Strategy:An Introduction to Game Theory
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Lecture - 31

Hello and welcome to mooc lectures on Strategy, An introduction to Game Theory. In this module we will primarily discuss Stackelberg model.

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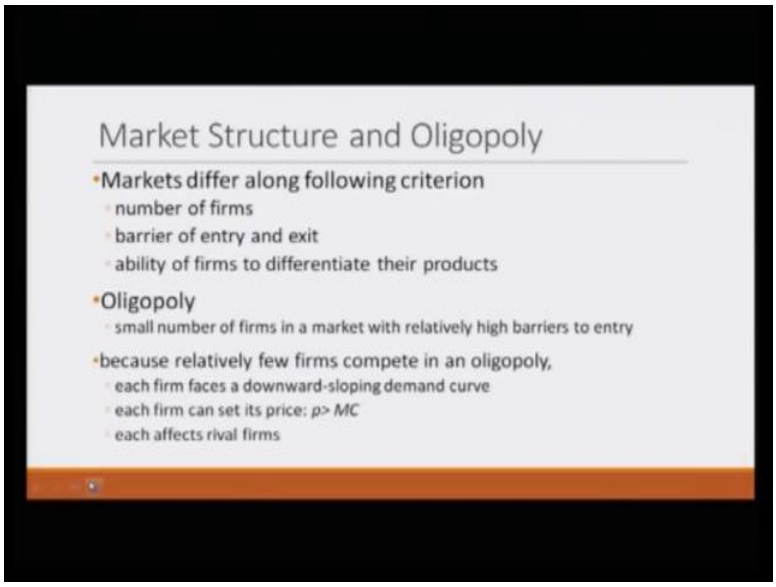

But, before we do so, let us talk about market structure and oligopoly. Markets are typically they are distinguished from other markets using some criteria like the presents of number of firms in the market or how difficulties to enter in the market or leave the market for a particular firm. Like remember, we discussed the entry game. So, there we were not modeling it, but typically when a firm tries to entering the market, it faces several barrier legal, administrative, financial.

So, how difficult it is to entering the market or exit in the market and it also depends on the ability of firms to differentiate their product. Oligopoly is a special kind of market, which is characterized by small number, presents of a small number of firms in the market with relatively high barriers to entry. High barriers to entry ensures that number of firm remains small in the market and because, there are very few firms in the market, which complete against each other.

Each firm faces at downwards sloping curve, what does it mean; that if price of the product is higher, demand for the product would go down, it may becomes 0 also, if someone is offering the same product at the lower price. You might have seen it in the case of better on competition. So, in oligopoly typically firms at it is price above it is marginal cost and most importantly for us, as we are studying the game theory here in oligopoly market, a behavior of a one firm or the action of one firm affects the decision of other firm, because all the firms are typically interested in maximizing their profit.

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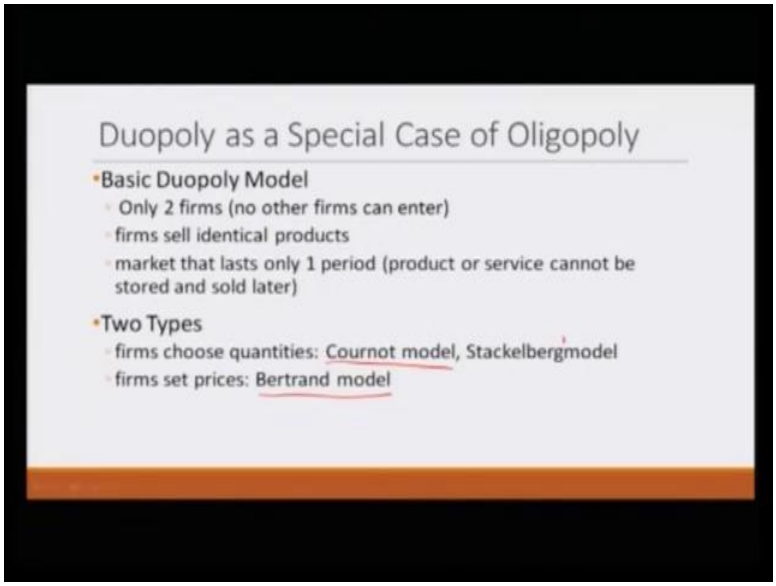

Now, what we are going to do, we are going to talk about duopoly briefly, duopoly is nothing, but a special case of oligopoly in which we have only two firms. The basic model that we have been discussing in this course is that, that there are only two firms and the firms sell identical products. Notice that should not be the case, even when we have Coke and Pepsi, their products are somewhat similar.

