Strategy: An Introduction to Game Theory
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 33

Hello, welcome to another module in this massive open online course Strategy, An Introduction to Game Theory. So, we have started looking at Bayesian games, we just formulated a Bayesian version of the battle of sexes game in the previous module. What we are going to start looking at now is to simplify the payoffs of this Bayesian battle of sexes and to infer the Bayesian version of the Nash equilibrium for this Bayesian game.

So, in this Bayesian, for any Bayesian game, similar to the games that we are consider earlier in pure strategies. Remember in a game with pure strategies, we had assigned one strategy to each player. Now, remember in a Bayesian game, there are players of each player can possibly have different types. So, we have to assign one strategy to each player of different each type.

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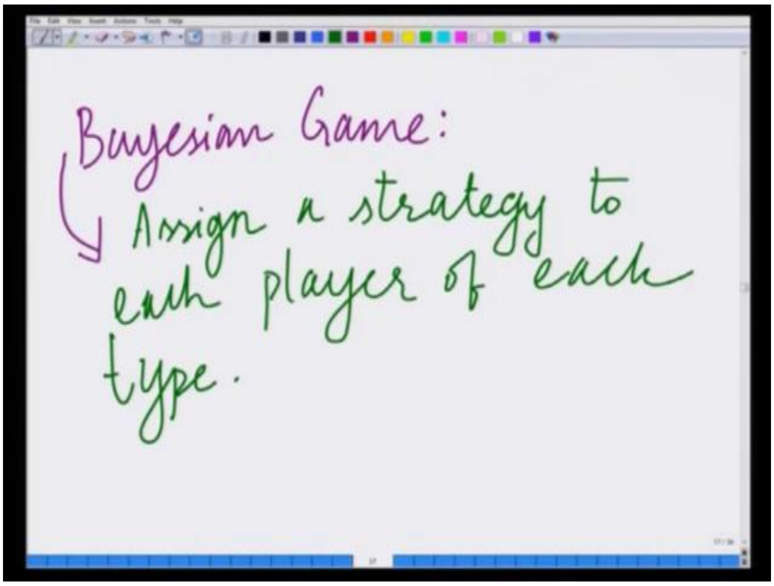

So, the most important thing to remember in a Bayesian game is to assign a strategy to each player of each type. So, the most important thing to remember in a Bayesian game is that; we have to assign a strategy to each player or each type. Remember, the boy and girl can choose in the Bayesian battle of sexes, the boy can either choose C or H, the girl of type I can either choose C or H and also girl of type U can also choose C or H. So, we have to assign a strategy to each player of each type.

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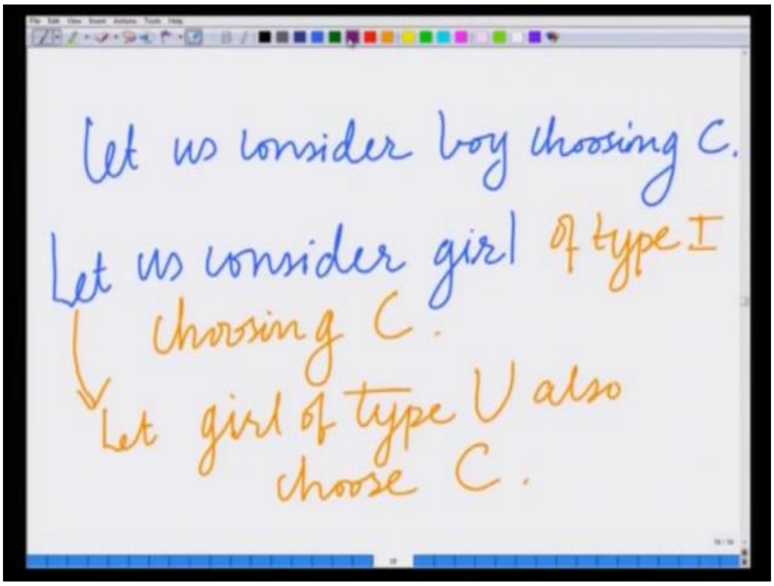

So, let us consider boy, that is player 1 choosing C, let us consider girl, choosing girl of type I, because remember we have to assign strategy to each player of each type. So, player girl to player 2 has two types; let us consider. So, let us consider girl of type I choosing C and similarly, let say girl of type U also chooses C. So, girl of type, so we are saying girl of type I is choosing C to watch cricket, girl of type U is also choosing C; that is to watch cricket.

