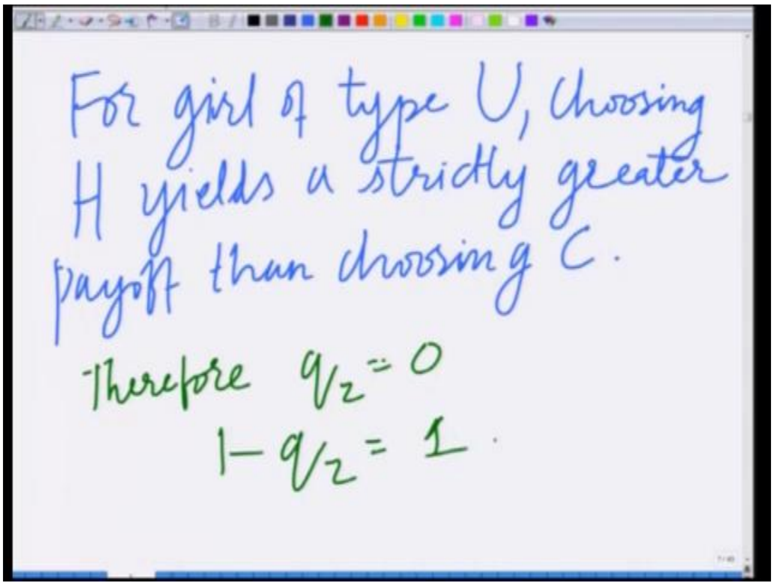
(Refer Slide Time: 08:30)



So, payoff to girl of type U for choosing C is well payoff to girl of type U for choosing C is 0 into 2 by 3 plus 5 into 1 by 3 equals 5 by 3. Similarly, payoff to girl of type U for choosing H is 10 into 2 by 3 plus 0 into 1 by 3, so payoff for choosing H is, her payoff to choosing H is well 10 times 2 by 3 plus 0 times 1 by 3 equals 20 by 3. Now, you can see for girl of type U choosing C yields a payoff of 5 by 3, but choosing H yields a payoff of 20 by 3.

Therefore, girl of type U or girl with an interested is always choosing H, because H is yielding a strictly a greater payoff than C.

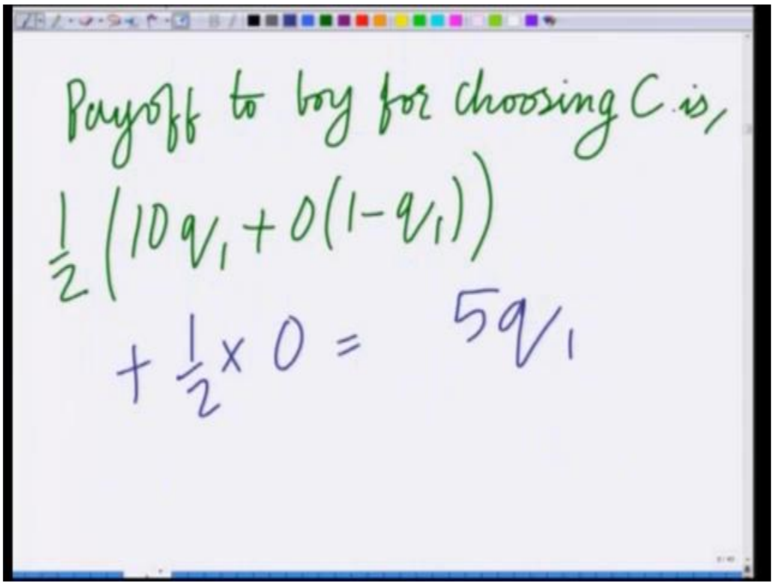
(Refer Slide Time: 10:12)



So, for a girl of type U choosing H yields a strictly greater payoff than choosing C therefore, she is always choosing H with probability 1, that is girl of type U is always choosing H with probability 1, which means if we go back over game tables ((Refer Time: 10:51)) it means that probability of choosing C is 0 therefore, q 2 equal to 0, 1 minus q 2 equals 1 therefore, q 2 equals 0 and 1 minus q 2 equals 1. So, we can write that down here, we have q 2 equals 0, 1 minus q 2 equals 1.

