Strategy: An Introduction to Game Theory
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Lecture – 37

Hello, welcome to another module in this online course Strategy, An Introduction to Game Theory. We are looking at Bayesian games in the past couple of modules. We have looked at two examples of Bayesian games, we have looked at a Bayesian version of the battle of sexes and we have also looked at another Bayesian game; that is a yield versus fight game between two people.

Now, let us look at another example of a Bayesian game, an interesting example. Let us look at a Cournot game, remember we have already looked at a Cournot game, but now let us look at a Bayesian version of the Cournot game.

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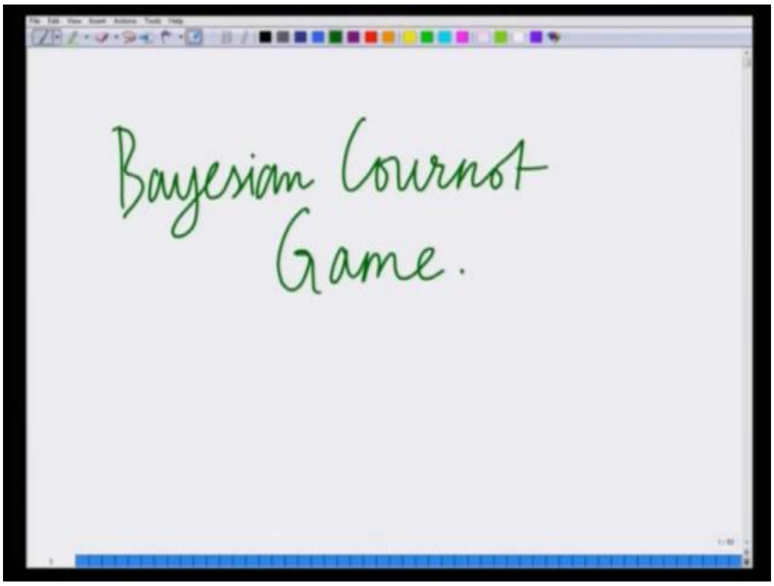

So, let us look at a Bayesian Cournot game or Bayesian version of the Cournot game.

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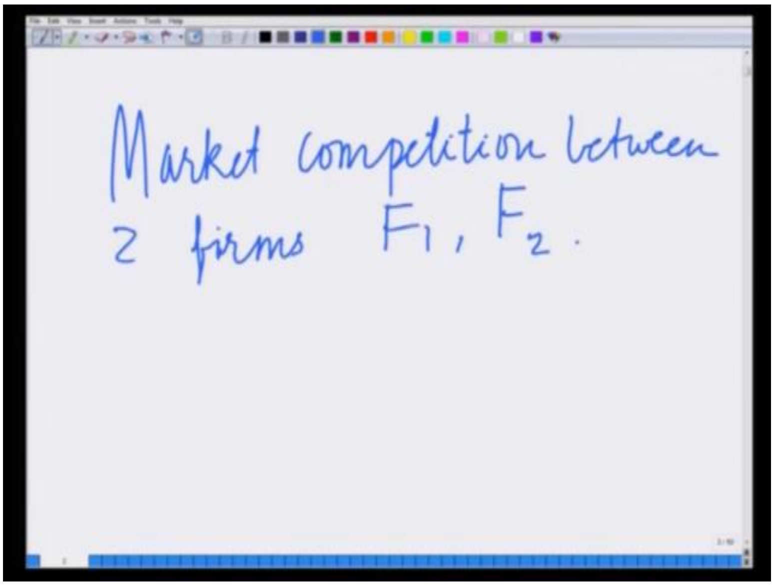

Remember, the Cournot game models market competition between two firms, F 1 comma F 2. So, the Cournot is a market game, which models the competition between two firms F 1 and F 2. However, now let us introduce uncertainty into this game, in terms of the uncertainty in the production cost of firm 2.

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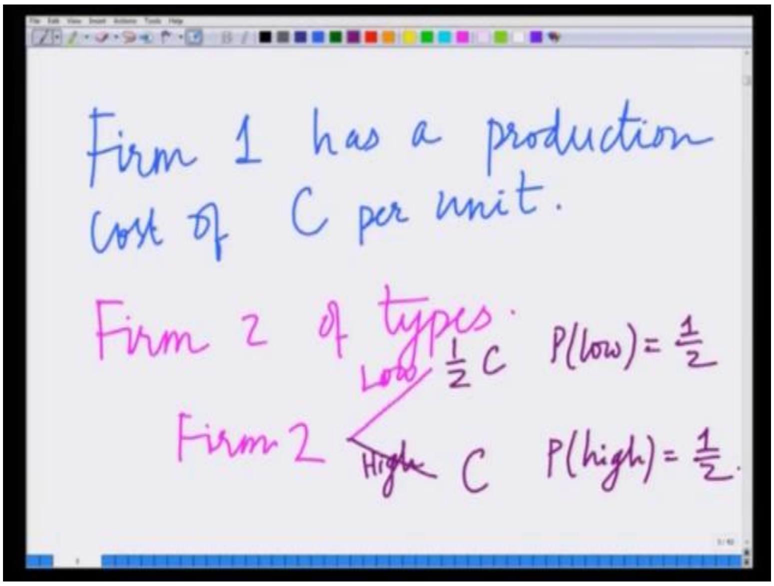

So, let us say that firm 1 has a production cost of C per unit. However, firm 2 is of two types, firm 2 of type low has a production cost. So, there is firm 2 of type low, which has a production cost of half C and probability of low equals half and term firm 2 of type high has a production cost of C and the probability of p high equals half. So, we are saying there are two types of firm 2.

