Strategy: An Introduction to Game Theory
Prof. Aditya K. Jagannatham
Department of Electrical Engineering,
Indian Institute of Technology, Kanpur

Lecture-35

Hello, welcome to another module in this online course Strategy, An Introduction to Game Theory. So, we have started looking at Bayesian games and Bayesian Nash equilibrium, what we are going to do in this module is to consider another example of a Bayesian game. We are going to consider a Bayesian version, a Bayesian game titled as yield versus fight.

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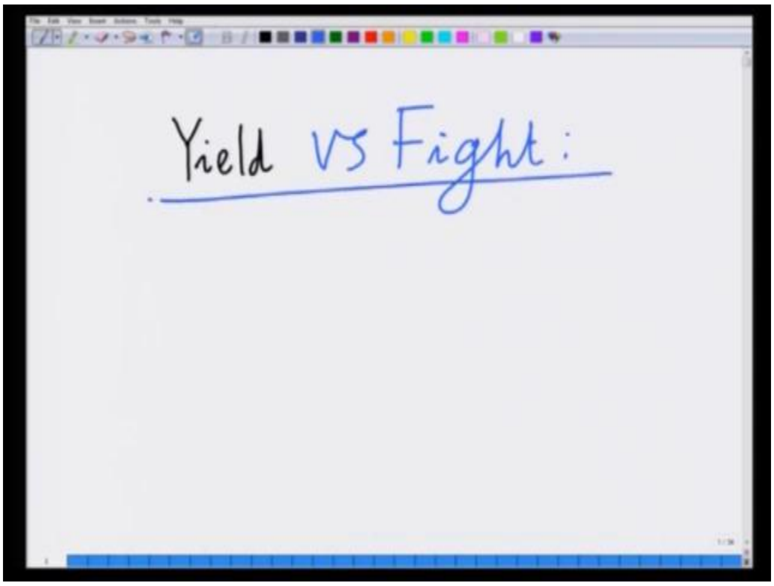

So, we are going to consider an example titled as yield versus fight, this is similar to an example of a confrontation that occurs between two people.

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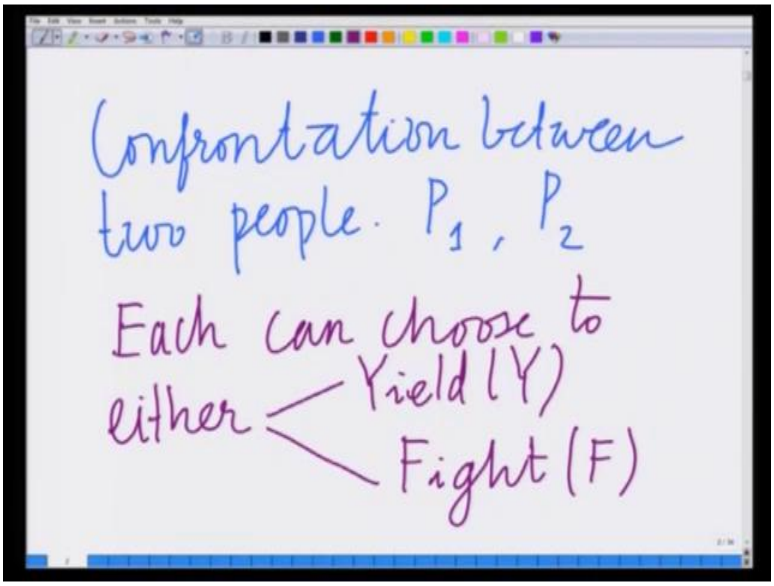

So, we will considering a confrontation a scenario of a confrontation between two, where each person can choose, so there are two people, let us term them, let us denote them by p 1, p 2; that is player 1 and player 2. And each person can choose, each can choose to either yield that is Y or fight; that is F. So, each person has two actions to either yield or fight. In addition, there is only one type of player 1 or person 1. However, there are two types of player 2, player 2 can either be strong or player 2 can either be weak.