So, in this Bayesian battle of sexes we have found the mixed strategy employed by the boy, that is we have found the probability p 1 minus p, we have found the mixed strategy employed by the girl of type U, we have found q 2 and 1 minus q 2. Now, what is remaining is to find q 1, so we have to still find the mixed strategy employed by the girl of type I. And, we can find q 1 as follows, remember we still have to look at the payoffs of the boy for choosing C and H and the payoffs of the boy for choosing C and H have to be equal, since he is employing a mixed strategy.

(Refer Slide Time: 12:03)

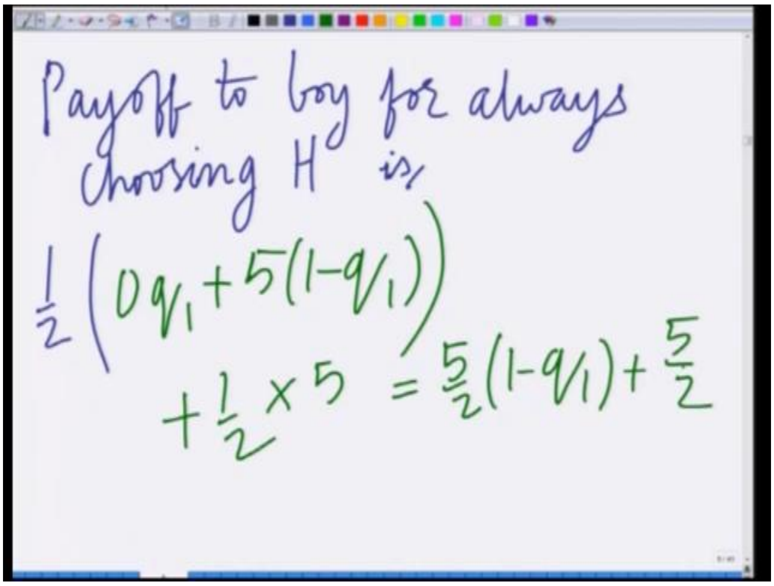


Now, the payoff of boy for choosing C is, what is the payoff to the boy, payoff to the boy for choosing C is well with probability half he meets an interested girl, when he meets an interested girl his payoff is 10 times q 1 plus 0 times 1 minus q 1. Because, interested girl, girl of type I is choosing C with probability q 1 and H with probability 1 minus q 1. So, the average payoff is 10 times q 1 plus 0 times 1 minus q 1 multiplied by the probability of girl of type I, which is half.

Therefore, his payoff corresponding to girl of type I is half into, well 10 times, let me write that down half into 10 times q 1 plus 0 times 1 minus q 1, which is equal to 5 times q 1. And, his plus his payoff corresponding to girl of type U is half with probability half he is meeting girl of type U, when he meets girl of type U this payoff, but girl of type U is always choosing H. So, his payoff corresponding to that is 0 when he meets girl of type U, girl of type U is choosing with she is always choosing H.

So, his payoff corresponding to girl of type U is 0 multiplied by the probability half. So, the net will be well this is 5 q 1 plus half times 0 equals, and what is this equal to this is equal to 5 times q 1, so the payoff of boy for choosing C is 5 times q 1. Now, the payoff for the boy for choosing H, remember if he chooses H, when with probability half he is meeting girl of type I, when he meets girl of type I his payoff is 0 times q 1 plus 5 times 1 minus q 1.

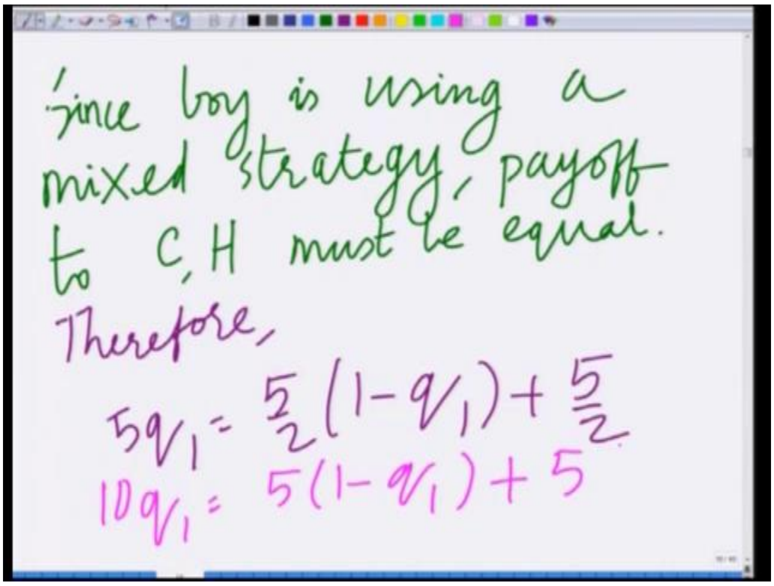
(Refer Slide Time: 14:29)