Firm 2 can be of type low, which has a low production cost of half C per unit and firm 2 can be of type high, which has a production cost of C per unit. And the probability of each of these firm, each of these types of firm 2 is half each; that is firm 2 can be of type low with probability half and firm 2 can be of type high with probability half. While, type low has production cost half C, type high has a production cost of C.

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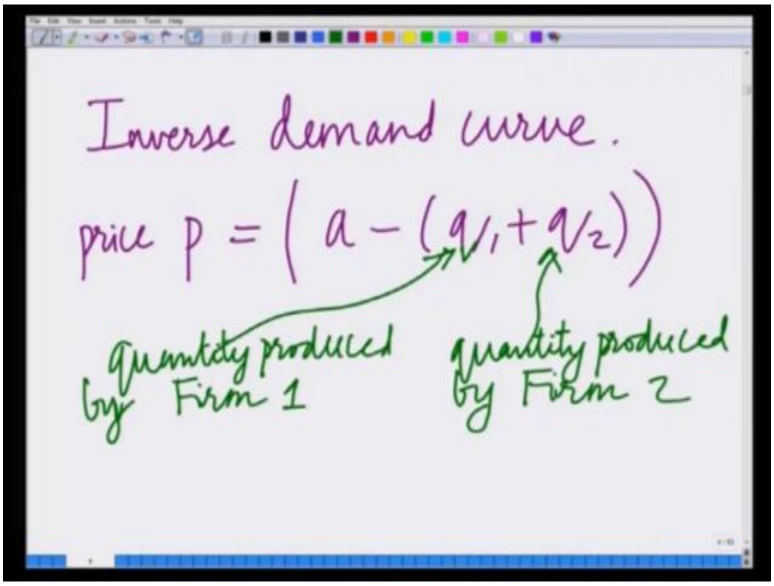

And the inverse demand curve, remember we will assume the same, the inverse demand curve; that is the price per unit p as the function of the quantities q 1 and q 2 produced by firm 1 and 2 is, well we can model that as a minus q 1 plus q 2. So, this is the price p, which is a minus q 1 plus q 2 as a function of q 1 is quantity firm 1 and q 2 is a quantity produced by firm 2. So, q 1 and q 2 are the quantities produced by firm 1 and firm 2.

So, inverse demand curve which gives the price as a function of the quantities is basically the price p equals a minus q 1 can plus q 2; that is what we are saying is that these quantities are closely related. Remember, with the Cournot game, if both the firms are producing substitutes, strategic substitutes, where one quantity can be more or less readily substituted for the other.

Therefore, the price decreases, price per unit decreases with the total quantity q 1 plus q 2 produced by both the firms. As q 1 plus q 2 is increasing, the price p which is equal to a minus q 1 plus q 2 is decreasing. So, this is the inverse demand curve, which gives the price as a function of the quantities q 1 and q 2.

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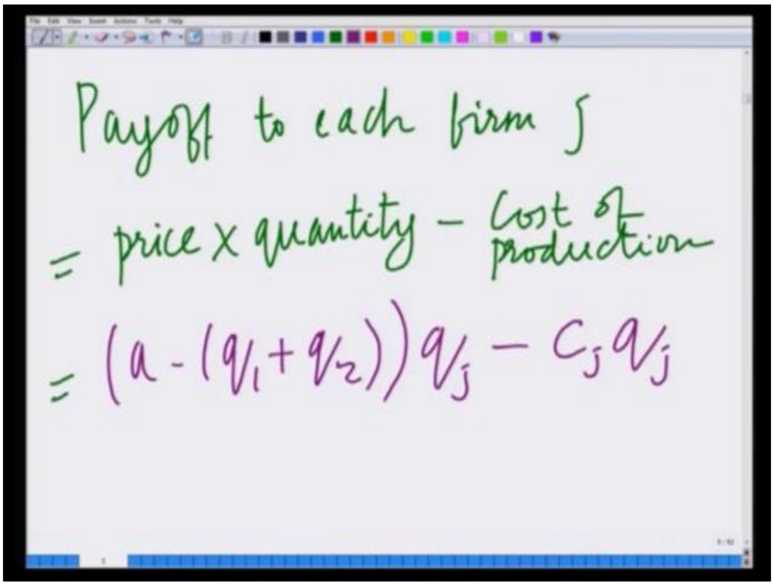

Now, therefore, the payoff to firm J, therefore, the payoff to each firm J equals price per unit times the quantity minus cost of production, which is equal to, remember the price is a minus q 1 plus q 2 times the quantity q J produced by the firm J minus C, the cost C J times q J. The payoff to each firm J is a minus q 1 plus q 2, which is the price times q J. The quantity this is the total revenue minus the cost, which is C J times q J, where C J is the cost per unit to firm J.

Now, what we want to do is, we want to find the Bayesian Nash equilibrium of this game, what we want to analyze, this is a Bayesian game. Since, there is an uncertainty regarding the payoffs of firm 2, regarding the types of firm 2 and payoffs of firm 2. Remember, we said that firm 2 can be of 2 types of type low or type high, each type occurs with probability half, this is not known to firm 1. Firm 1 does not know which type of firm 2 easy is playing against. So, there is uncertainty regarding the type of firm 2, hence the payoffs involve.

So, this game is Bayesian in nature, since there is uncertainty. So, we want to analyze the Bayesian Nash equilibrium for this game, let us start by assuming the quantities produced by the firm 1 and each type of firm 2.