But, they are not selling exactly same product, but we want to study Coke and Pepsi, we study typically in the case of duopoly market. And also we assume that market last only one period. The products or services produced by the firms cannot be stored and sold later. There are two types of duopoly market, one firms uses quantities to complete against each other and I believe that you have already studied Cournot modeling that context.

And firms set price to complete against each other and that is better and modulating, that also you studied earlier. If not then, it is very similar to Cournot model and only you have to think about that firms are deciding prices as there is strategy, when they are competing against each other. But, there is another model that we are interested in and that is Stackelberg model.

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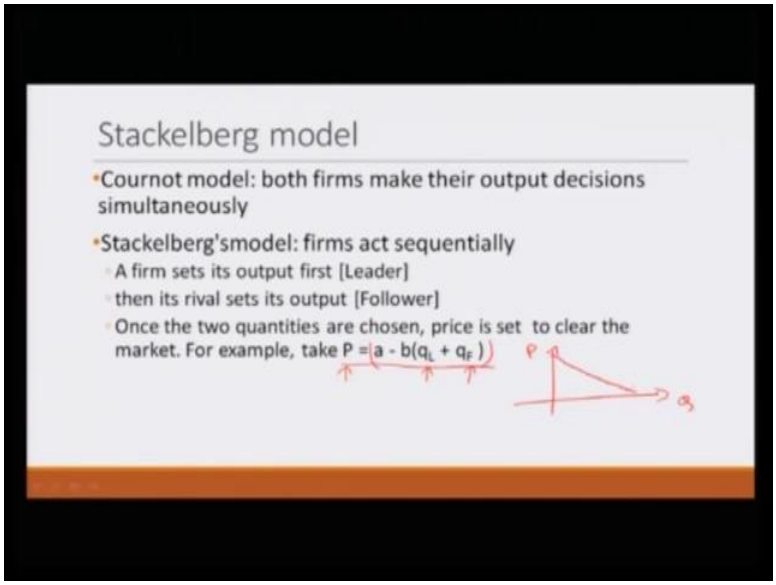

So, in Stackelberg model also firm chooses quantity to complete. What is the difference between Stackelberg model and Cournot model? In Cournot model both firms make their output decisions simultaneously, both of the firms choose Q; that they are going to produce simultaneously. But, in Stackelberg model, firms acts sequentially, there is a firm which we will call leader, it sets it is output first.

Then, the other firm which we can call follower or their rival of the leader, follower is the better word. Follower which observes the output decided by firm 1 and then, sets it is price. How can we justify it in real life? One can say that leader is a bigger firm and follower is a smaller firm, a smaller firm is a nimble, can weight tell with larger firm call leader in this case, decides it is output.

But, there can be some other reasons also, right now our interest is in the mathematics of the problem. So, once the two quantities are chosen price is set to clear the market. We have already discussed in the case of oligopoly market, the both the firms face downwards sloping demand curve and here for just for simplicity, we are taking a case,

we have inverse P is equal to a minus b multiplied by Q L plus Q F, where Q L is the quantity decided by leader and Q F is the quantity decided by follower and this is the price that would be present in the market. So, what we are basically saying that here we have price and here we have total quantity, the demand curve is a downward sloping linear curve.