So, payoff to boy for always choosing H is well, payoff to boy for always choosing H is half into 0 times q 1 plus 5 times 1 minus 1 minus q 1. So, this is 0 times q 1 plus 5 5 into 1 minus q 1 plus half with probability half his meeting girl of type U, girl of type U is always choosing H. Since, girl of type U is always choosing H his payoff corresponding to choosing h is 5, therefore this is going to be equal to half into 5, which is equal to 5 by 2 into 1 minus q 1 plus 5 by 2.

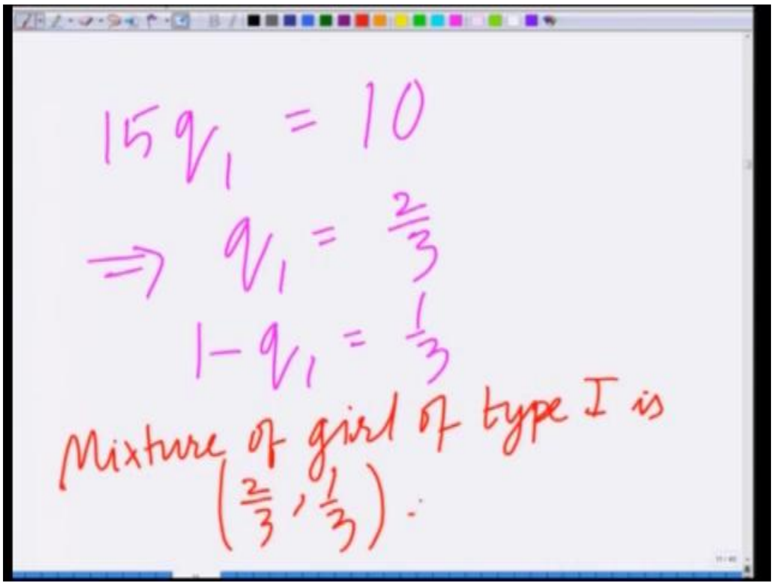
So, the payoff to boy for always choosing C is 5 q 1 payoff to boy for always choosing H is 5 by 2 times 1 minus q 1 plus 5 by 2 and since boy is choosing the mixed strategy; that means, his payoff of to C should be equal to payoff to H, since boy is choosing the mixed strategy or this player 1.

(Refer Slide Time: 15:52)



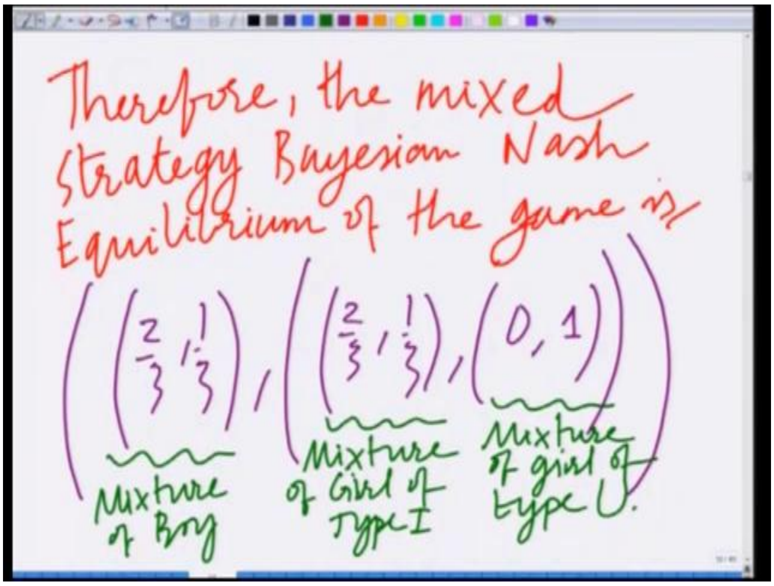
Since, boy is using a mixed strategy payoff to C comma H must be equal, therefore we must have 5 q 1 equals 5 by 2 1 minus q 1 plus 5 by 2. Therefore, 5 q 1 equals 5 by 2 1 minus q 1 plus 5 by 2 and that implies 10 q 1 equals 5 times 1 minus q 1 plus 5.