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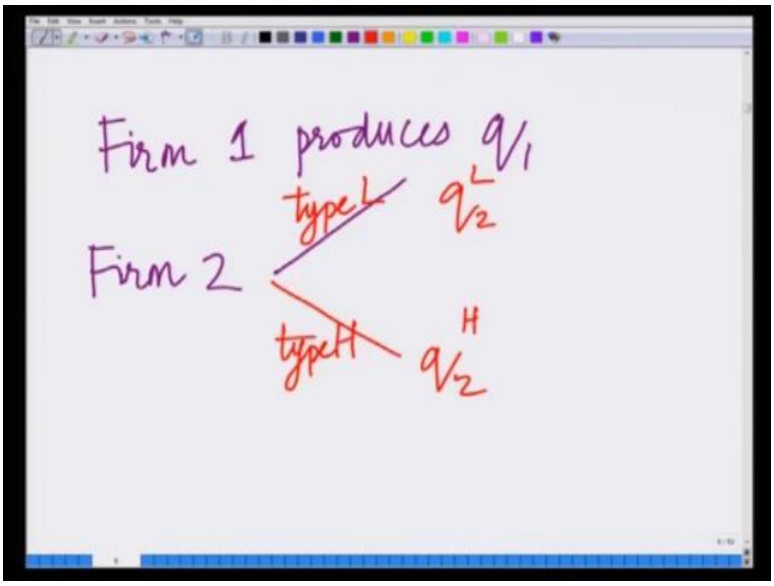

So, let us say firm 1 produces q 1 and firm 2, well firm 2 there are two types. So, let us say firm 2 of type low produces q 2 L and firm 2 of type high produces q 2 H. So, there are two types of firm 2. We are saying firm 2 of type low produces a quantity or chooses the strategy or action q 2 L and firm 2 of type H, high production cost chooses the quantity q 2 H. So, these are so we are describing quantities to the different firms, we are saying firm 1 is producing q 1, firm 2 of type low is producing q 2 L and firm 2 of type H is producing q 2 H.

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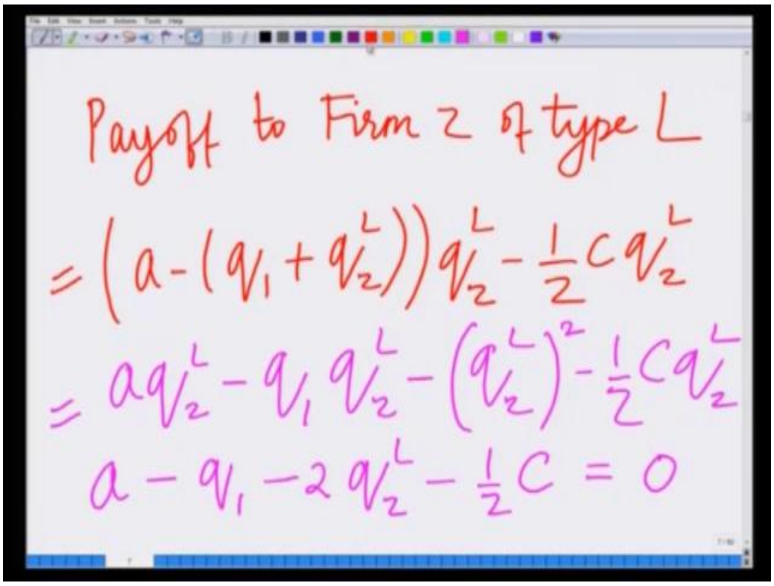

And therefore, the various payoffs are given as follows, the various payoffs to firm 2, for instants the payoff to firm 2 of type L is equal to, well a minus q 1 plus q 2 L times q 2 L price times the quantity minus the cost. Remember, the cost per unit is only half C for firm 2 of type L. So, this is minus half C times q 2 L. So, this is the price, which is a minus q 1 plus q 2 L times the quantity q 2 L. This is the revenue minus half C, which is the price per unit times q 2 L, which we can simplify as a q 2 L minus q 1, q 2 L minus q 2 L square minus half C q 2 L.

So, we have derived the payoff, we have derived an expression for the payoff for firm 2 of type L. What do we have to do now; we have to differentiate this expression set it equal to 0 to find the best response of firm 2 of type L. So, we want to find the best response of firm 2 of type L, therefore, I am going to differentiate this expression and set it equal to 0. When, I differentiate this expression and set it equal to 0, I get a, I am differentiating with respect to q 2 L minus q 1 minus twice q 2 L minus half C equal to 0.

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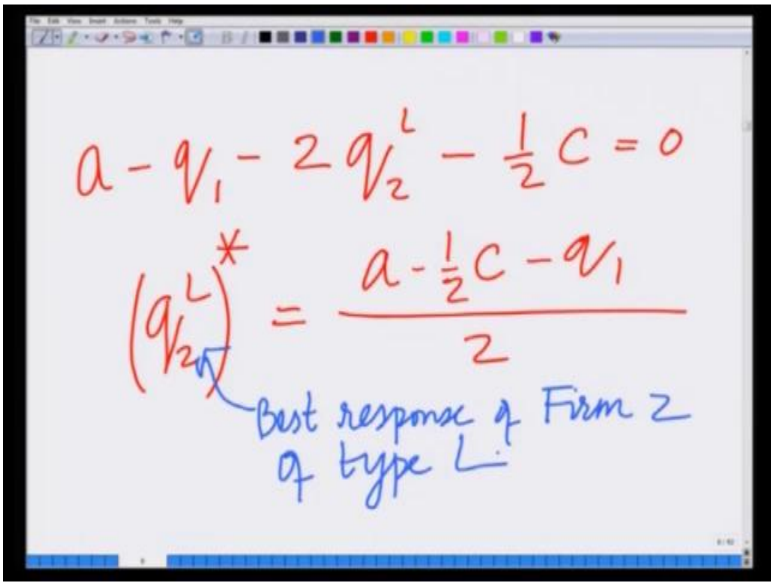