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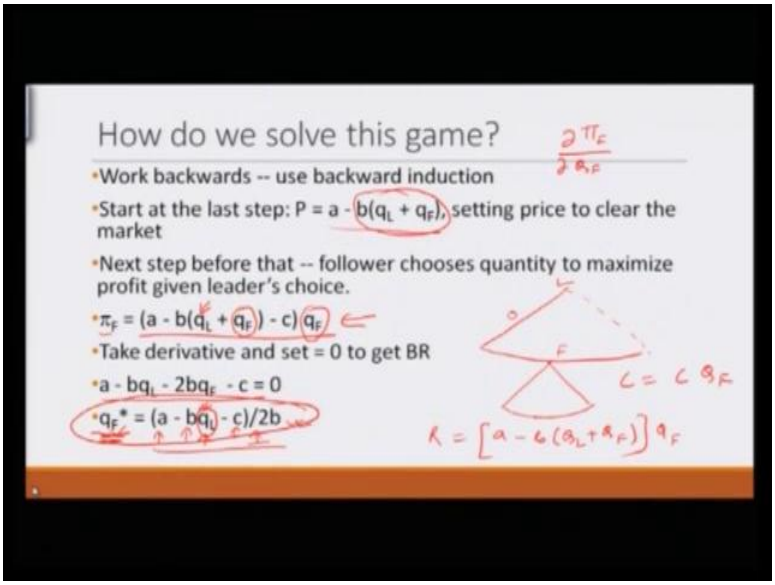

How should we solve such game? Again the techniques that the technique that we have learn is of backward induction, work backward and that is we are going to solve this. Start at the last step, P is equal to a minus q, Q L plus Q F. Now, the thing is, the next step before that follower chooses quantity to maximize given leaders twice. Notice, if I denoted in a game 3, this is a situation like this, all though this can be really very high.

So, we can say it is not a line, but a dotted line just to indicate that it is 0 and this is really increasing in this direction and here is the leader and then, follower decides and here the payoffs gets realized. So, here follower is deciding. What follower does? When follower is deciding about quantity to produce, it would take leaders decision as given. Again, notice that we have two different kinds of sub games in this game tree, one is the whole game and second, sub game in which second type of sub game in which the follower gets to move. Follower does not have any strategic interaction, he just has to decide. So, the problem is simpler.

How would he decide? He is interested in maximizing his payoff. What would be his payoff? Payoff of firm is equal to his, the firms profit and that is equal to total revenue minus total cost. How much will be the total revenue? The total revenue would be again price multiplied by the quantity, it produces. Quantity is it is decision, but price gets determined not only because of the quantity of this, follower is going to produce, but also the price is influenced by the quantity that leader has decided to produce.

So, what do we have? How much is their revenue? Price a minus b, a and b are arbitrary constant, both are positive, so this is what Q F, this is P multiplied by Q F, this would be the revenue for the follower. We assume that the cost is linear, so we can say linear cost means higher the quantity follower is producing, higher would be the cost, it would change linearly. So, let say that q is c multiplied by Q F.

So, this is what is written here, here follower has absolutely no control over Q L, what follower can decide, this Q F follower has to decide and how can be decide, basically he is interested in maximizing this entity. So, simple calculus if you use, if you differentiate pi F; that is the profit of follower with respect to Q F. What do you get? You get this equation, if you set it equal to 0, the follower would be able to figure out how much quantity it has to produce.

And of course, it depends on the parameters a, b and also c. More importantly, it depends how much leader has decided to produce, notice, this is the strategy of follower, because it does not only give followers action in one of the instances. But, in fact in all the eventualities, it does not matter how much quantity firm 1 is willing to produce given that quantity; this is what firm 2 is produce, come to should produce. Also of course, we have to be careful; I am not taking care of it clear, but think about it if firm one produces really high quantity.

In that case, we will have to modify this that price has to be a non negative number, if this becomes larger than a. In that case, we should assume that price is equal to 0 or in that case, the response from follower would be, the best response from follower would be do not produce any think when price is equal to 0. Otherwise, this is the best response from the follower.