(Refer Slide Time: 17:02)



Which means 15 q 1 equals 10, which implies q 1 equals 2 by 3 and therefore 1 minus q 1 equals 1 by 3. So, mixture of girl of type I equals, so mixture of girl of type I is well 2 by 3 comma 1 by 3

(Refer Slide Time: 17:35)

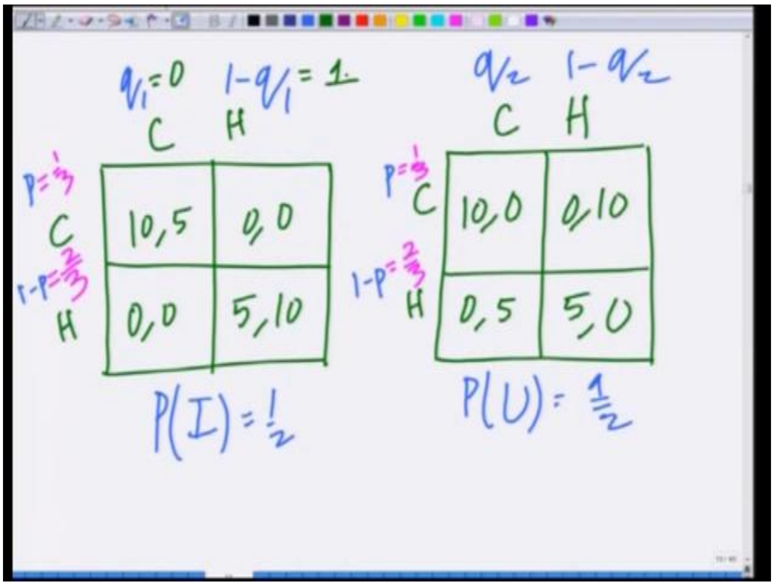


Therefore, the Bayesian Nash equilibrium, therefore the mixed strategy Nash equilibrium mixed strategy Bayesian Nash equilibrium of this game, is well we have already seen the mixture of the boy is 2 by 3 comma 1 by 3 mixture of girl of type I is 2 by 3 comma 1 by 3 and mixture of girl of type U is 0 comma 1. Because, girl of type U is always choosing H and this is the Bayesian mixed strategy Nash equilibrium this is the mixture or mixing probabilities of boy this is the mixture or mixing probabilities of girl of type I and this is the mixture of girl of type U.

So, this is the Bayesian mix strategy Nash equilibrium, in which each player of each type can po10tially use a mixed strategy. We, have derived the mixture of the boy these are the mixture of the boy is 2 by 3 comma 1 by 3 mixture of the girl of type I is 2 by 3 comma 1 by 3 and mixture of girl of type U is 0 comma 1. Therefore, this is the mixed strategy Bayesian Nash equilibrium of this game.

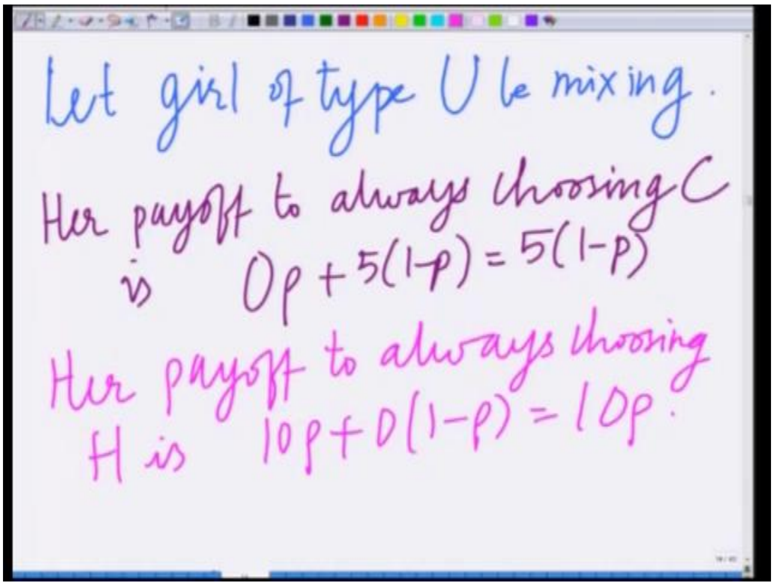
We can now, derived another Bayesian mixed strategy Nash equilibrium of this game and that can be d1 as follows, we have assume in this game we have assumed or standard with this assumption that the girl of type I is mixed. Let us, start with the another assumption instead then the girl of type U is mixing.