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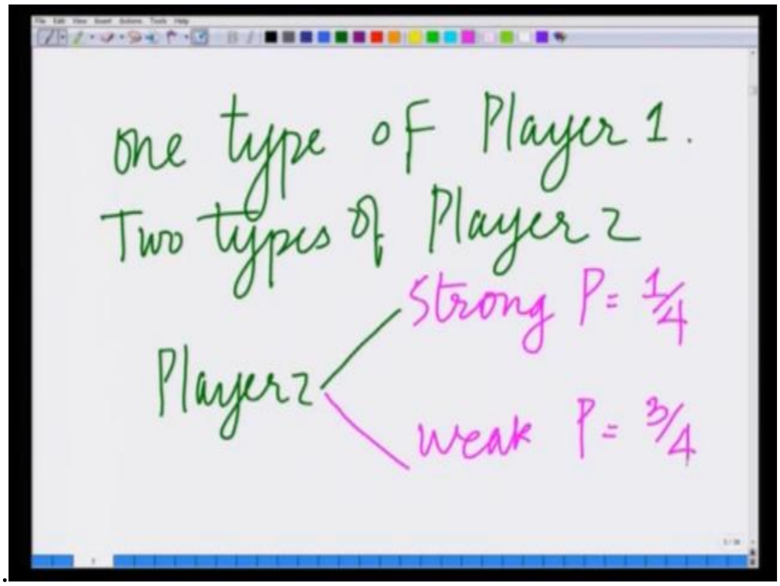

So, there is let us introduce uncertainty, there is one type of player 1. However, there are two types of player 2, player 2 can either be strong which occurs with probability equals one-fourth or player 2 can be weak, which occurs with probability, regarding probability 3 by 4. So, what we are saying is that, there are two types of player 2, there is uncertainty regarding the nature and therefore, the payoffs of player 2. Player 2 can either be of types strong which occurs with probability 1 by 4 or player 2 can be of type weak, which occurs with probability 3 by 4.

So, there are going to be two game tables, similar to what we had seen in the battle of sexes is going to be one game table, corresponding to player 2 of type strong. There is going to be one game table corresponding to player 2 of type. Therefore, this game is Bayesian in nature, why is this game Bayesian in nature, because there is uncertainty regarding the payoffs of player 2, the type of player 2 or the payoffs of player 2. So, therefore, let us write the game tables for this game.

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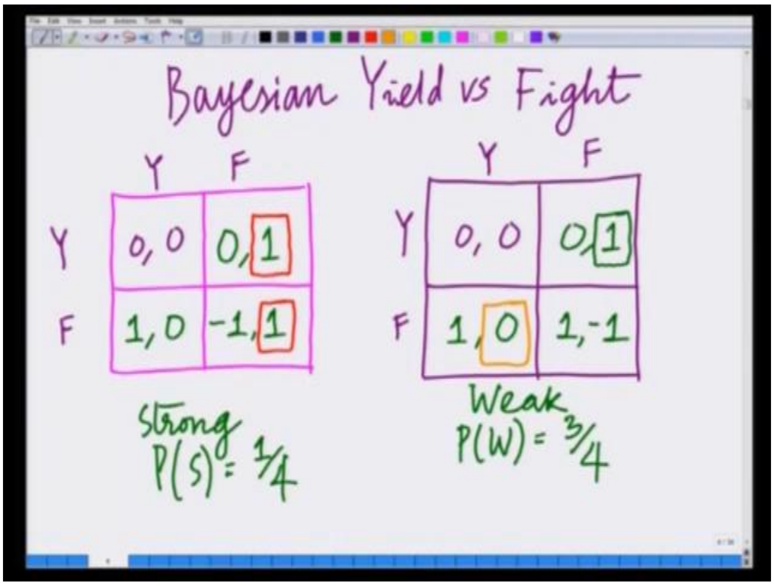

There are going to be again two game tables for this game, one game table corresponding to, well player 2 of type strong and one player, one game table corresponding to player 2 of type weak. Each player can choose to either, yield or fight, yield or fight, yield or fight, yield or fight, if both of them yield, both of them get 0. If player 1 yields and player 2 fights, then player 1 gets 0 and player 2 gets 1.

Similarly, if player 2 yields and player 1 fights, then player 1 gets a payoff of 1 and player 2 gets a payoff of 0. Basically, what we are saying is, if one person chooses to yield and other person chooses to fight, then the person, who is yielding gets 0 and the person, who is fighting gets a payoff of 1. So, the person who is yielding are basically chooses not to fight or chooses to walk away from that confrontation, walks away with the payoff of 0, when the person, who indulges in the confrontation or the person who fights gets a payoff of 1.