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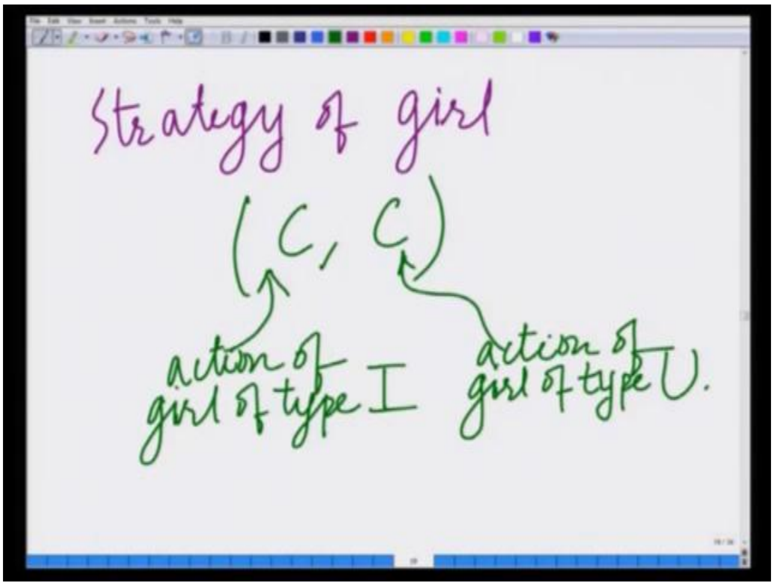

And this can be represented as strategy of girl can be represented as C comma C, where the first one is the strategy or action girl of type I and second one is strategy or action of girl of type U. So, now, because there are two types of player 2; that is girl. We have two actions in the strategy that is one for girl of type I, one for girl of type U and we are saying in this particular strategy, girl of both types; that is payoffs of both types is choosing C.

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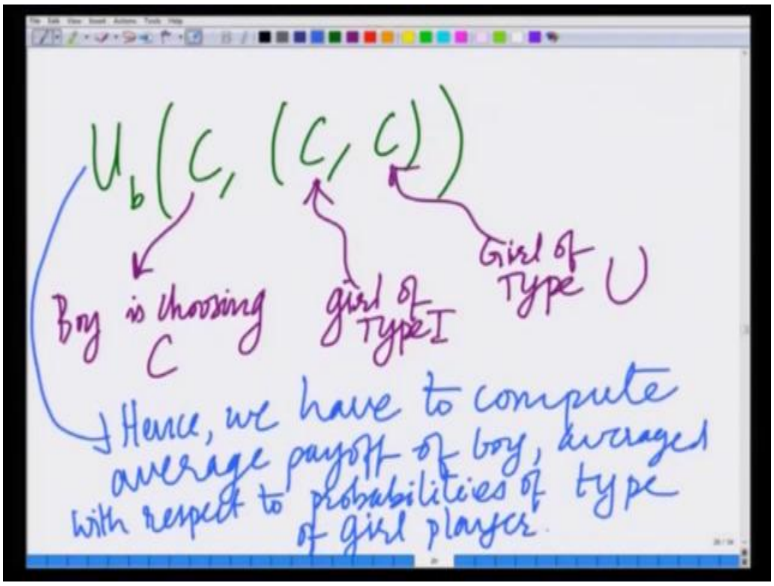

Now, what we have to do is, we have to find the payoff of the boy corresponding to the strategy of the girl. So, what we have to do is, we want to compute the payoff to the boy corresponding to the strategies C C of the girl; that is we are saying boy is choosing C, this C is for and this C is for… So, we have to find the net payoff of the boy choosing C, because even if the boy chooses C, there is uncertainty regarding who is playing against. He might be playing against girl of type I or you might be playing against a girl of type U.