So, I have a minus q 1 minus 2 q 2 L minus half C equal to 0. So, therefore, we have the best response q 2 L star equals a minus half C minus q 1 divided by 2. So, what have we done? We have first derived an expression for the payoff of firm 2 of type L as a function of the quantity q 1 produced by firm 1 and q 2 L produced by firm 2 of type low. Then, we have differentiated this with respect to q 2 L, set it equal to 0 to find the maximum; that is we are finding the best response. That is we are finding the q 2 L star corresponding to which this payoff is maximized and that expression we have shown is q 2 L star equals a minus half C minus q 1 divided by 2.

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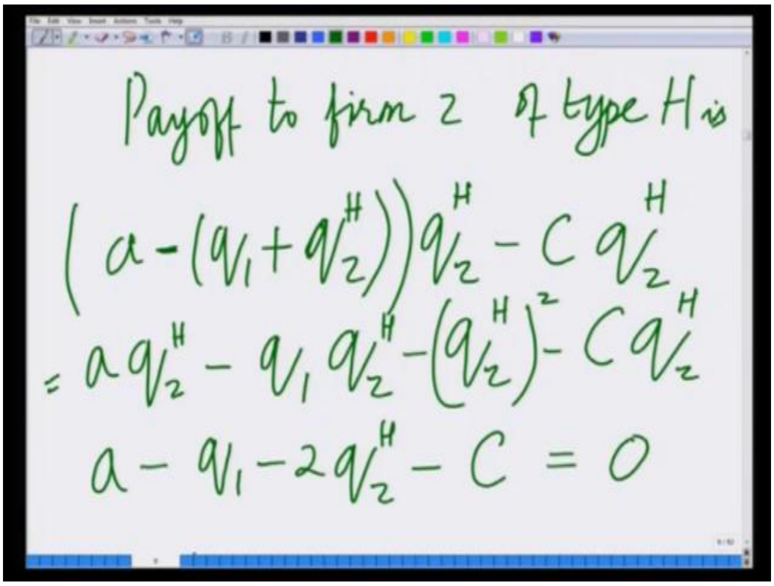

Now, similarly payoff to firm 2 or payoff to firm 2 of type H is well the price, which is a minus q 1 plus q 2 H. Remember, firm 2 of type H produces quantity q 2 H times q 2 H minus it has a production cost of C, therefore C times q 2 H. So, payoff to firm 2 of type H is the price, which is a minus q 1 plus q 2 H times q 2 H the quantity minus the cost, cost per unit is C. Because, firm 2 of type H has a cost C times q 2 H, which is equal to a times q 2 H minus q 1 q 2 H minus q 2 H square minus C q 2 H.

Now, again to find the best response q 2 H, we can differentiate it and set it equal to 0 to find the q 2 H corresponds to which this payoff is maximized. If I differentiate this with respect to q 2 H, I have a minus q 1 minus 2 q 2 H minus C equals 0.

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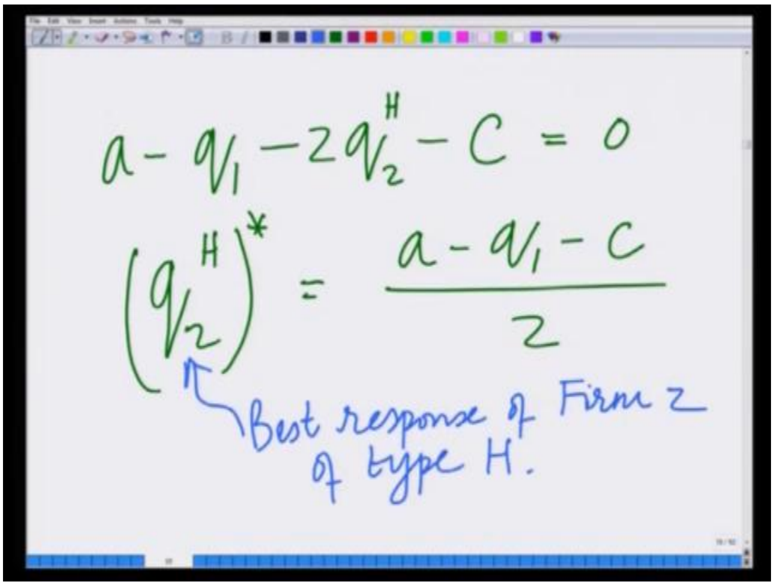

So, I have a minus q 1 minus 2 q 2 H minus C equals 0, which means q 2 H star equals well a minus q 1 minus C 1, 2. This is the best response, what is this q 2 H star, this is the best response of firm 2 of type H. So, the best response q 2 H star of firm 2 of type H is a minus q 1 minus C divided by C as a function of the quantity q 1 divide provided or manufacture by firm 1.

So, what have we done, we have derived q 2 L star which is the best response firm 2 of type L and we also derive the best response q 2 H star of firm 2 of type H. Now, we have to derive the payoff of firm 2, remember for firm 1, there is uncertainty and recording the type of firm 2. So, for firm 1, we have to compute the average payoffs.