So, if player 1 yields and player 2 fights, then player 1 gets a payoff of 0, player 2 gets a payoff of 1. On the other hand, if player 1 fights and player 2 yields, player 1 gets a payoff of 1, player 2 gets a payoff of 0. Similarly, in the other table corresponding to the type weak of player 2, if player 1 yields and player 2 fights, we have 0, 1. If player 1 fights and player 2 yields, then we have 1 comma 0.

Now, the only remaining option and by the way let me remind you, this first table corresponds to type strong of player 2 and the probability are strong equals 1 by 4 and the second table corresponds to weak type, weak of player 2. And the probability of weak in naturally 1 minus 1 by 4 equals 3 by 4. Now, if player 1 chooses to fight and player 2 also chooses to fight and player 2 is of type strong, then obviously, the strong person is going to win, strong player 2 is going to win.

Therefore, the payoff of player 1 is going to be minus 1 and 1; that is if both choose to fight and player 2 is strong, then the payoff of player 1 is minus 1, payoff of player 2 is 1. And however, if both choose to fight and player 2 is weak, then obviously, the weak player 2 is going to lose the fight. There in that scenario, the payoff of player 1 is going to be 1 and the payoff of player 2 is going to be minus 1. So, these are the game tables for this game.

When both of them yield, both of them get a payoff of 0, when one yields and the other fights, the person who is yielding gets the payoff of 0 and the person who is fighting gets a payoff of 1. This happens in player 2 of both types. However, what is different corresponding to player 2 of the two different types is that, if both fight, then if player 2 is strong, he gets a payoff of 1, player 1 gets a payoff of minus 1.

While, if both fight and player 2 is weak, then player 2 gets a payoff of minus 1 and player 1 gets a payoff of 1. So, this is our Bayesian yield versus fight game and these are the game tables for the Bayesian yield versus fight game. So, these are the two game tables, one game table corresponding to each type of player 2, the left game table corresponds to player 2 of type strong, which occurs with probability 1 by 4. The right table corresponds to player 2 of type weak, which occurs with probability 3 by 4, therefore, this is the Bayesian yield versus fight game.

Since, there is uncertainty regarding the type and payoffs of player 2. Now, how do we analyze this game? We know from the battle of sexes game, that to start analyzing this game we have to first compute the average payoffs to player 1 as a function of the strategy or as a function of the actions of player 2 of different types.

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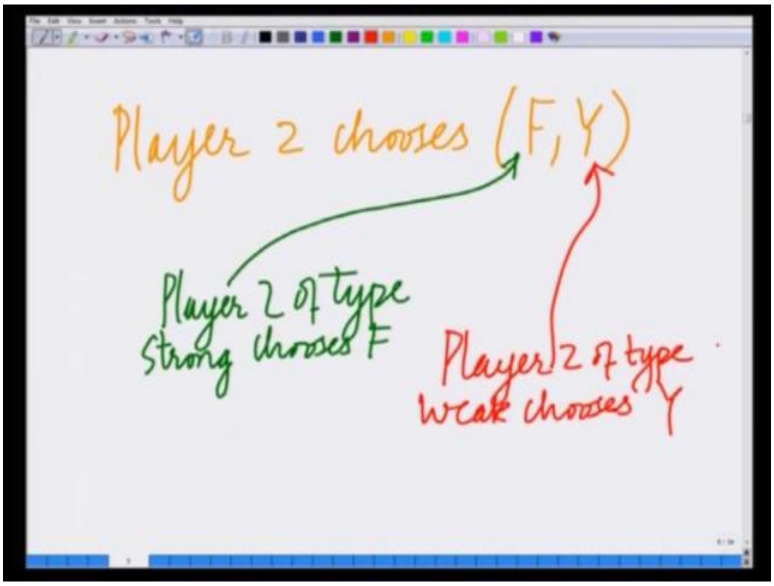

So, let say player 2 is choosing Y F comma Y; that is player 2, let say player 2 chooses the strategy, let me remind you again, F comma Y. What is this mean? This means player 2 of type strong chooses F and player 2 of type weak chooses Y and player 2 of type weak chooses Y, player 2 of type strong chooses F and player 2 of type weak chooses Y. So, player 2 is choosing F comma Y.

