

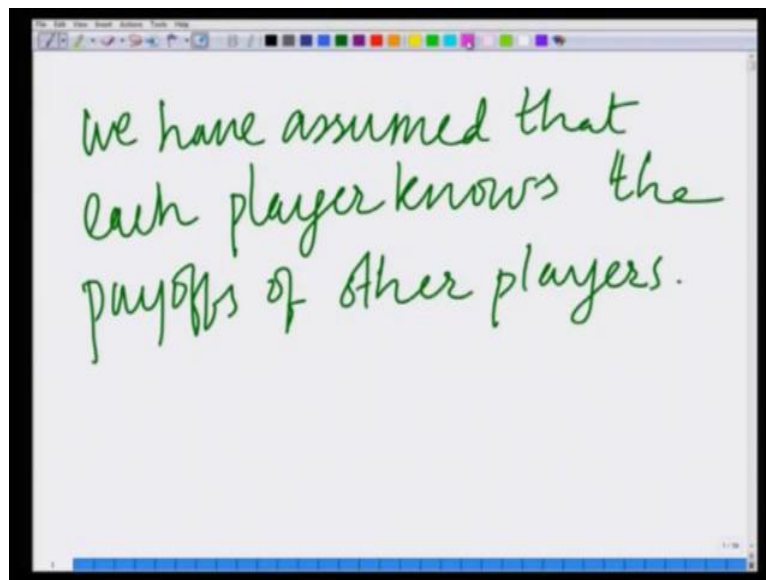
Strategy: An Introduction to Game Theory
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Lecture - 32

Hello and welcome to another module in this massive open online course Strategy, An Introduction to Game Theory. And, so far we have looked at different kinds of games, we have looked at the basic forms of games would be in game tables and then, we have looked at games with mixed strategy. So, we have looked at games with pure strategy and we also looked at games with mixed strategy.

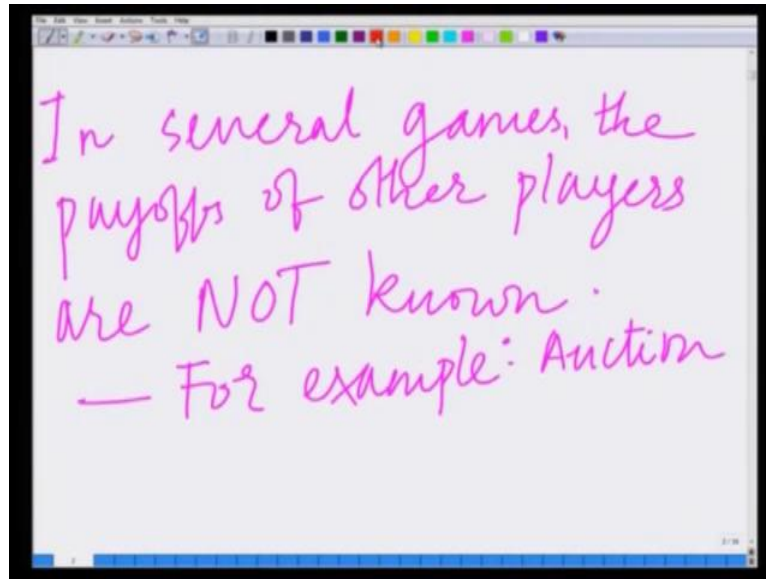
What we are going to start doing today is start trying to look at a different kind of game and what is the motivation behind this. So, far we have assumed in all the games that we have seen, so far we have assumed that each player goes exactly the payoffs of the other player. So, far we have assumed that, the player knows that each player that the game table is known exactly, that is each player knows the payoff of all the other players.