Therefore, we have to compute the average payoff of the boy, average with respect to the probabilities of the different types of the opponent player or the player or the girl player. So, we have to compute the average payoff, average over the different types of the other player or the girl player. So, we have to hence, we have to compute average payoff of boy, average with respect to probabilities of type of the girl player.

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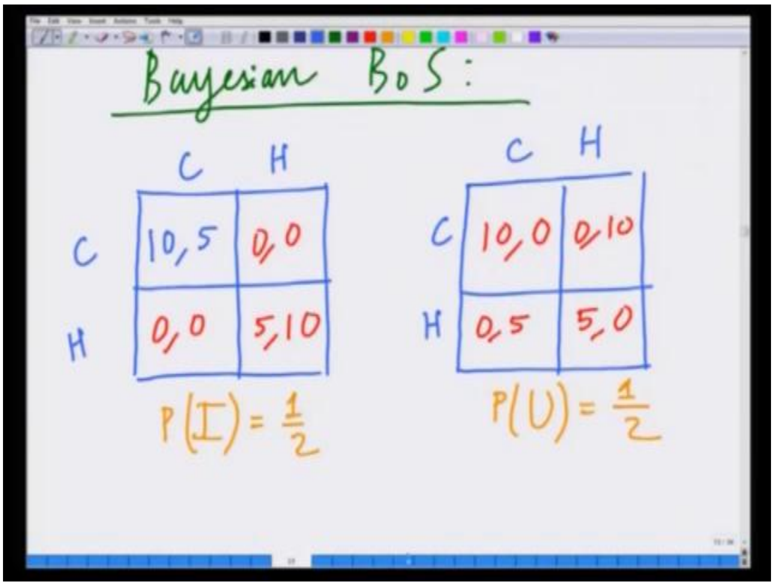

Because, if you go back to the game table ((Refer Time: 06:19)) and you look at the Bayesian battle of sexes in the ((Refer Time: 06:22)). Then, the probability half is going to encounter an interesting, interested game in which case the payoffs is 10 and with probability half is going to encountered, encounter an uninterested, in which case also his payoff is 10. So, is average payoff is half into 10 plus half into 10, which equals 10.

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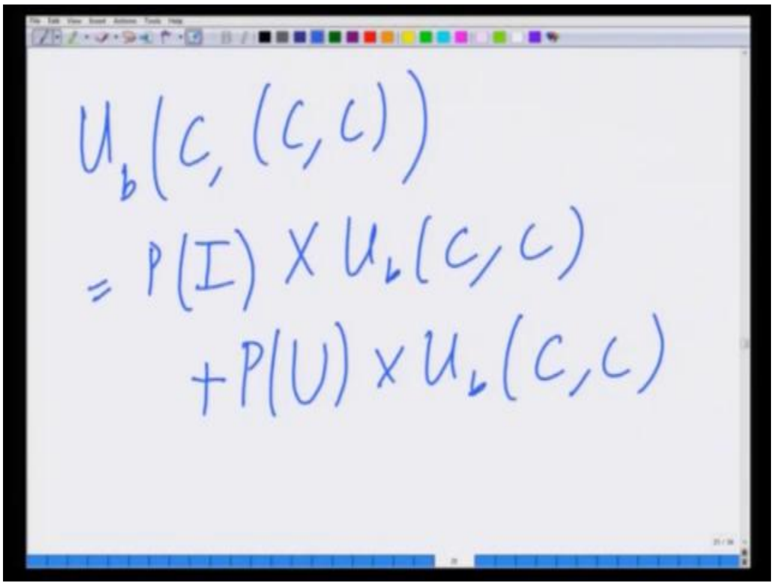

So, therefore, the average payoff of the boy, when we chooses C comma C is a, well probability of meeting a girl, who is interested, times is average payoff corresponding to C comma C. Because, girl was interested in choosing C plus the probability of uninterested times again C comma C, because girl was uninterested she is also choosing

C.