(Refer Slide Time: 19:51)



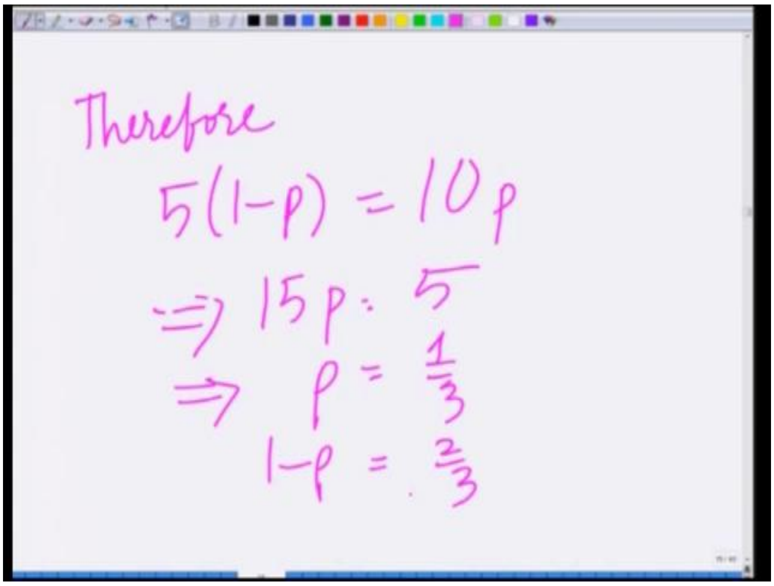
Let us, redraw these tables the game tables for this the Bayesian and the various payoffs or 10 5 0 0 0 0 5 comma 10 10 comma 0 0 comma 10 0 comma 5 5 comma 0. Let, say boy is mixing probabilities p minus 1 minus p 1 minus p girl of type I is imply q 1 1 minus q 1 girl of type U is employed q 2 1 minus q 2 and probability of girl of type I equals half probability girl of type U equals half. Now, let us start with assumption that girl of type U is mixing.

(Refer Slide Time: 20:57)



Let girl of type U, if girl of type U is mixing her payoff to always choosing C is well 0 into p plus 5 into 1 minus p her payoff to always choosing C is 0 into p plus 5 into 1 minus p equals 5 times 1 minus p. Her payoff to always choosing H is 10 into p plus 0 into 1 minus p that is 10 into p her payoff to always choosing H her payoff to always choosing H is 10 into p plus 0 into 1 minus p. Now, she we know that she is mixing, because of both her payoff from C and H are equal, therefore we must have 5 into 1 minus p equals 10 into p.

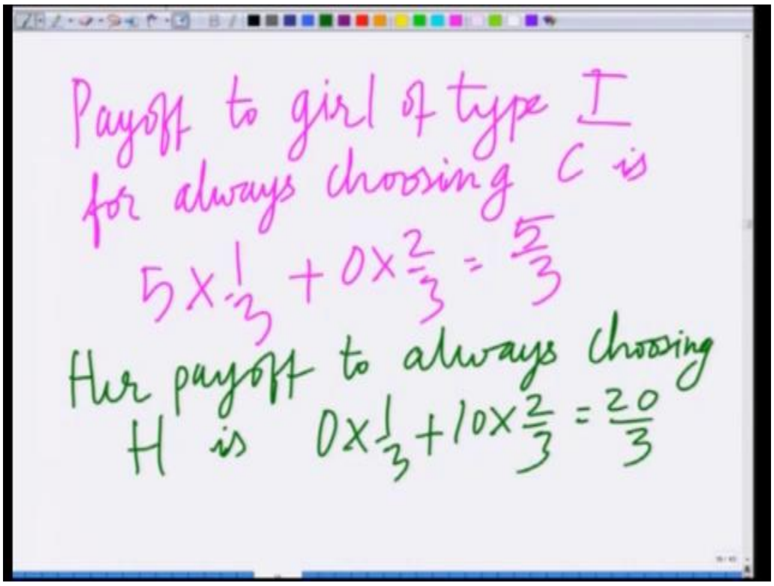
(Refer Slide Time: 22:34)



Therefore, 5 into 1 minus p must be equal to 10 times p. which implies that 15 p equals 5,which implies that p equals 1 by 3 and 1 minus p, therefore equals 2 by 3. Therefore, this means that the mixed strategy employed by the boy is p equals 1 by 3 1 minus p equals 2 by 3 p equals 1 minus 1 by 3 1 minus p equals 2 by 3.