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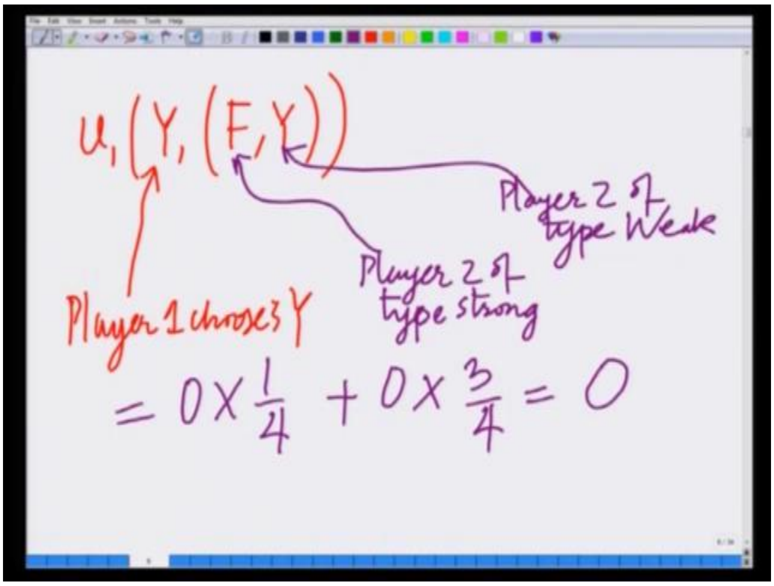

If player 2 is choosing F comma Y and let say player 1 is choosing Y. So, if player 1 is choosing Y and player 2 is choosing F comma Y, then what is the average payoff of player 1; that is denoted by u 1; that is player 1 choosing Y. Player 2 of type strong chooses F, player 2 of type weak is choosing to yield. What is the average payoff? Well the average payoff is, we can write this as now let us go back to the game table ((Refer Time: 11:06)), player 1 is choosing yield, player 2 of type strong is choosing fight.

Therefore, we are in this box the average payoff is player 1 is 0 multiplied by the probability 1 by 4. So, it is 0 times 1 by 4. So, this is equal to 0 times the probability 1 by 4 plus we also have player 1 choosing Y and player 2 of type weak is choosing Y, in which case the payoff to player 1 is 0 multiplied by the corresponding probability 3 by 4. So, this is again another 0 into the corresponding probability 3 by 4.

So, let me repeat that argument ((Refer Time: 12:02)) we are considering the case, where player 1 is choosing Y, player 2 of type strong is choosing F, player 2 of type weak is choosing Y. So, corresponding to player 1 and player 2 of type strong, who is choosing F player 1 is choosing Y, player 2 of type strong is choosing F, payoff of player 1 is 0, 0 multiplied by the probability of type strong into 1 by 4 plus.

Again, player 1 is choosing Y player 2 of type weak is choosing also choosing Y, corresponding to that the payoff is 0 multiplied by the probability 3 by 4. So, we have 0 into 1 by 4 plus 0 into 3 by 4 equal to 0. So, we have the utility of player 1, when he is choosing Y and player 2 is choosing F comma Y. That is player 2 of type strong is choosing F player 2 of type weak is choosing Y is 0, the average payoff is 0.

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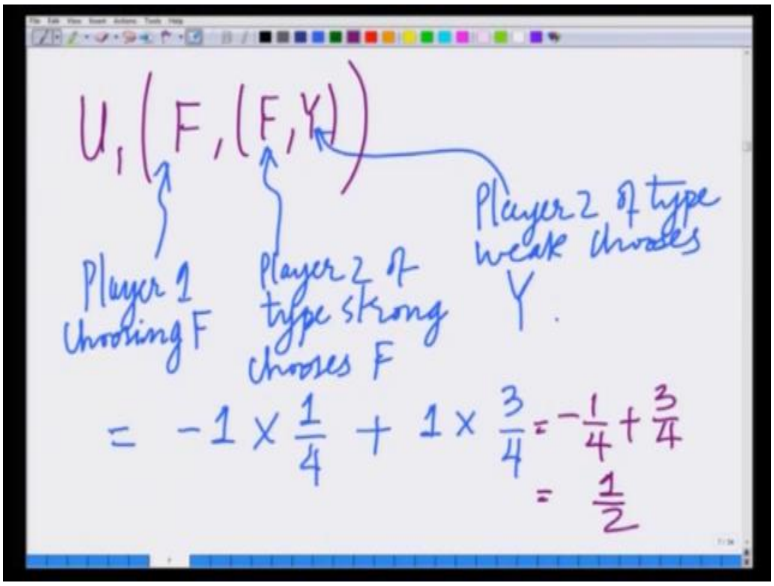

Now, let as consider compute the average payoff of u 1 of F comma F comma Y; that is player 1 choosing F, player 2 of type strong chooses F, player 2 of type weak chooses Y. So, we are considering now the average payoff of player 1 corresponding to his choice of F, player 2 of type strong choosing F and player 2 of type weak choosing Y. This payoff is given as now if you look at it, ((Refer Time: 14:12)) player 1 is choosing F, player 2 of type strong is choosing F.