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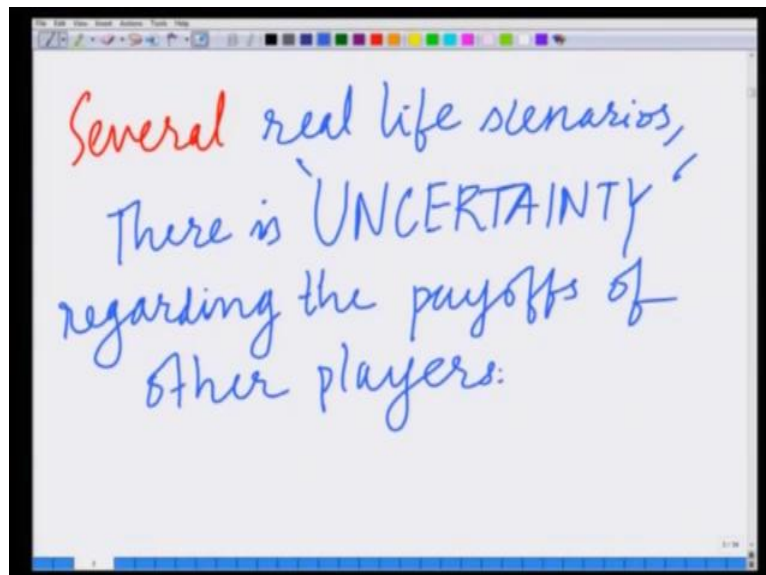
So, so far, we have assumed that each player knows the payoff, knows the exact payoffs of other players. However; this assumption might not also true in several scenarios, for example, in an auction with the several bidders, each bidder might not know the valuation and therefore, the payoffs to the other player.

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So, in several games the payoffs of other players are not known. For example an auction, in an auction, why we each bidder or each player knows his known valuation, his own payoff it might not, he might not know in the payoffs in the valuations of the other players. Therefore, there is uncertainty regarding the payoffs of the other player.

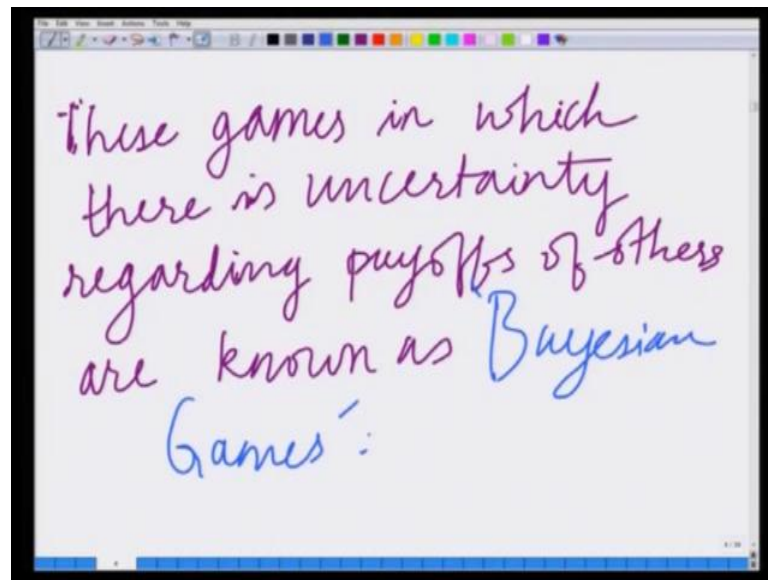
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So, in several scenarios, in several games, in several real life scenarios, in several real life scenarios, there is what is known as uncertainty, there is that is the key word uncertainty regarding the payoffs of the other players. Such a game that is these kinds of

games, in which there is uncertainty regarding the payoffs of the other player that is each player knows his payoffs. But, he is uncertain about the payoffs of the other players or for are known as the easier games at these models, these games are studied under the framework of the easier game theory.