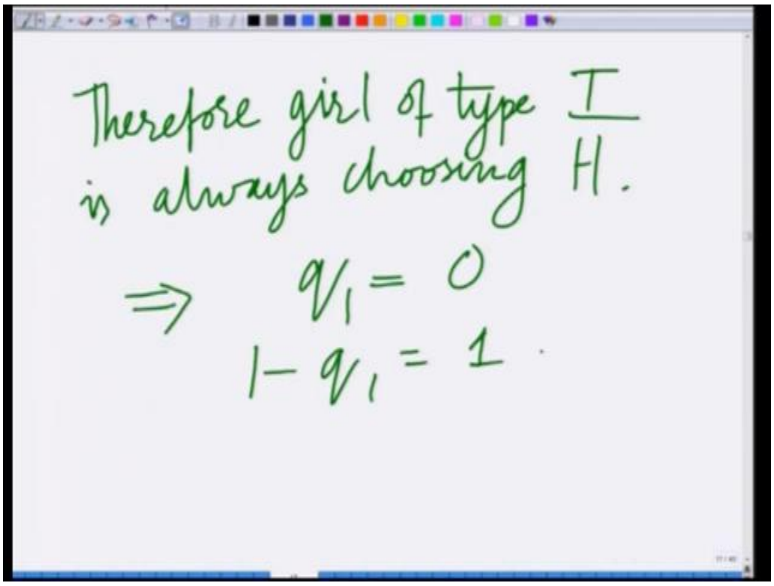
Therefore, we have derived the mixture of the boy assuming that the girl of type u is mixing. Now, if p equal to 1 by 3 and 1 minus p equal to 2 by 3, let us see what the girl of type I is to it, now if p equals 1 by 3 1 minus p equals 2 by 3 payoff to the girl of type I for always choosing C equals 5 into 1 by 3 plus 0 into 2 by 3, which is 5 by 3.

(Refer Slide Time: 23:35)



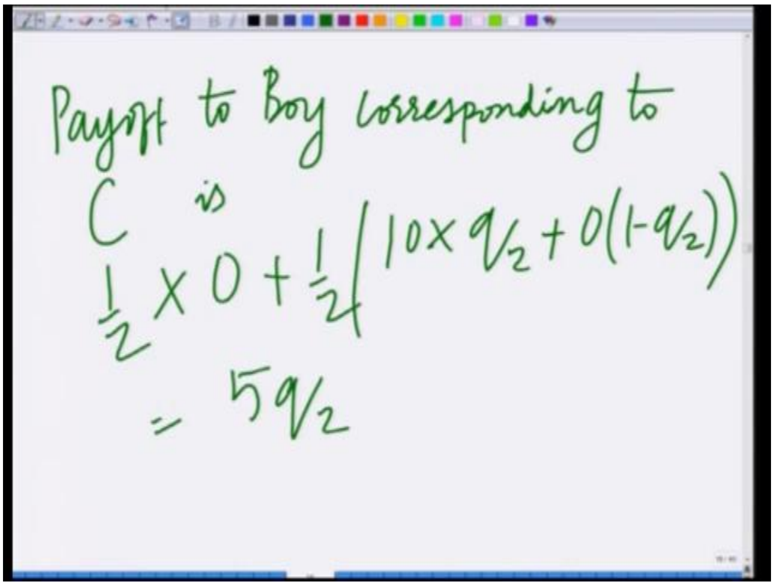
Payoff to girl of type I for always choosing C is 5 into 1 by 3 plus 0 into 2 by 3, which is equal to 5 by 3 and payoff for all and her all payoff always choosing H is 0 into 1 by 3 plus 10 into 2 by 3 her payoff always choosing H is 0 into 1 by 3 plus 10 into 2 by 3 equals twenty by 3. Now, you can see the payoff of girl of type I her payoff to C is 5 by 3, her payoff to H is 20 by 3, therefore her payoff to H is strictly greater than or payoff 2 C. Therefore, she is always choosing H, which means the probability with each he is choosing H is 1 the probability with, which she is choosing C is 0.