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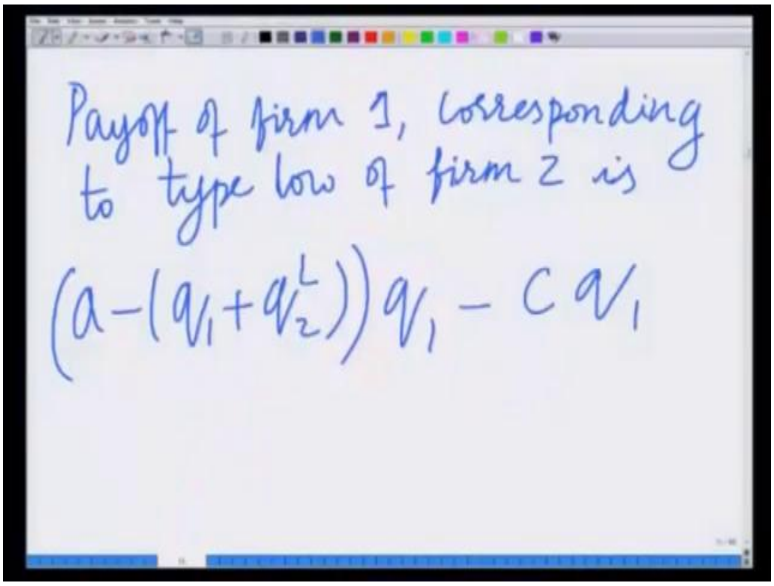

So, firm 1, payoff a firm 1 corresponding to type low of firm 2 is firm 2 of type low is produced using q 2 low the payoff corresponding of firm 1 corresponding to that is a minus well q 1 plus q 2 low price times q 1 minus cost C times q 1. Because, q firm 2 of type low is producing q 2 low, therefore, price is a minus q 1 plus q 2 low. Therefore, revenue is a minus q 1 plus q 2 low times q 1 minus the cost, which is C times q 1.

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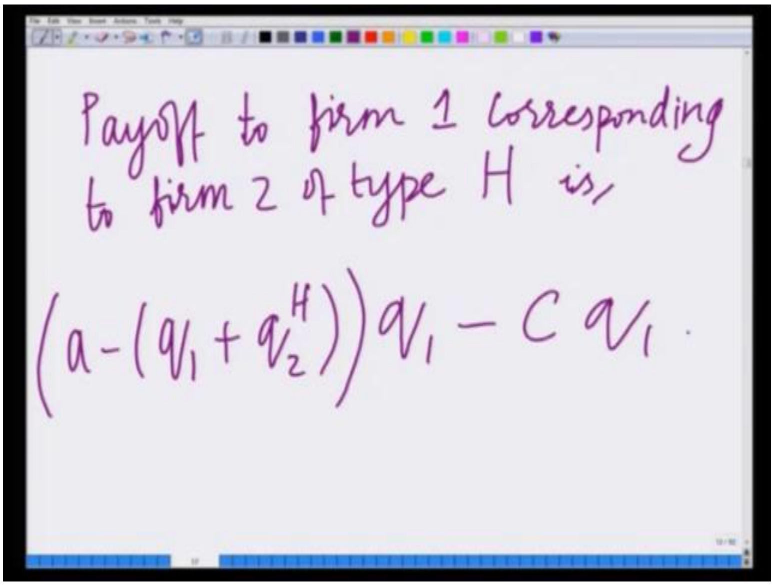

Similarly, payoff of firm 1, similarly payoff 2, firm 1 corresponding to firm 2 of type H is well a minus q 1 plus q 2 H times a minus q 1 plus q 2 H times q 1 minus C times q 1, which is the price, which is a minus q 1 plus q 2 H times the quantity q 1 minus C times q 1. Therefore, the average payoff, remember the probability of firm 2 of type low and firm 2 of type high is half each.

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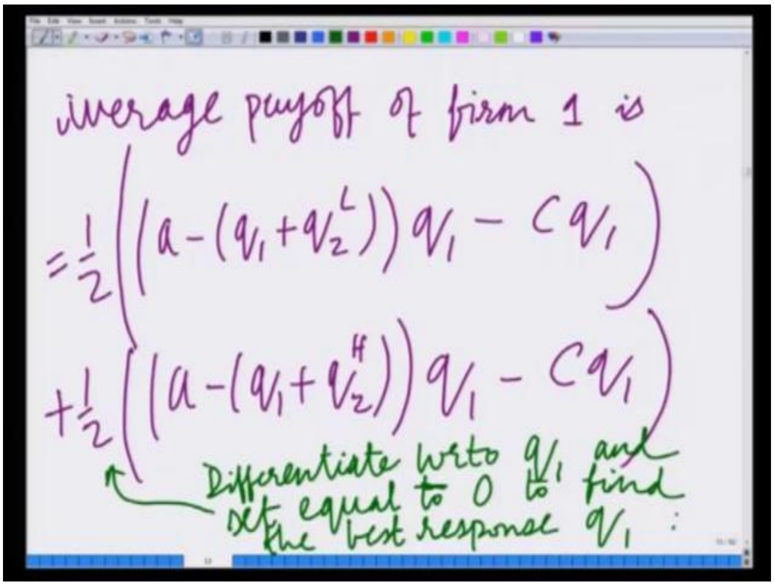

Therefore, average payoff average payoff of firm 1 is half times payoff from firm 2 of type low, which is a minus q 1 plus q 2 low times q 1 minus C times q 1 plus half times payoff corresponding to firm 2 of type I which is a minus q 1 plus q 2 high into q 1 minus C times q 1. And now what I can do is I can differentiate this with respect to q 1 differentiate this average payoff of firm 1 with respect to q 1 to find the best response q 1. So, differentiate let me write that differentiate with respect to q 1 and set equal to 0 to find the best response q 1.