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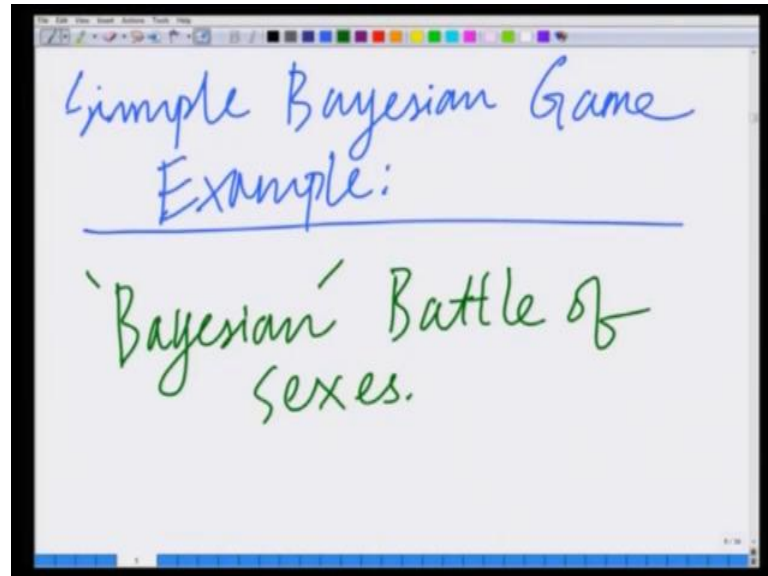
So, such games, so these games in which there is uncertainty, other is such games are in which there is uncertainty regarding the payoffs of others are known as the Bayesian games. So, we are saying that there is uncertain game that is, the player does not exactly know, what are the payoffs of other players that is there might be different kinds of other player that is, there might be different factors, related to the players there might be different types of other players.

And, each type of the other player has a certain payoff for instance, we are going to look at a, later we are going to look at an example of a Bayesian Cournot game, where each for might not know the cost of production of the other firm. There is uncertainty while each for most it is one production cost, it does not know the production cost of its opponent firm. So, there is an uncertainty regarding the cost and therefore, the payoffs of the other.

So, these are broadly the kind of scenarios, which are captured under Bayesian game theory. So, Bayesian game theory basically captures those scenarios or used to model

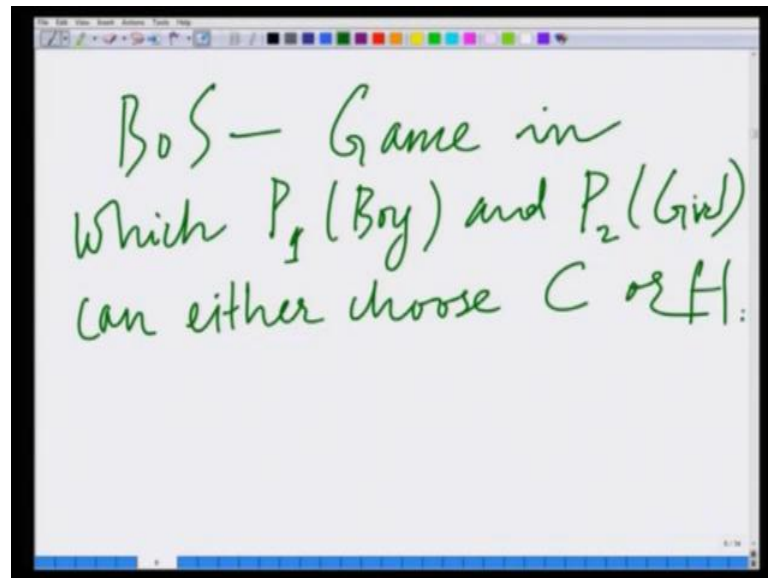
those games, in which there is uncertainty regarding the payoffs of the other player. So, such games are modeled under these frame work of Bayesian game theory.