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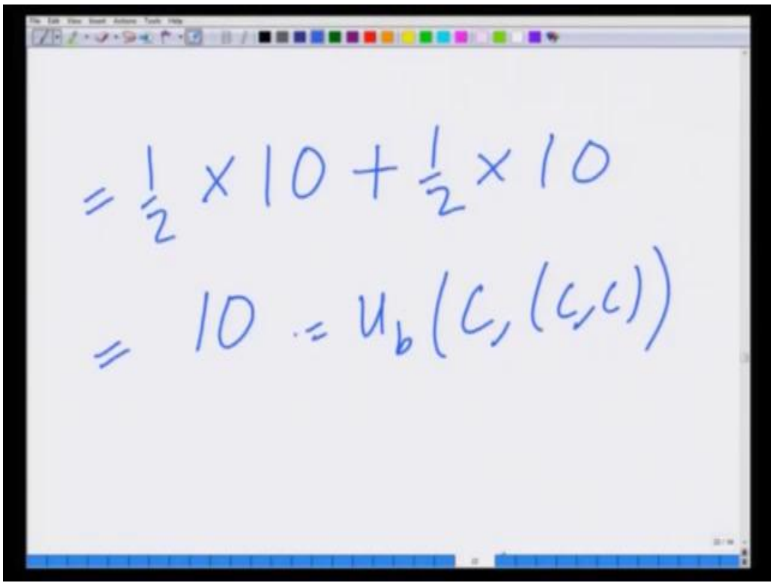

And this, you can get as this is equal to well the probability, again if you go back here, ((Refer Time: 07:34)) the probability that the girl is interested is half and if the boy is choosing C and interested girl is choosing C. Then, therefore, the contribution is half times 10 plus. Now, we have to look at C of the boy versus the girl of type U is also choosing C, in which case the payoff of the boy is 10. So, it will be half into 10.

So, therefore, this will be equal to this average payoff will be equal to half into 10 plus half into 10, which is equal to 10. Let me repeat that again, let me just go through that argument once more, ((Refer Time: 08:27)) we were considering a scenario, where the boy is choosing to watch C; that is cricket girl of type I is choosing to what C, girl of type U is choosing to watch C.

And now also remember from this Bayesian game tables ((Refer Time: 08::41)), girl is of type I with probability half; girl is of type U with probability half. So, half of the time payoff of the boy is, he is choosing C, girl of type I is also choosing C. So, half of the time is payoff is 10, half of the time, when his choosing C, he is encountering girl of type U, who is also choosing C is payoff is 10. So, it is half into 10 plus half into 10, which is equal to 10.

So, payoff average offer his choosing C verses girl of both type choosing C is 10, which is equal to u b of C comma C comma C. Now, let us look at is payoff, if he is choosing H; that is to watch Harry Potter movie, well the girl of both types is choosing to watch C comma C.

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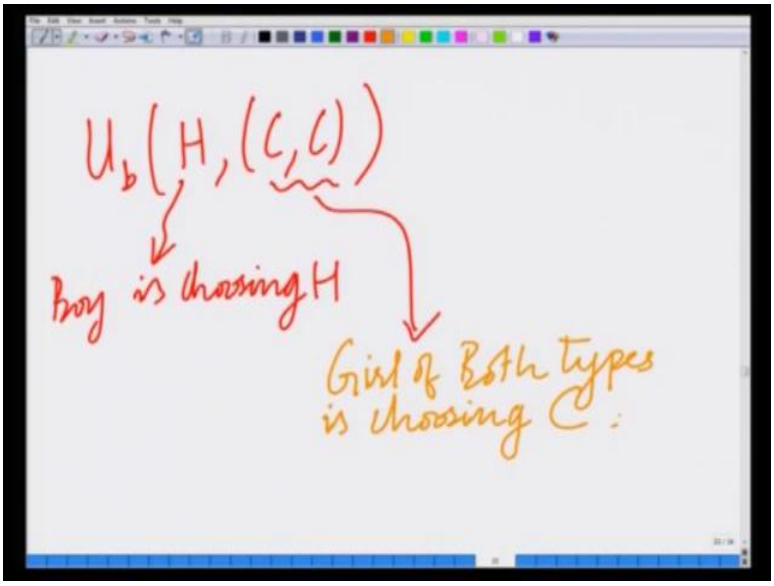