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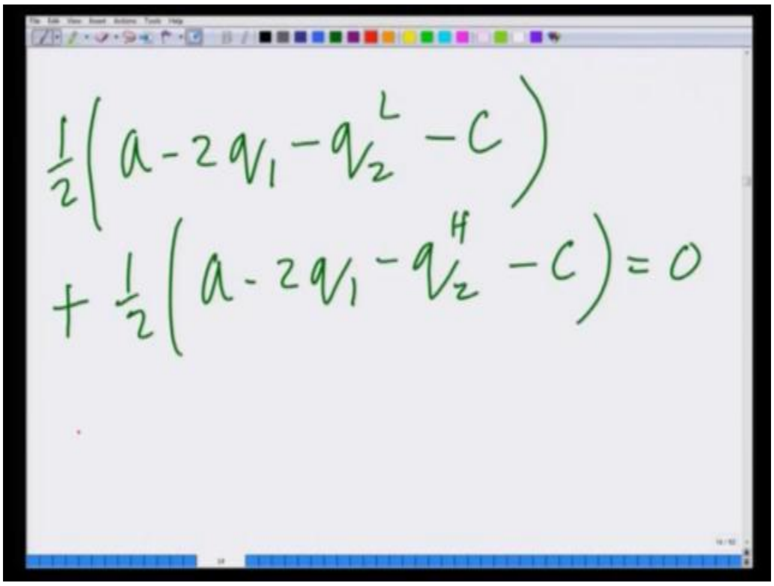

And when I differentiate and set this equal to 0, I will have you can check half of a minus 2 q 1 minus q 2 L minus C plus half of a minus 2 q 1 minus q 2 H minus C equal to 0.

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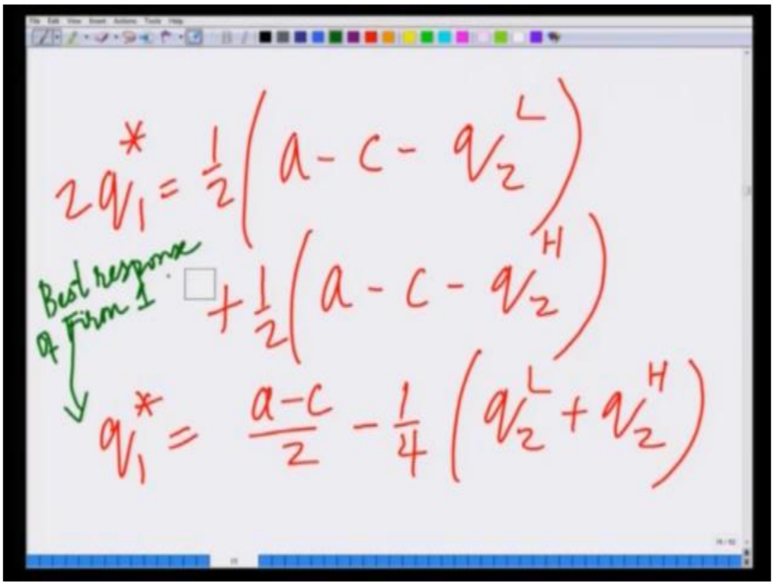

Which means, I will be solving this I will have 2 q 1 star equals half times a minus C minus q 2 L plus half times a minus C minus q 2 H. Therefore, I can write q 1 star equals well I can write this as a minus C by 2 minus 1 by 4 q 2 L plus q 2. So, what are we derived, we have derived q 1 star, which is the best response. By differentiating the payoff average payoff of firm 1, which is function of q 1, by differentiating this with respect to q 1 and setting it equal to 0.

We have derived the best response q 1 star of firm 1 as a function of the quantities q 2 L and q 2 H produced by firm 2 of type low and high respective. So, this is the best response of firm 1, this is the best response of firm 1. How do you find the Bayesian Nash equilibrium, well a Bayesian Nash equilibrium each firm of each type is playing it is best response. Therefore, we will have firm 1 producing q 1 star, firm 2 of type low producing q 2 L star, firm 2 of type H producing q 2 H star.

Therefore, if you look at this ((Refer Time: 20:27)) firm 2 of type low will be producing q 2 L star, firm 2 of type H will be producing q 2 H star and firm 1 will be producing q 1 star. Therefore, now we can write the equations for this as well everyone will be producing there, everyone will be playing their best responses.