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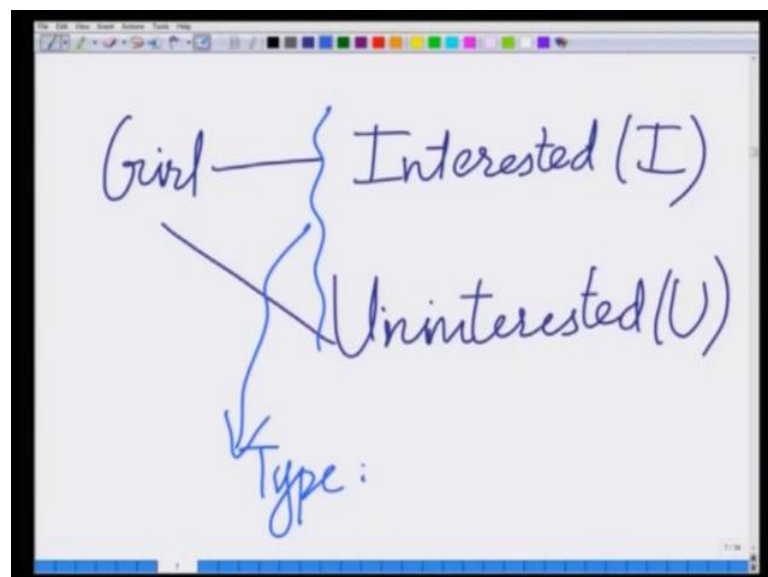
So, let us start by looking at a simple example, a simple Bayesian game example and in the simple Bayesian game example, we are going to start by looking at Bayesian version of the battle of sexes. Remember, we have already looked at the conventional battle of sexes. We are going to now start looking at the Bayesian, the Bayesian battle of sexes and let me remind you, the battle of sexes again is a game between a boy and girl, where the boy and girl can either choose to watch C that is cricket.

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Battle of sexes is a game, in which the player 1 that is the boy and player 2 that is, the girl can either choose C that is to watch the game of cricket or H that is to watch the Harry Potter movie. And, however, we are going to now slightly modify this scenario in, to consider a situation, where the boy is uncertain about the mood of the girl or about the payoffs to the girl.

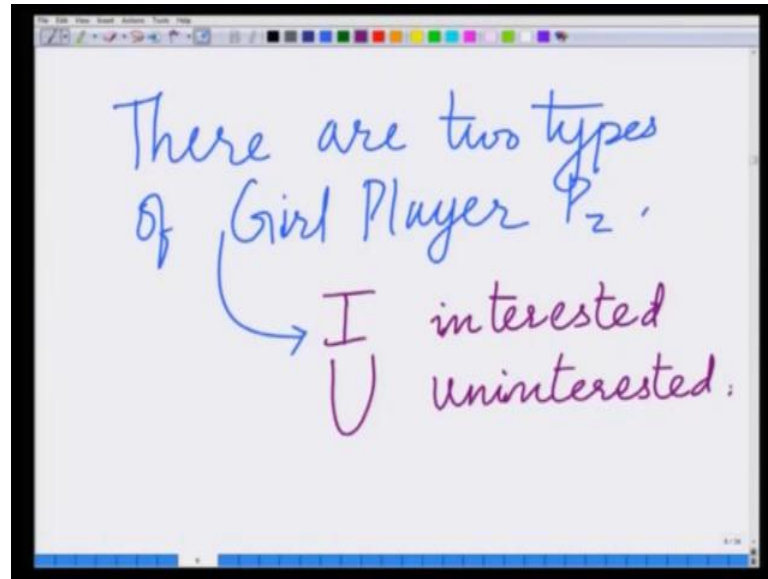
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For, instance the girl can be either be interested or uninterested in watching the game cricket or the movie with the boy. So, we are saying that the girl can have two different