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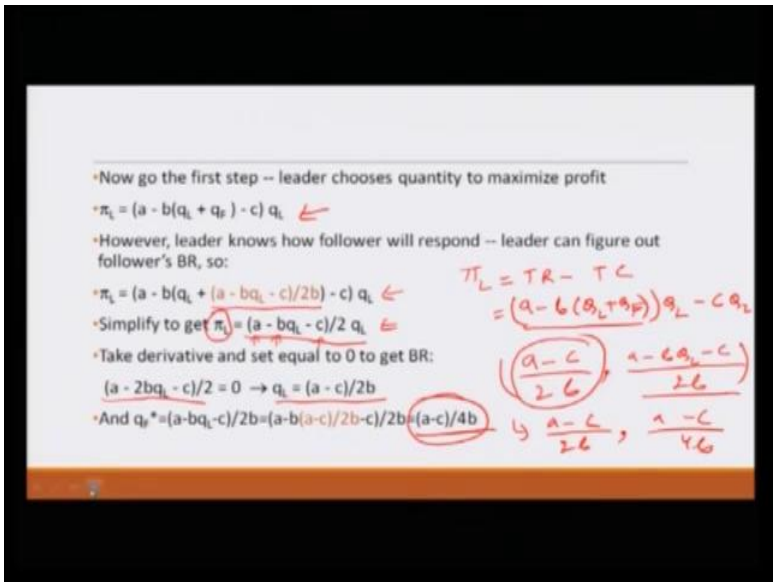

Now, let us go back one step, leader, now leader has to choose the quantity that it would produce to maximize it is profit. Now, what will be happen, again similar equation we should write pi L; that is the profit of leader, it again depends on how much is the total revenue and total cost. And how much is the total revenue, total revenue is equal to price multiplied by quantity. So, price is a minus b Q L plus Q F multiplied by this is price multiplied by quantity, Q L minus total cost.

And how much is the total cost, c multiplied by Q L; that is what is return here. This way is very simple the logic the game is of common knowledge, all the players know that all the players are rational and all the player know that all other players are rational. So, leader knows that how follower will be spot and that we have already obtained using backward induction.

If Q L is a particular given Q L, this is the way follower would respond. So, why because that is the rational for player 2 to do or that is the rational for follower to do and this is known to leader. So, leader will take it into a count. So, this Q F or this Q F will be replaced by the formulation that we had on the earlier page and that we have put in the different color pink. So, the equation gets modified. If you simplify the equation this what you will obtain.

Now, if you look at this equation this equation does not have Q F, this a is known, b is known, c is known. So, if only variable present in this equation is Q L and what player

the leader has to do, leader same is to maximize poly of L and how can be maximized, again we can take derivative with respect to Q L and he will obtained the first order condition and he will obtained the Q L.

So, this is the derivative you get, if you solve it Q L is comes out to be a minus c divided by 2 b. Once, we know the Q L, let us go back to the equation for Q F is star, once you know the Q L, you plug the value of Q L here and you will obtain the Q F star. And this is what you will get a minus c 4 b divided by 4 b. So, what would be the equilibrium here, equilibrium would be that player 1 would produce a minus c to b. And player 2 will use the strategy a minus b Q L minus c divided by a minus b Q L minus c divided by 2 b. This would be the strategy used by player 2.

Notice, here I am writing how much player 1 would produce, because that is player 1s strategy, but here I am not writing a minus c by 4 b, this is the outcome not the strategy. So, when we are writing the equilibrium, we have to be careful that we write only strategies for the player and not the outcome. What would be the outcome that outcome would be that player 1 or the leader would produce a minus c divided by 2 b and player 2 would produce a minus c divided by 4 b.