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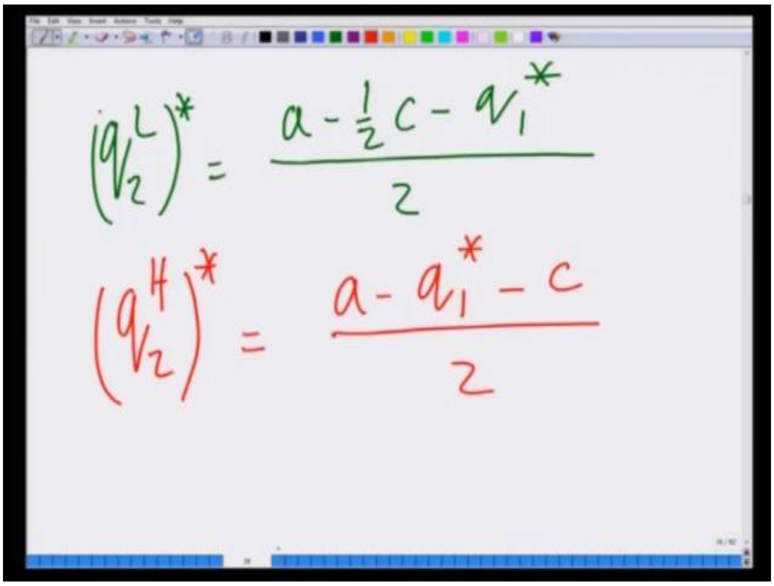

So, we have q 2 L star equals, well a minus half C minus q 1 star divided by 2 and q 2 H star equals a minus q 1 star minus C divided by 2.

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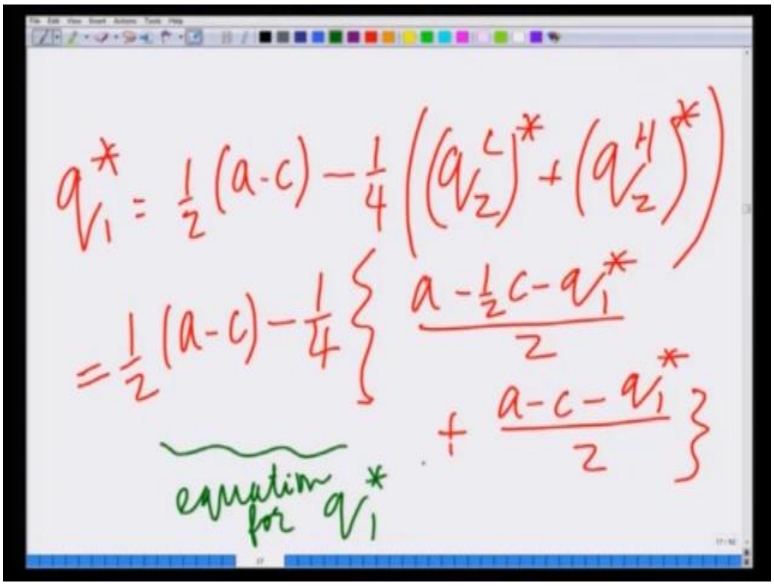

And also q 1 star equals half a minus C minus one-forth q 2 L star plus q 2 H star and from the previous equations given here we can substitute the expression for q 2 L star and q 2 H star in this. And we get half a minus C minus one-fourth, well q 2 L star equals a minus half C minus q 1 star divided by 2 plus a minus C minus q 1 star divided by 2. So, we get an equation in q 1 star. So, this is an equation for q 1 star.

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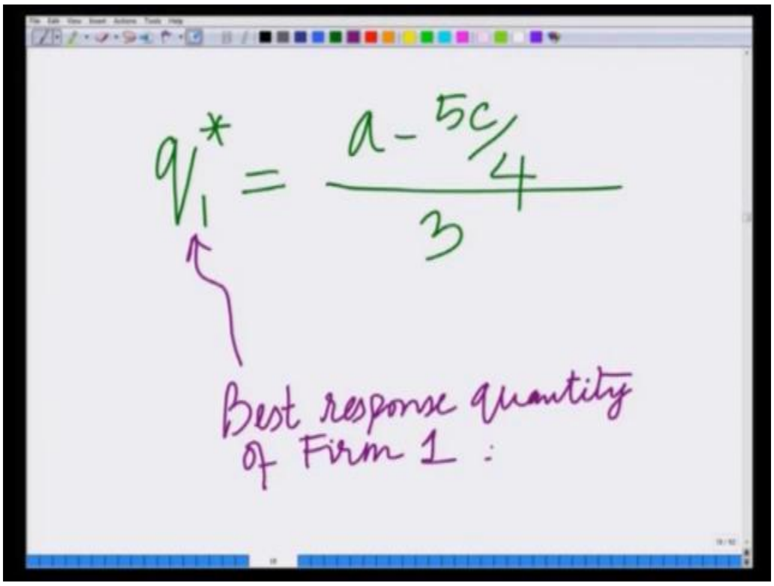

And we can solve this, we can obtain by solving this, one can obtain q 1 star equals a minus 5 c by 4, 5 c by 4 divided by 3. So, this is the best response quantity of firm 1. Now, we can substitute this q 1 star in these expressions, we can substitute this q 1 star in these expression for q 2 L star and q 2 H star to derive the expressions for q 2 H star and q 2 L star.

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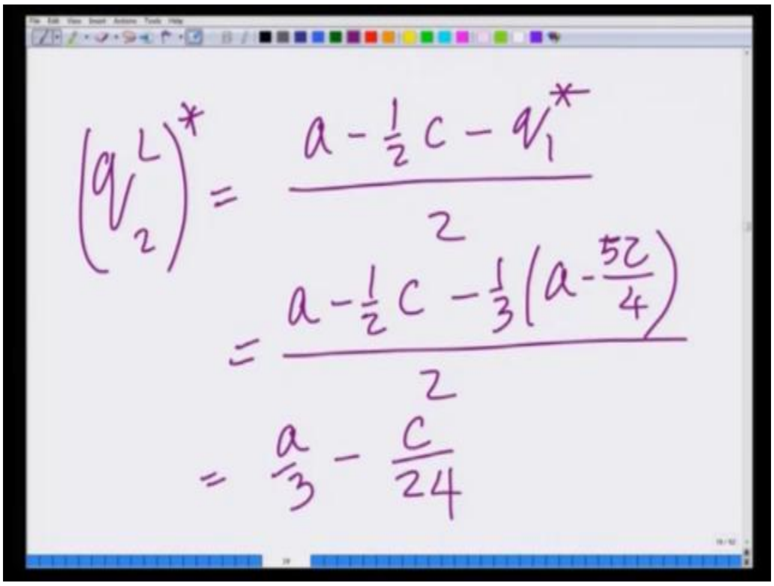