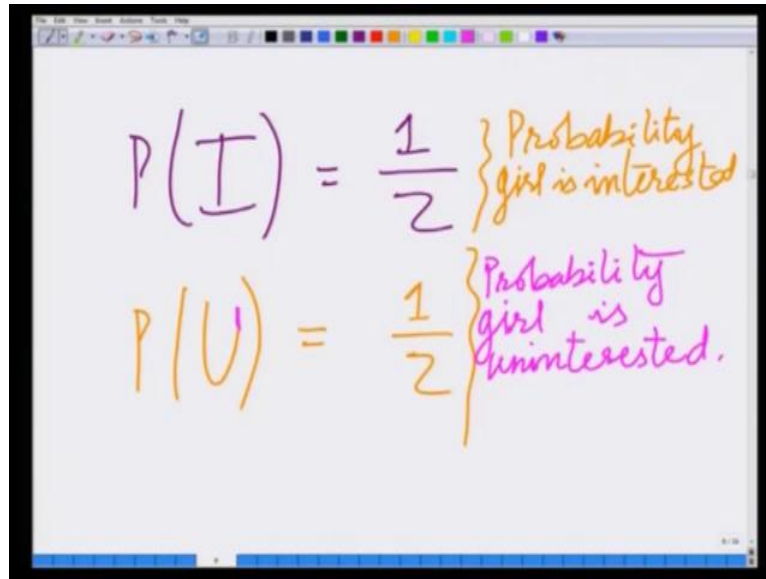
moods that is you can either be interested, which I am denoting by I or she can be uninterested, which we are denoting by U. And, the boy is uncertain about the mood of the girl and this, we are also going to formally know call as the type. These are known as, both these are known as the type. So, we are saying the types, there are two types of player or player 2 or the girl there.

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So, we are formally say, there are two types of girl player or p_2 , there are two types of the girl player represented by I either, that is interested or U uninterested. And, how do we characterize the girl of these two different types? You characterize them by a probability, because the boy is uncertain about that type of the other player that is the girl. So, all he knows is the probability of the different types of the other player that is a probability of the different types of the girl.

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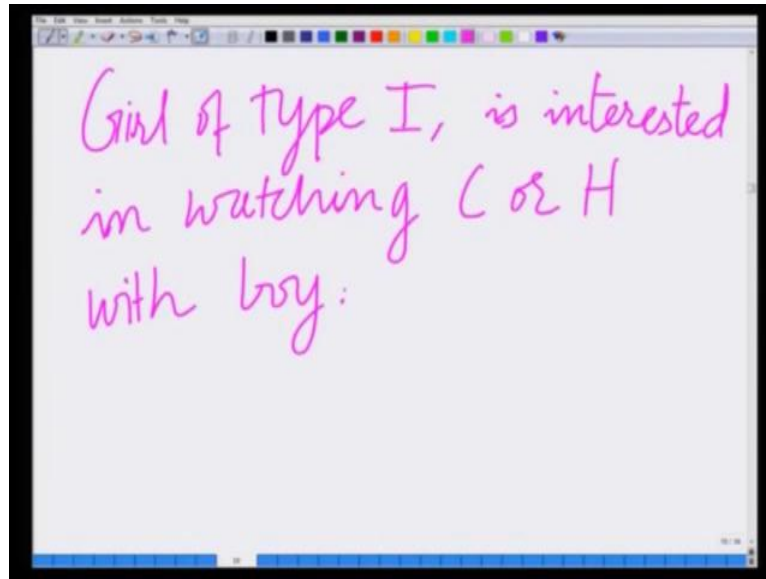


The image shows a whiteboard with two equations written in purple and orange. The first equation is $P(I) = \frac{1}{2}$ with a bracket to the right that says "Probability girl is interested". The second equation is $P(U) = \frac{1}{2}$ with a bracket to the right that says "Probability girl is uninterested".

We can assume or let say it is even that the probability that the girl is interested is equal to half, that is with randomly one half of the type the girl is of time interested and probability half of the time the girl is interested. So, probability I equal to one half, this is basically probability and this is also the probability girl is uninterested, probability that the girl is uninterested. So, there are two types one is probability I that is probability with the girl is actually interested is half and the probability, in which there will all uninterested is half and, each girl of each type has around preferences and around payoffs.