(Refer Slide Time: 25:18)



Therefore, girl of type I is always choosing H, which implies q 1 equals 0 and 1 minus q 1 equals 1, therefore we must also have that q 1 equals 0 1 minus q 1 equals 1. So, in this problem this Bayesian battle of sexes game, we have found that the mixture employed by the boy is, because 1 by 3 1 minus p equals 2 by 3 the mixture employed by girl of type I is q 1 equal to 0 1 minus q 1 equal to 1. It now, remains to find the probability the mixture probabilities q 2 1 minus q 2 of the girl of type U and this can be found by once again looking at the payoffs of the boy.

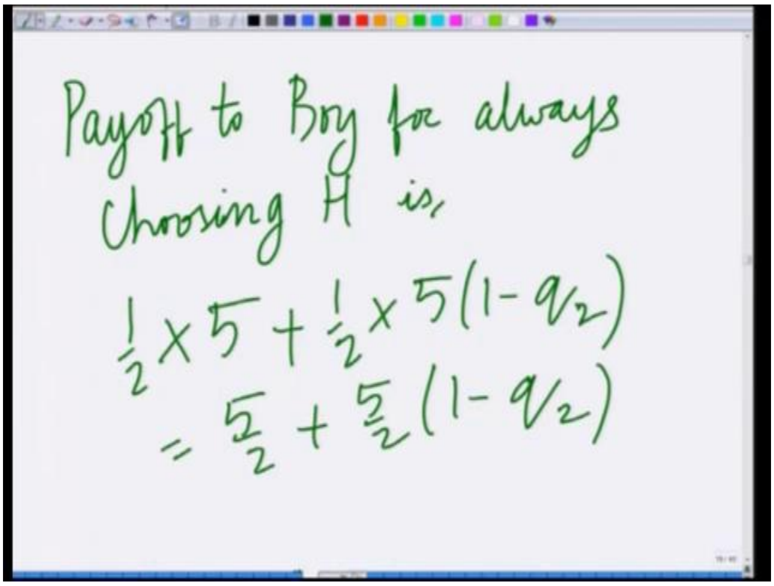
(Refer Slide Time: 26:31)



Payoff of the boy corresponding payoff of the boy are player 1 corresponding to C or payoff to boy corresponding to C is well payoff to boy corresponding to C is with probability half is meeting girl of type I, when he meets girl of type I, is payoff is 10 girl of type I is always choosing H. Therefore, his payoff is 0 and with probability half is meeting girl of type U, when you meets girl of type U his payoff is 10 times q 2 plus 0 times 1 minus q 2. Therefore, his net average payoff is half times well, where he meets girl of type H his payoff is 0.

So, this is half into 0 plus when we meets girl of type U his payoff is half into 10 into q 2 plus 0 into 1 minus q 2 half into 10 into q 2 plus 0 into 1 minus q2, which is equal to 5 times q. So, the payoff to the boy corresponding to always choosing C is 5 times q 2, payoff to boy for always choosing.

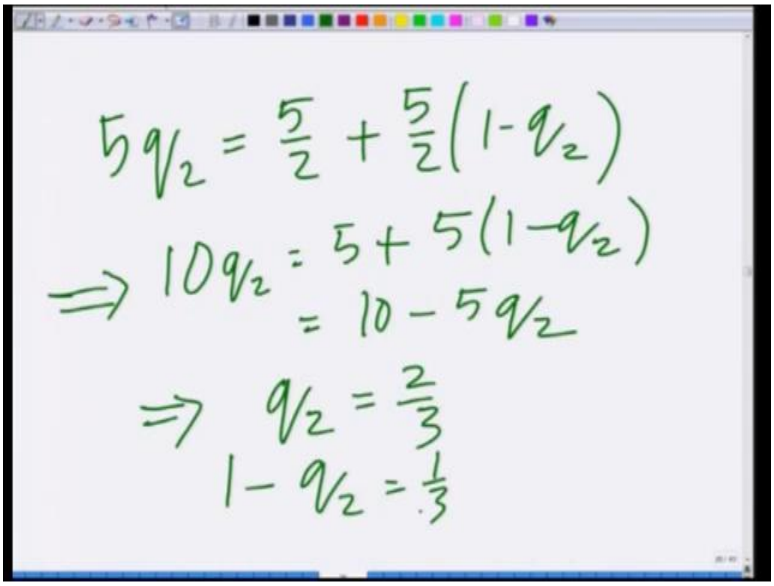
(Refer Slide Time: 28:22)



Let us, look at payoff to boy for always choosing H is well with probability half is meeting girl of type I girl of type I is always choosing H, so his payoff is half times 5. So, his payoff is half times 5 plus with probability half is meeting girl of type U girl of type U is choosing well girl of type is mixing with the probability q 2 and 1 minus q 2 corresponding to, which payoff is 0 times q 2 plus 5 times 1 minus q 2, which is basically 5 times 1 minus q 2.