So, we have q 2 or q 2 L star equals a minus half C minus q 1 star divided by 2, which is a minus half C minus one-third a minus 5 C by 4 divided by 2 equals a by 3 minus C by 24 star; that is q 1 star that is the best response of or the Nash equilibrium quantity of firm 2 of type low.

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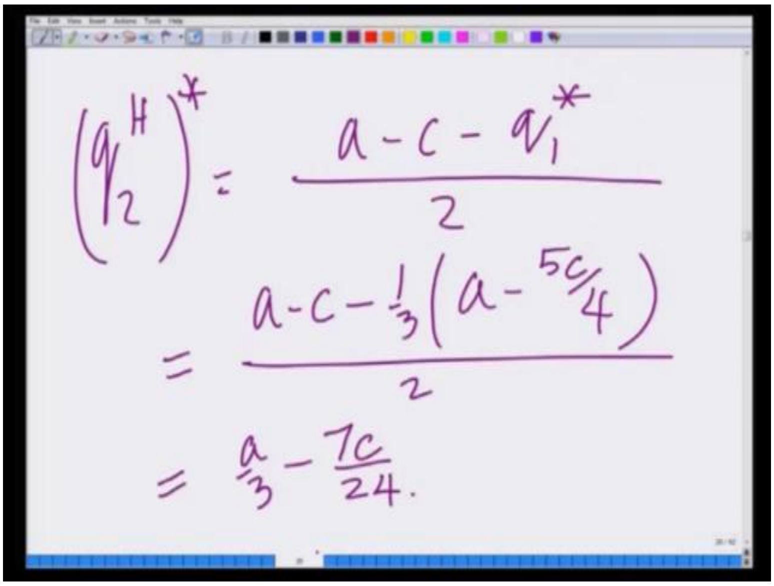

And similarly, q 2 H star is equal to, once second substituting q 1 star, we have a minus C minus q 1 star divided by 2, which is equal to a minus C minus one-third a minus 5 c divided by 4 divided by 2, which is equal to a by 3 minus 7 C by 24. Therefore, we have derived the expressions for q 1 star, q 2 L star and q 2 H star.

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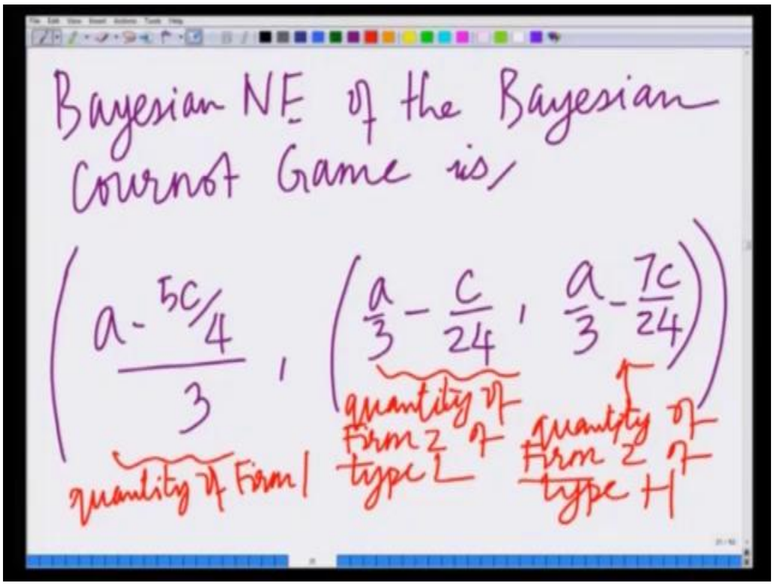

So, we have derived the Bayesian Nash equilibrium, Bayesian Nash equilibrium of the Cournot, Bayesian version of the Cournot game is therefore, well q 1 star equals a minus 5 C by 4 divided by 3 q 2 L star equals a by 3 minus C by 24 and q 2 H star equals a by 3 minus 7 C by 24. And what are these, this is the high quantity of firm 1, this is the quantity firm 2 of type low and this is the quantity H.

So, we have derived the Bayesian Nash equilibrium or the BNE of the Bayesian version of the Cournot game, what I have derived, we have derived the equilibrium quantities of each type at the Bayesian Nash equilibrium. We are saying that the quantity produce by firm 1 in the Bayesian Nash equilibrium is a minus 5 C by 4 divided by whole divided by 3. The quantity produce by firm 2 of type low is a by 3 minus C by 24 and the quantity produced by firm 2 of type H is a by 3 minus 7 C by 24.

Therefore, we have analyze a Bayesian version of this Cournot game in which, there is uncertainty regarding the type and payoffs of firm 2 and what we have derived is way of derived the Bayesian Nash equilibrium, this game by computing the quantities producing by each firm of each type at the Bayesian Nash equilibrium. So, this concludes, this example on a Bayesian version of the Cournot game.

Thank you very much.