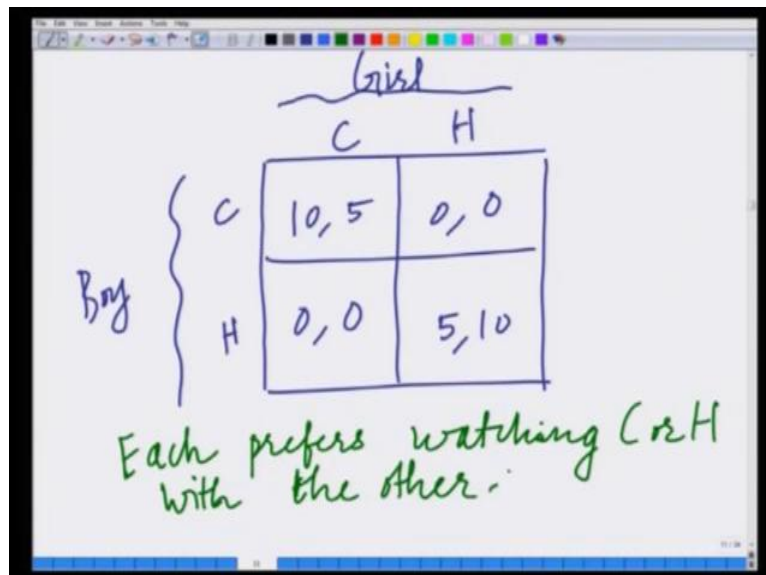
So, next what we have to do is let we have to model the payoffs for this way and by the way, we have also assuming that there is only one type of player 1 or point. In this simple example, we are going to later extended to an example, we have there are multi types of both place. But, to keep this scenario simple at this point in the beginning, we are assuming that there is only one type of player 1 that is boy, why there are two types of the girl that is the girl, can be of type interested or the girl can be of time are uninterested.

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Now, in the girl is interested she is interested girl of type is interested in watching cricket or potter movie with a boy. So, remember or this are conventional battle of sexes game, where their payoff is higher, if they are both that is each prefers watching the movie or the game of merging the game of cricket and that is watching C or H together.

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So, in this scenario the game table that is corresponding to type I of the girl the game table again is given by the standard battle of sexes, where each has an option C or H, C or H, 10 comma 5, 5 comma 10, 0 comma 0. And, this is the column is for the girl player

the row is for the boy player, so each prefers, so in this each prefers watching C or H with the other player. So, this is the conventional battle of sexes this is the game table for the conventional battle of sexes game.

However, when the girl is of type U in the girl is uninterested she does not prefer watching C or H, when the boy prefers watching C or H with the girl, the girl does not prefer watching C or H with the boy.

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		Girl	
		C	H
Boy	C	10, 0	0, 10
	H	0, 5	5, 0

Corresponding to girl of type U.
 girl prefers to watch C or H alone,
 while Boy prefers to watch C or H together.

So, for this scenario the game table can be modeled as follows, the game table can be modeled as C H, C H when they are both watching cricket together the boy gets a payoff of 10, but the girl get payoff of 0. Because, you prefers not to watch C, with the boy therefore, when they are choosing C, C when boy and girl are both choosing C, C payoff the boy is 10, then the payoff the girl is 0, but when they are choosing C H.

Because, the girl is doing thing differently from the boy the girl gets a payoff of 5, but the boy gets a payoff of 0, because the boy is interested in watching H with the girl. Similarly, when there are choosing H C the payoff of the boy is 0, because e should watching C, when the girl is watching H while the girl is watching C, so I am this payoff has, because the board boy is girl is watching H her payoff has to be 10 was she prefers H.

And, when then choosing H comma C their payoff is again 0 comma 5, because the boy is watching H and the girl is watching C. So, they do not think differently, so the payoff to the boy is 0, when the girl prefers to watch something different, then the boy is over, but she is watching cricket which she prefers lefts. So, are payoff is 5 and finally, when they are both watching H comma H, there payoff is given as 5 comma 0, because the boy is watching H with the girl.