So, let us compute u b of H comma C comma C; that is boy is choosing H girl of both types, boy is choosing H, girl of both types is choosing C. Then, what is it payoff, well if we go back to here, ((Refer Time: 10:13)) if we consider boy is choosing H, girl of type I is choosing C is payoff is 0, multiplied by the probability half and again a boy is choosing H, girl of type U is choosing C, his payoff is again 0, multiplied by half.

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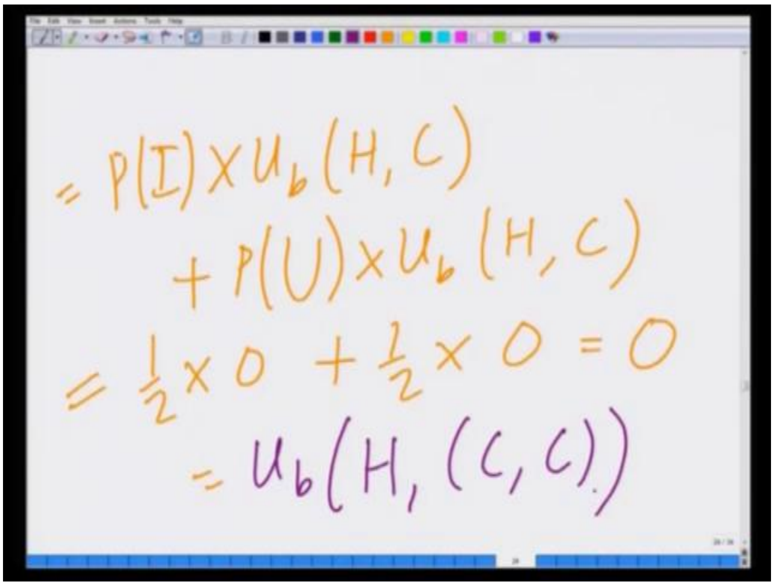

If I write is payoff is going to be, well that is going to b equal to half into u b of that is probability. That girl is interested into payoff corresponding to when boy is choosing H and interested girl is choosing C plus probability, where is uninterested into u b H comma c; that is boy is choosing H, girl who is an interested is choosing C. In both cases is payoff is 0. So, this is simply half into 0 plus half into 0, which is equal to 0. Therefore, corresponding to the scenario, let me also denote this as which is equal to corresponding to u b of H comma C comma C.

So, this is the average payoff corresponding to the choice H by the boy and the strategy C C by the girl, where girl of type I is choosing C, girl of type U; that is uninterested is also choosing C. And this is the average payoff of the boy, which means it has to be average with respect to the probabilities of the different types of the other player or player 2. That is the girl player.

And here, if both the cases when he is choosing H and girl player of type I is choosing C is getting a 0. When, he is choosing H and girl player of type U is choosing C, he is also getting a 0. Therefore, the average is half time 0 plus half time 0, which is equal to 0. Let we do one more to illustrate, how to compute it.