In which case the payoff to player 1 is minus 1 multiplied by the probability 1 by 4, so therefore, the first component is minus 1 multiplied by the probability 1 by 4 plus. In addition, we have player 1 choosing F, player 2 of type weak choosing Y, corresponding to which payoff of player 1 is 1, multiplied by the probability 3 by 4. So, this will be 1 into 3 by 4. So, this is minus 1 into 1 by 4 plus 1 into 3 by 4; this is basically equal to minus 1 by 4 plus 3 by 4, which is equal to a half.

Therefore, the payoff to average payoff to player 1 corresponding to his choosing fight F and player 2 of type strong choosing fight and player 2 of type weak choosing Y equals a half. And how did we arrive at this, we arrived at this because, let me repeat that argument ((Refer Time: 15:22)) if player 1 is choosing fight, player 2 of type strong is also choosing fight, payoff of player 1 is minus 1 multiplied by the probability 1 by 4 plus.

If player 1 is choosing fight player 2 of type weak is choosing yield, payoff of player 1 is 1 multiplied by the probability, which is 3 by 4. So, minus 1 into 1 by 4 plus 1 into 3 by

4, which is 3 by 4 minus 1 by 4 equals a half. So, therefore, we have computed the average payoffs of player 1 choosing Y and F comma Y of player 2 and player 1 choosing F and F comma Y half player 2.

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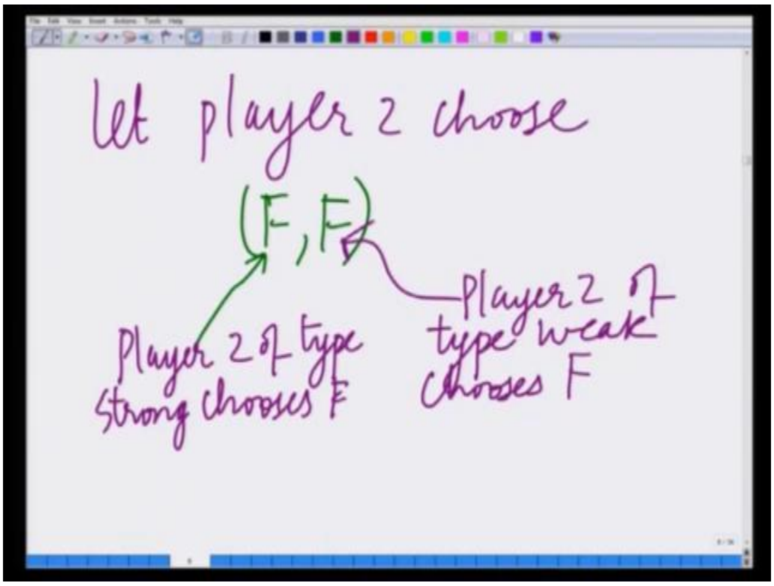

Similarly, let us also now compute, let us choose another strategy. Let us choose, let player 2 choose F comma F; that is player 2 of type strong chooses F and player 2 of type weak also chooses F. Let player 2 the strategy of player 2 be F comma F; that is player 2 of type strong is choosing F and player 2 of type weak is also choosing F.

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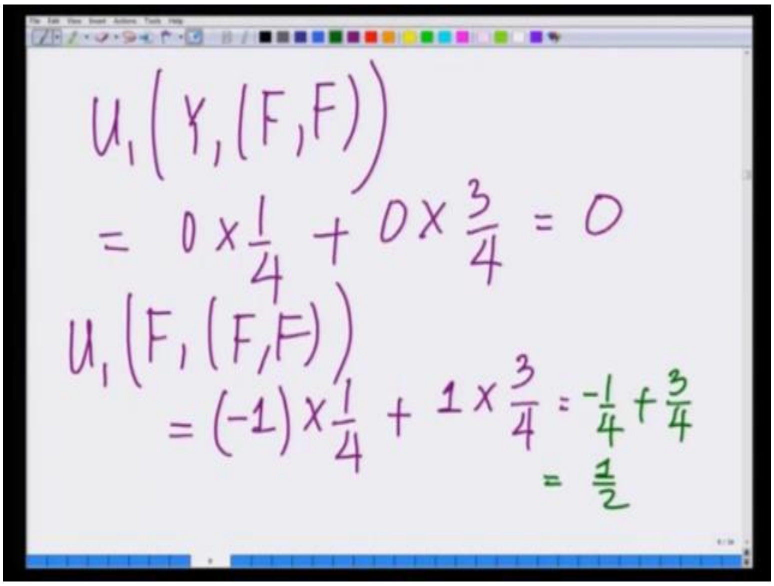