So, his payoff is 5 and the girl prefers not to watch anything together with the boy, so, her payoff is 0. So, this is the game table or again these are the payoffs, player 2 the column player is the girl, the row player is the boy and these are the payoffs corresponding to corresponding to girl of type U, that is girl, when girl prefers to watch either C of H C or H alone by the boy prefers to watch C or H together along with the girl. So, here girl prefers to watch C or H alone while boy prefers to watch C or H, C or H together, so these are the game tables.

So, there is one type of boy, but there are two types of girl, girl of type I girl of type U and there is a gif different, what you have to observe here, it is that there is a different game table corresponding to the girl of each type. So, there is uncertainty regarding the move are regarding the type of the way, way to stated formally is that there is a different type, there are different types of player 2 each type of player 2, as a roll or one preferences. And, there is a different game table will the payoffs are different and therefore, the game tables are different corresponding to a different players of the different types.

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Bayesian BoS:

	C	H
C	10,5	0,0
H	0,0	5,10

$P(I) = \frac{1}{2}$

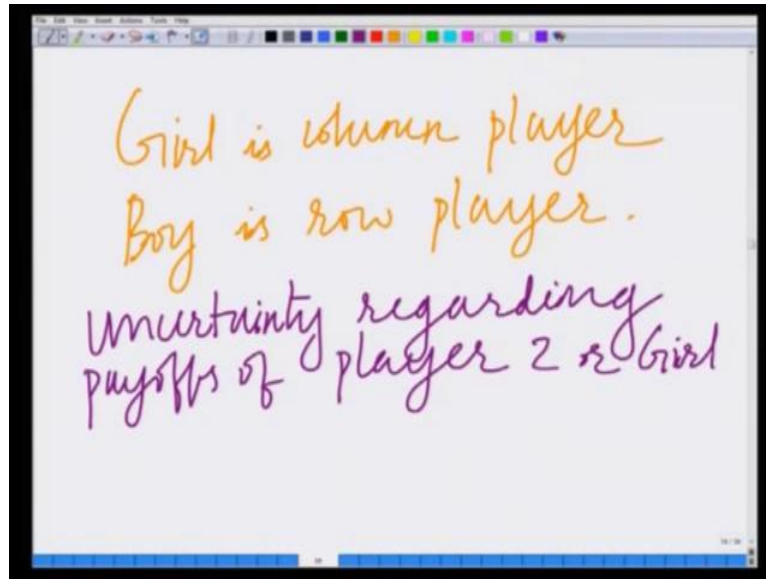
	C	H
C	10,0	0,10
H	0,5	5,0

$P(U) = \frac{1}{2}$

And, now what we are going to do is let us right these pay these tables side by side, so what I am going to do is right for this Bayesian battle of sexes would this Bayesian battle of sexes let us write these game tables side by side. So, what we here is we have the table corresponding to girl of type interested and we have another table corresponding to the girl of type uninterested. So, both her C H C H C H C H or the girl of typing interested the payoffs are the conventional payoffs that is 10 comma 5, 0 comma 0, 0 comma 5, 5 comma 10, to the girl of type uninterested the payoffs are slightly different from the conventional battle of sexes.

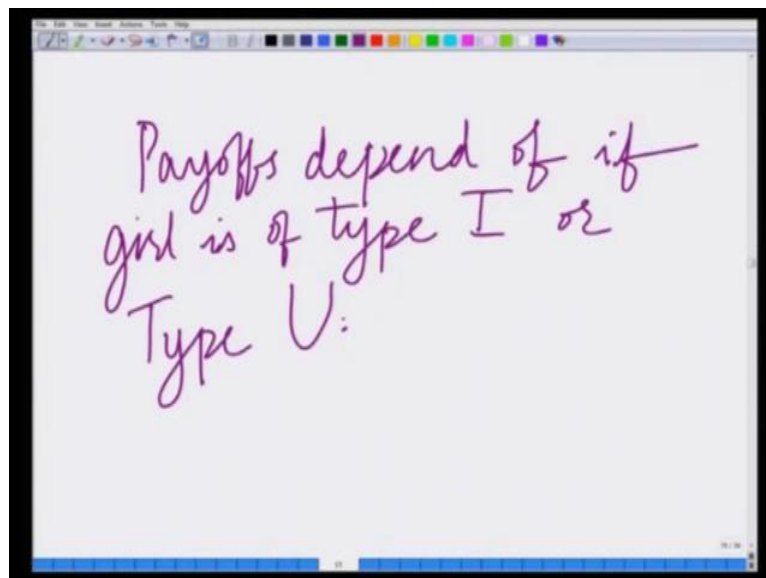
So, 0 comma 10, 0 comma 5, 5 comma 0 and therefore, and also this is for the girl of type interested and the probability, that the girl is type of interested as you already said is half probability that the girl is uninterested, we have already said is half.