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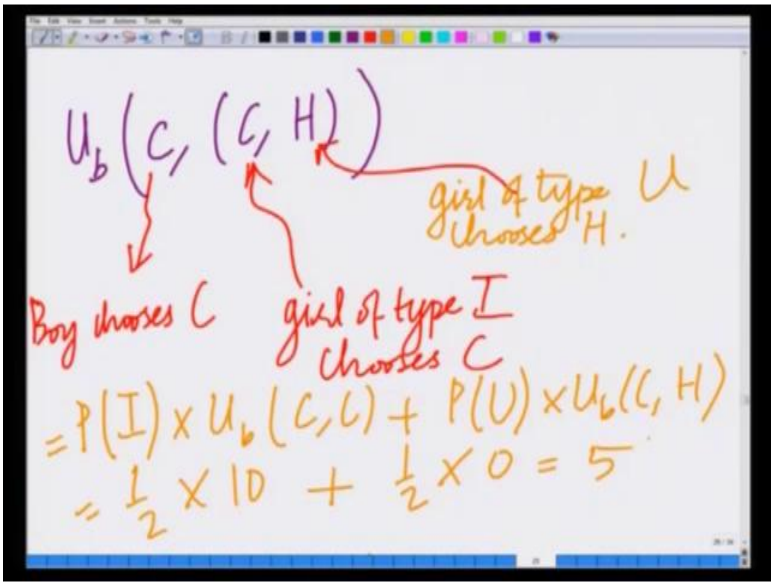

Let us consider this scenario, where the boy is choosing C and the girl is choosing C comma H. So, this is a scenario, where boy is choosing C, girl of type I chooses C and girl of type H or girl of type U chooses H girl of type U chooses H. So, we are consider now difference scenario, where boy is choosing C, girl of type U I is choosing C and girl of type U; that is uninterested is choosing H.

Now, when we consider this scenario, let us again go back to the game table, when boys choosing C girl of type Interested is choosing C, multiplied by the probability half plus, when boy is choosing C, girl of type uninterested is choosing H. And corresponding to that payoff to the boy is 0, multiplied by the probability half. So, again going back ((Refer Time: 13:50)), we have this can be express as probability that the girl is interested times u b of C comma C plus probability.

That the girl is uninterested times u b of C comma H, which is equal to half times u b of C comma C; that is when boy is watching cricket, girl is watching cricket and girl is of type of i is 10 plus half into probability of u. When the girl is type U times when boy is watching cricket and girl of type U is watching H is payoff 0, which is equal to 5.

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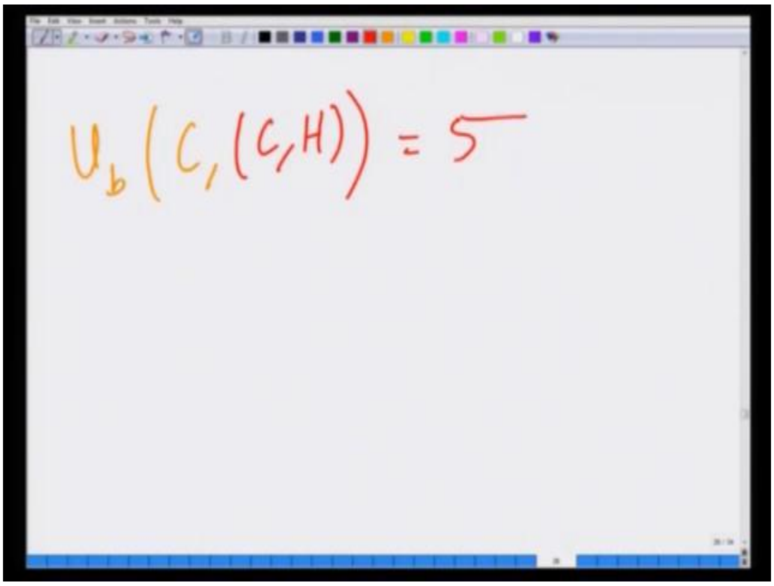

So, we have u b of C comma C comma H is equal to 5; that is in the boy is watching cricket girl of type interested is choosing cricket girl of type uninterested is watching H or the movie, the payoff the average payoff of the boy is 5.