Then, what is u 1 of well u 1 of Y comma F comma F; that is what is u 1 of Y comma F comma; that is player 1 is choosing to yield, player 2 of type strong is choosing fight and player 2 of type weak is also choosing to fight. And this quantity is given as this is equal to well, let us go back to our game table, player 1 is choosing yield, player 2 of type strong is choosing fight. So, we pay a payoff of player 1 is 0, 0 multiplied by the probability 1 by 4.

Again, player 1 is choosing yield, player 2 of type weak is also choosing to fight, payoff to player 1 is again 0. So, 0 multiplied by the probability 3 by 4. So, this is well 0 into 1 by 4 plus 0 into 3 by 4 equals 0. So, the average payoff correspondent to player 1, who is yielding and player 2 of both types was fighting is basically 0. Because, player 1 is yielding is average payoff, his payoff corresponding to both player 2 of type strong and player 2 of type weak is 0.

Because, he is choosing to yield or walk away from that fight is payoff, average payoff is that therefore 0 into 1 by 4 plus 0 into 3 by 4 equals 0. On the other hand, if player 1 chooses to fight and player 2 of both types is also choosing to fight, then the payoff is again let us go back to this player 1 is choosing to fight, ((Refer Time: 18:51)) player 2 of type strong is choosing to fight. In which case payoff of player 1 is well minus 1 multiplied by the probability, which is 1 by 4.

And again player 1 is choosing to fight, player 2 of type weak is also choosing to fight corresponding to which payoff of player 1 is 1 multiplied by the probability 3 by 4. So, the average payoff can be obtained as well minus 1 multiplied by the probability 1 by 4 plus 1 multiplied by the probability 3 by 4, which is equal to again minus 1 by 4 plus 3 by 4, which is equal to half.

So, the pay average payoff to player 1 correspond to his strategy choice his action choice of F and action choice F comma F of player 2 of type; that is where player 2 of both types is choosing to fight is equal to a half. How did we arrive at this? Let us again look at that, ((Refer Time: 20:02)) player 1 is choosing to fight player 2 of type strong is choosing to fight.

In which case payoff of player 1 is minus 1 minus 1 multiplied by the probability 1 by 4 plus player 1 is choosing fight, player 2 of type weak is also choosing to fight corresponding to which payoff of player 1 is 1, 1 into 3 by 4. So, it from good payoff is minus 1 into 1 by 4 plus 1 into 3 by 4, which is equal to a half. So, therefore, we can now write the average payoff, we can now compute an average payoffs for the player 4, player corresponding to the strategy combinations F Y and F F.

So, we have computed the average payoffs of player 1 corresponding the strategy combinations F Y and F F of player 2. Now, we also have to compute the strategy, the payoffs, the average payoffs of player 1 corresponding to, you also have to compute the average payoffs of player 1 corresponding to Y F and Y Y of player 2.

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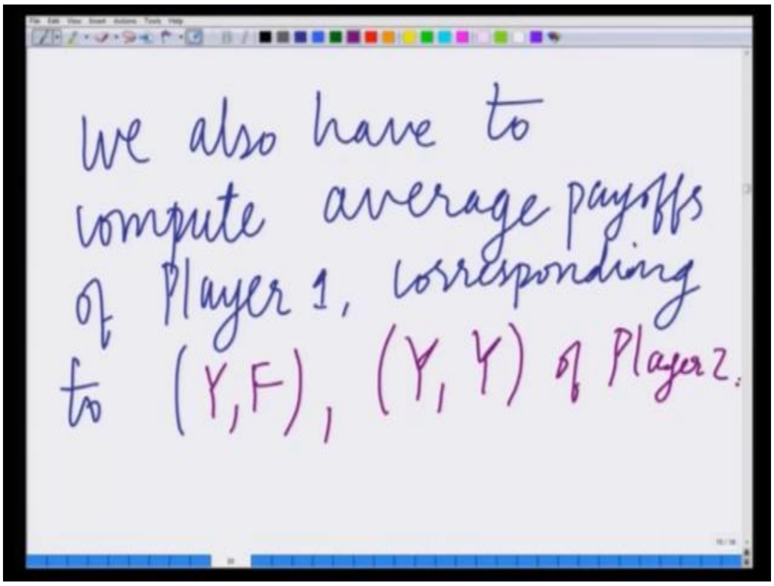