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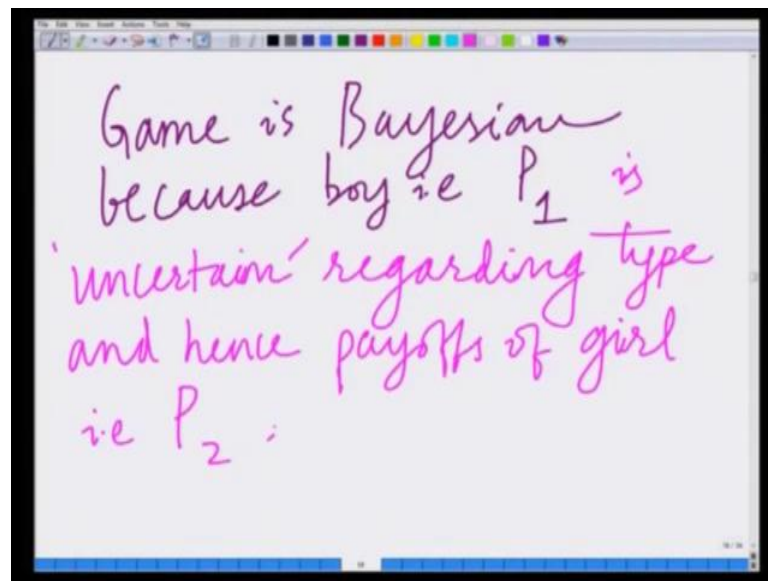
And the girl is the column player and the boy is in the row player girl is a column player and the boy is the row player. And therefore, we have now modeled the Bayesian battle of sexes and also, let me right it in specifically there is uncertainty regarding payoffs of there is uncertainty, why is this Bayesian there is uncertainty regarding payoffs of there is on uncertainty regarding payoffs of player 2 or the girl player. The payoffs depend on the type of the girl player that is payoffs depend on if the player 2 the girl is of type I or girl is of type U that is payoffs.

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If, girl is of type I that is interested or of type U that is uninterested, so the payoffs depend on the type of the girl, girls you are the player 2 girl of different types has different payoffs. And, the boy player 1 does not know the type of the other player, that is why there is uncertainty, that is why this game is Bayesian nature, because there is uncertainty, because the player 1 that is the boy is, is uncertain regarding the payoffs or they have to the type in the other player.

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Game is Bayesian, because boy that is P 1 is uncertain the key word is uncertain regarding type and hence payoffs of girl that is player 2. So, what you saying is the game is Bayesian in nature, because player 1 that is boy is uncertain regarding the payoffs of player 2. He only knows that randomly half of the time girl is of time I, that is interested an half of the time girl, girl is of type U and that is an interested, but it does not know at any given instance, if we playing the game with the girl of type I or the girl of type U and therefore, and there is uncertainty regarding the payoffs of the other player.

So, we are going to now start ((Refer Time: 21:29)), so let us stop this module here and in the next module. We are going to start analyzing this game to compute the payoffs are to simplify the payoffs and then compute the Bayesian Nash equili, because this is the Bayesian game, we have to compute the Bayesian Nash equilibrium of this game, which is something, we are going to start doing in a next module.

Thank you.