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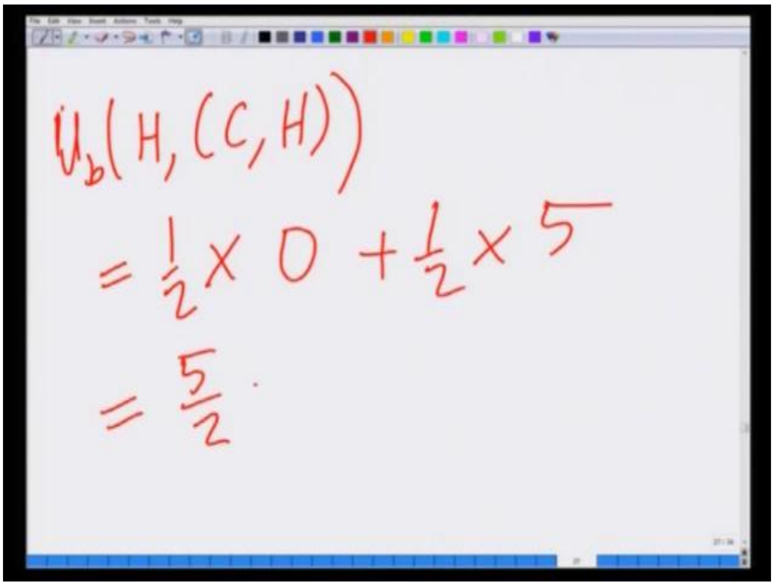

Similarly, now let say the boy chooses H u b; that is boy is choosing H, girl is choosing C comma H; that is girl of type I is choosing C, girl of type U is choosing H, then the payoff is given as if we go back again here. You can see boy is choosing H girl of type I is choosing C in which cases payoff a 0, multiplied by half plus when boy is choosing H girl of a girl of type U also choosing H in which cases payoff is 5, multiplied by half.

So, the net payoff will be if we can go back to this ((Refer Time: 15:53)) this net payoff will be given as you can verify this will be given as half times 0 plus half times 5, which is equal to 5 by 2. So, his average payoff that is when the boy is choosing to watch h girl of type I is choosing to watch C, girl of type U is choosing to watch H. In this sort of scenario is average payoff, average with respect to the probabilities of different types of the other player; that is the girl is given as half times 0.

Corresponding to when, he is watching H, girl is watching C and girl is of type I plus half into 5, corresponding to when he is choosing to watch H, girl of type U is choosing to also watch H. And therefore, the net payoff is given as half into 0 plus half into 5 is equal to 5 by 2. Similarly, we can consider other strategy. So, right now, we have consider C C and C H for the girl.

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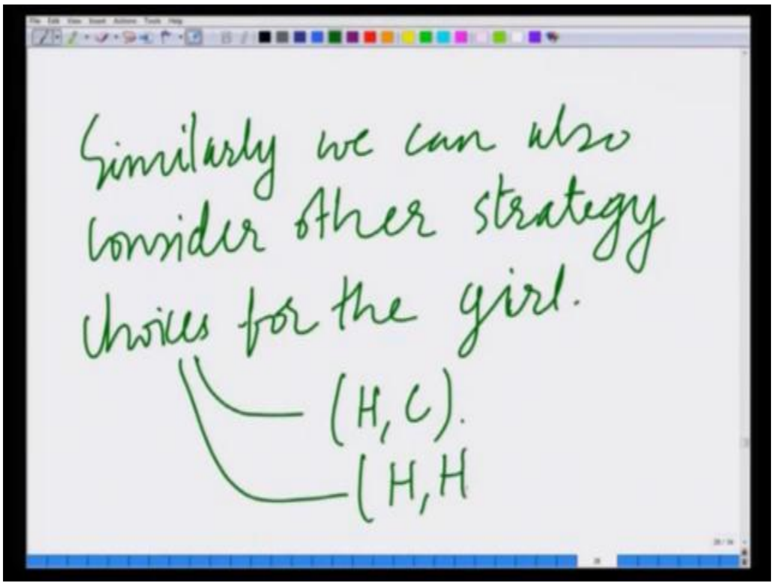

Similarly, we can also consider other strategy choices for the girl; that is way of consider C C and C H. We can also consider H C and H H and again derive the average payoff corresponding to different actions of the boy.