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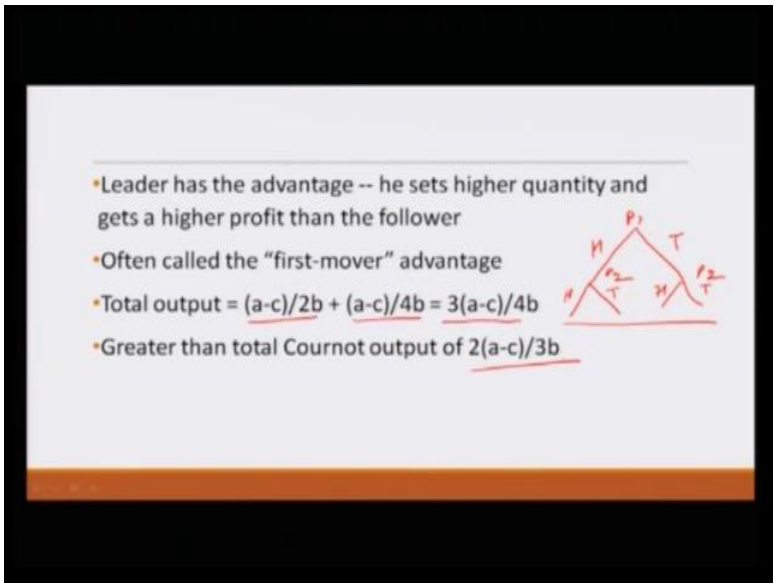

What happens, the leader has the advantage, why if you compare these two leader produces more price would be same for both that firm. So, leader would have higher profit. So, leader has the advantage, he sets higher quantity and gets a higher profit than

the follower. This is often called the first mover head one advantage, but do not you think; that in all sequential game, the players would the first mover would have advantage. There are many situations in which the later mover would have advantage.

Like for example, very simple example that we take off is of matching penny and in this the players are playing this game sequentially. So, player 2 knows, what player 1 has decided H or T. So, H or T and here you have player 1 and after observing player 2 have to move here of course, player 2 would win, because player 2 knows what player 1 has done.

So, in this case, you do not have first mover advantage; although this is not an interesting game, but I just wanted to tell you, so that you do not thing that first mover will always happen advantage. How much is the total output, the total output is the output produce by the leader and output produced by the follower and it comes out to be 3 multiplied by a minus c divided by 4 b. If you use the same formulation and if you obtain the Cournot output, this is what you would get. So, in Stackelberg model the output is higher.

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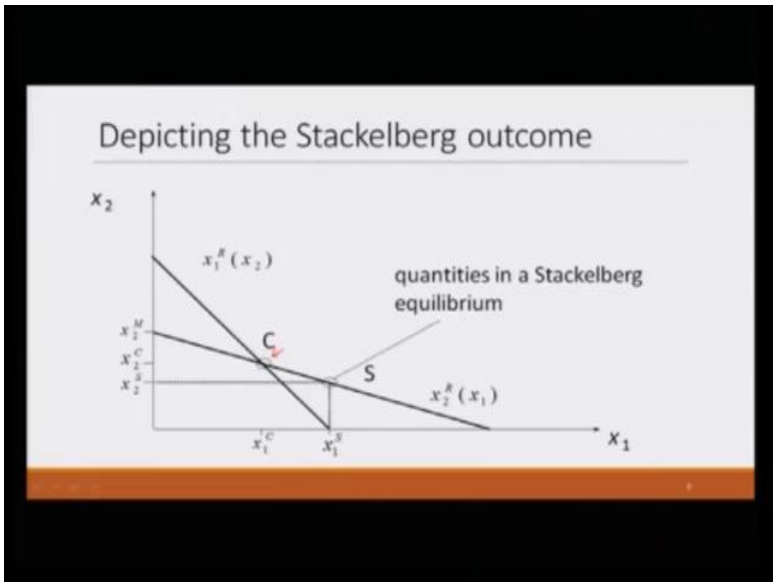

Why this is a graph that gives you the Cournot outcome as well as Stackelberg outcome. What happens in the Cournot outcome, in Cournot outcome both the players decide simultaneously. So, the two equations that we solved one after another, what you have to keep in mind that you should solve those two equations simultaneously. In that case, you

have two equations in Q L and Q F. So, you should be able to solve. But, here you are solving one after another.

So, player 1 can pick and then, player 2 has to take this into account. So, in the Stackelberg outcome as I have already told you; that the leader has advantage, but overall consumers are happier. Because, you have more output in the market and because of more output you have lower price.

Thank you.