So, let me write that here, we also have to compute, so of player 1 corresponding to well Y comma F and Y comma Y of player 2.

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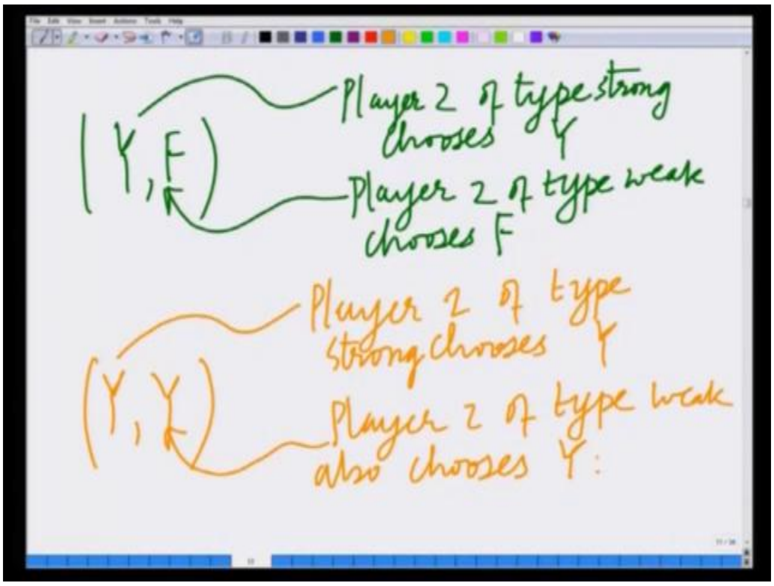

Well, what is Y comma F Y comma F means that player 2 of type strong chooses Y and player 2 of type weak chooses F. So, Y comma F strategy Y comma F of player 2 denotes well where player 2 of type strong chooses to yield and player 2 of type weak chooses to fight. And the combination Y comma Y, if for player 2 denotes, player 2 of type strong chooses to yield and player 2 of type weak also chooses Y; that is where player 2.

So, Y Y strategy Y Y of player 2 denotes a strategy, where player 2 of both types; that is player 2 of type strong and player 2 of type weak, both of them choose yield. Now, here in this game interestingly, we do not need to compute the average payoffs of player 1 corresponding to these strategies of player 2. The reason being if you are go back to the game table, ((Refer Time: 23:50)) let us now try to find the best responses of player 2.

Now, the best response, remember player 2 knows is time, the best response of player 2 of type strong is, if we have to look at the table on the left, if player 1 is choosing yield the best response of player 2 of type strong is to fight. Because, fighting is in the payoff of 1, yielding is in a payoff of 0. Similarly, if player 1 chooses to fight, the best response of player 2 of type strong is to again fight, because fighting gives in the payoff of 1, yielding gives in the payoff of 0.

Therefore, interestingly if you look at this game table, you will see that Y is never a best response for player 2 of type strong. So, another words the other way to say this is that the action F 4 player 2 of type strong dominates the action Y for the player 2 of type strong, because F always yields in a higher payoff corresponding to Y. Therefore, in any equilibrium any Nash equilibrium of this game player 2 of type strong is not choosing Y, because Y is dominated by F.

That is for player 2 of type strong y always yields a lower payoff of that F. So, both this strategies remember Y F and Y Y assume that player 2 of type strong is choosing Y, but Y is a dominated strategy for player 2 of type strong.