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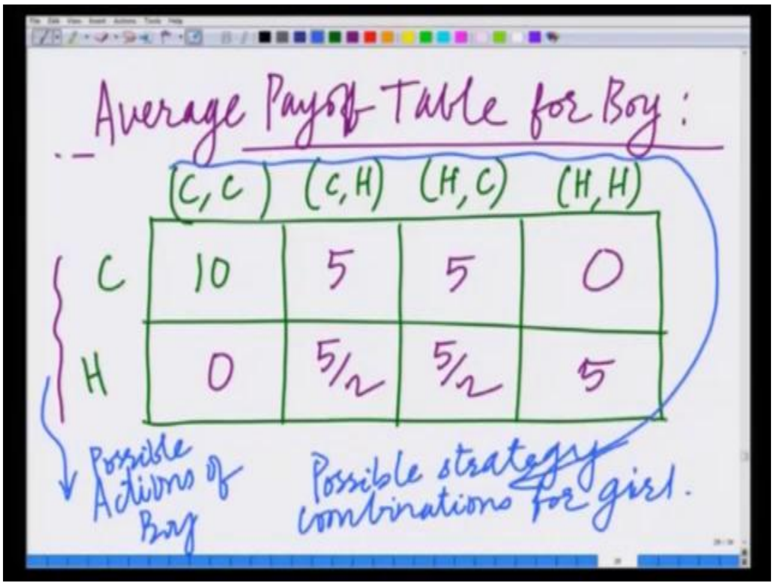

So, the net, if you draw this now, this average payoff table for the boy, so what I am going to draw now is, if I draw the average payoff table for the boy, it is going to look like this. Well, I am going to have four different possible strategy combinations for the girl. So, for the boy can choose there C of H, C or H, girl can choose either C or C or C or H or H or C or H comma H.

We already computed the average payoff for boy choosing C girl choosing C comma C that is girl of type I choosing C, girl of type U choosing C and this way calculated as 10 boy choosing H girl choosing C C this you calculated as 0. Now, we also calculated, boy choosing C girl choosing C H, this is 5, boy choosing H, girl choosing C H; that is 5 by 2, boy choosing C girl choosing H comma boy choosing C girl choosing H comma C; that is a game 5.

We are not computed this, but you can computed similarly boy choosing H girl choosing H comma C; that can also be computed as 5 by 2. Finally, boy choosing C girl choosing H comma H; that is girl of type I choose girl of type U also choosing H that can be computed as 0. Boy choosing H, girl choosing H comma H; that is girl of type I choosing H, girl of type U choosing H; that can computed as 5.

So, now, we have computed the average payoff table for the boy. What is this average payoff table, this is the average payoff the girl and boy is going to see, for each of the strategies C or H verses the various strategy combinations of the girl. That is where you are assigning one action for each girl or of each type. Therefore, the different strategy combinations of the girl or either C C or C H or H C or H H and therefore, this is the average pay of table for the boy.

So, the rows here are the actions of the possible actions and the columns here possible strategy, these are the possible strategy combinations for them. So, these are the possible strategy combinations for the girl, because we are assigned at one strategy for each girl of each type. In this order pair in the first action represents the action of the girl of type I, a second action represent the action of the girl of type U. So, therefore, we have computed this average payoff table.

So, let us take a break at this point and the next module, we are going to talk about, how to derive the Bayesian Nash equilibrium for this game. So, what we have done now is, if derived the or we have computed the average payoffs for both the boy, for the boy player and next we are going to talk about, how to derive the Bayesian Nash equilibrium for this game

Thank you very much.