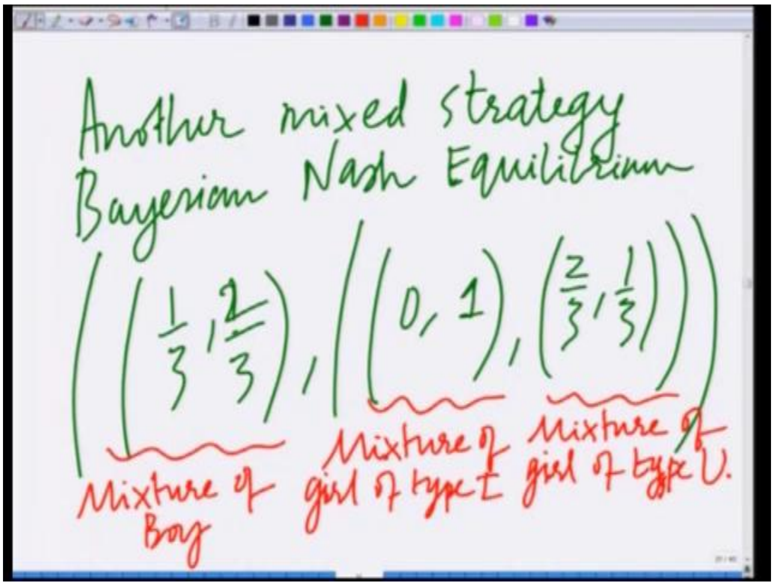
So, this will be half multiplied by 5 times 1 minus q 2, which is equal to 5 by 2 plus 5 by 2 1 minus q. So, his payoff from C is 5 times q 2 and his payoff from H is 5 by 2 plus 5 by 2 times 1 minus q 2, since he is mixing we must have these two payoffs equal.

(Refer Slide Time: 29:47)



Therefore, it must be the case that 5 times q 2 equals 5 by 2 plus 5 by 2 times 1 minus q 2, which implies that 10 q 2 equals 5 plus 5 times 1 minus q 2 equals 10 minus 5 q 2, which implies q 2 equals 2 by 3 1 minus q 2 equals 1 by 3. Therefore, mixture of girl of type U is mixture of girl of type U is 2 by 3 comma 1 by 3.

(Refer Slide Time: 30:33)



So, therefore, we have the mixed strategy another mixed strategy another mixed strategy Bayesian Nash equilibrium for this game is, where boy is mixing with probabilities well 1 by 3 comma 2 by 3, girl of type I is well choosing the mix mixed strategy 0 comma 1.

Because, girl of type I is always choosing H and girl of type U is choosing the mixed strategy 2 by 3 comma 1 by 3, this is what we found and therefore, this is mixture of boy this is mixture of girl of type I this is mixture of the of mixture of girl of type.

So, we have found to mixed strategies Bayesian Nash equilibrium mixed strategy Bayesian Nash equilibrium of the game, in the first Bayesian mixed strategy Bayesian Nash equilibrium the boy is employing mixed 2 by 3 1 by 3 girl of type I is employing mixture 2 by 3 1 by 3 girl of type U is employing the mixture 0 comma 1, which means she is always choosing H.

In the second Bayesian mixed strategy Nash equilibrium the boy is employing the mixture 1 by t 1 by 3 2 by 3 girl of type I is a employing mixture 0 comma 1, which means she is always choosing H and girl of type U is employing the mixture 2 by 3 comma 1 by 3. And therefore, we have found the mixed strategy Bayesian Nash equilibrium for this Bayesian battle of sex's game. And, this is the Bayesian game and further, we have found the Bayesian mixed strategy Nash equilibrium, because each player of each type is employed a mixed strategy, at this big strategy are the best responses for each player of each type.

At the Bayesian Nash equilibrium each player of each type is employing his or her best response mixed strategy. Therefore, this is the Bayesian these are Bayesian mixed strategy in Nash equilibrium for this game. I hope this clarified the idea of Bayesian mixed strategy Nash equilibrium in Bayesian games.

Thank you, thank you very much.