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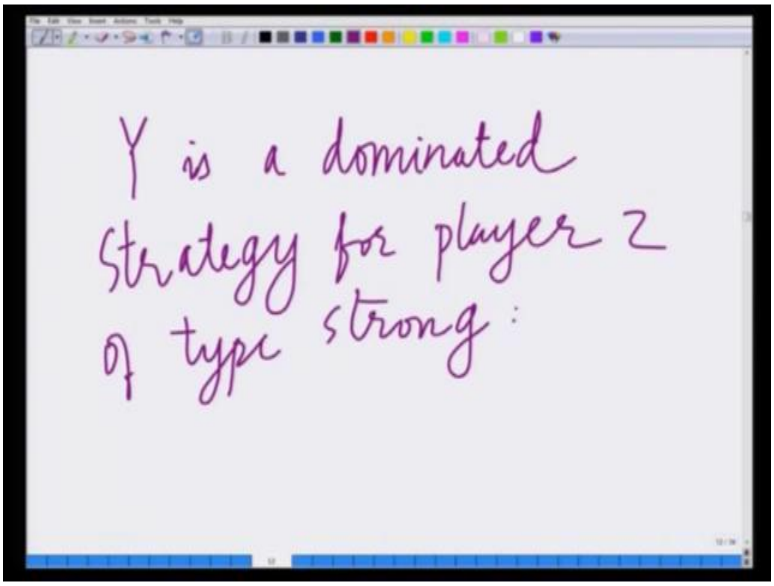

So, let me write that idea, summarize that idea are here Y is a dominated strategy for player 2 of type strong.

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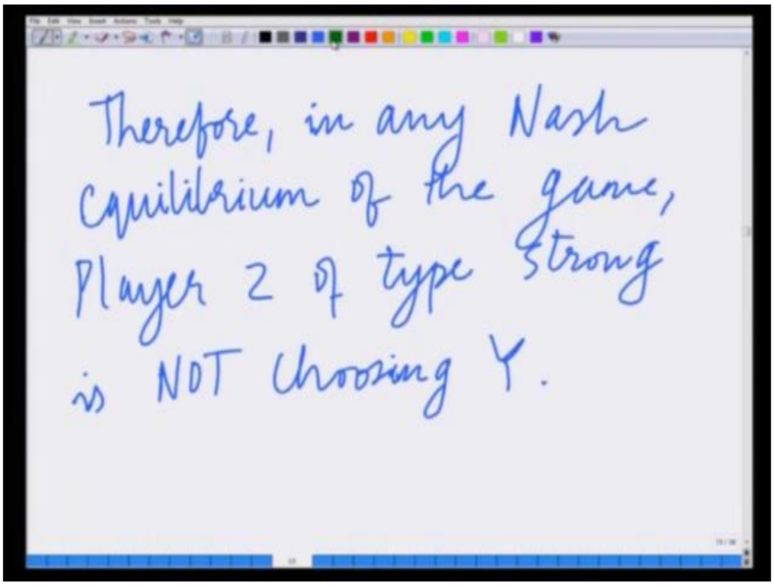

So, in any Nash equilibrium of the game, therefore in any Nash equilibrium of the game player 2 of type strong is not choosing to yield for player 2 of type strong yielding always yields a lower payoff then fighting. So, we all we saw that for player 2 of type strong yield is dominated by fight. So, player 2 of type strong is never choosing to yield therefore, we do not need to consider the strategies Y F and Y Y, in both of which player 2 of type strong is choosing to yield.

I hope this point is clear, therefore, we do not need to consider these strategies; that is Y F and Y Y which player 2 of type strong is choosing to yield, because player 2 of type strong never chooses to yield, because yield is dominated by fight, Y is dominated by F Therefore, we can ignore the strategies Y F and Y Y are player 2. So, this basically summarizes Y, we are going to ignore Y F and Y Y.

Let us now complete the best responses for player 2 corresponding to weak, ((Refer Time: 27:47)) if player 1 chooses to yield, the best response of player 2 is weak is to fight, which gives in the payoff of 1. And if player 1 chooses to fight, the best response of player 2 of type weak is to choose yield, because yield gives in the payoff of 0, while fight gives in the payoff of minus 1. So, these are the best responses of player 2, F type weak.

So, now, what have we done, we have derived the best responses of player 2 of both types. So, we have derive the best responses of player 2 of type strong, the best response of player 2 of type strong is always to fight, because yield is dominated by fight or fight dominates yield. The best response of player 2 of type weak is to basically fight, if player 1 chooses to yield, yield if player 1 chooses to fight, this summarizes the best response of player 2. So, let as stop here and let as continue with this game in the next module to derive the Bayesian Nash equilibrium of this game.