Strategy:An Introduction to Game Theory
Prof. Vimal Kumar
Department of Humanities and Social Sciences,
Indian Institute of Technology, Kanpur

Lecture – 30

Welcome to next lecture on Strategy, An Introduction to Game Theory.

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In this module, I am going to talk about the ultimatum game, so what do you mean by the ultimatum game.

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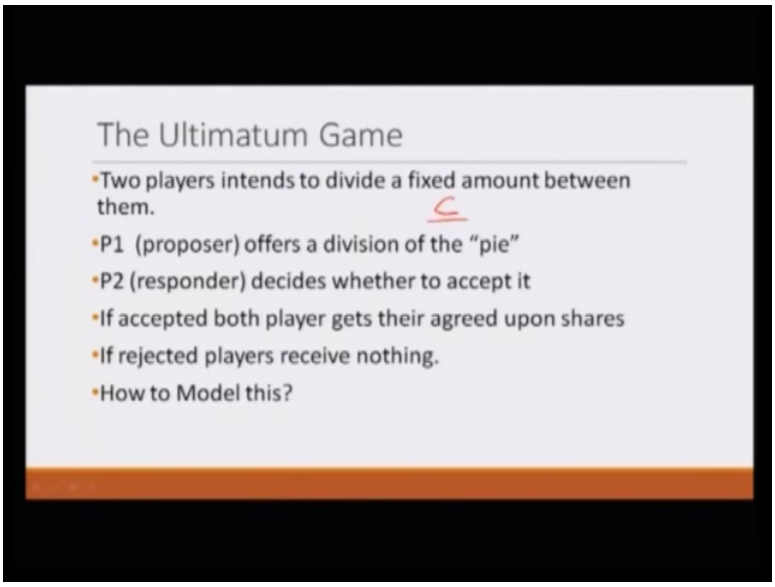

Basically, two players intend to divide a fixed amount between them, let us say size of

that fixed amount is, the size of that fixed amount is c. And, this is what they intend to divide and there is a very particular way of dividing it, may not finds here, but that is not the aim, the aim is very simple to study an extensive from game and what happens. And, it on, we will realize the usefulness of ultimatum game, because it will form the basic foundation of our bargaining chapter.

So, what happens there is a player P 1, who we call professor basically, he offers a particular way of dividing the pie. Player 2 this responding, player 2 decides, whether to accept the offer or reject it, if player 2 accepts the offer whatever player 1 has proposed gets implemented and if player 2 rejects, then both of them do not get anything. How should we model this? One way to model it is simply, that use the extensive form game and for simplicity we will say, we will allow that only finite number of offers are possible.

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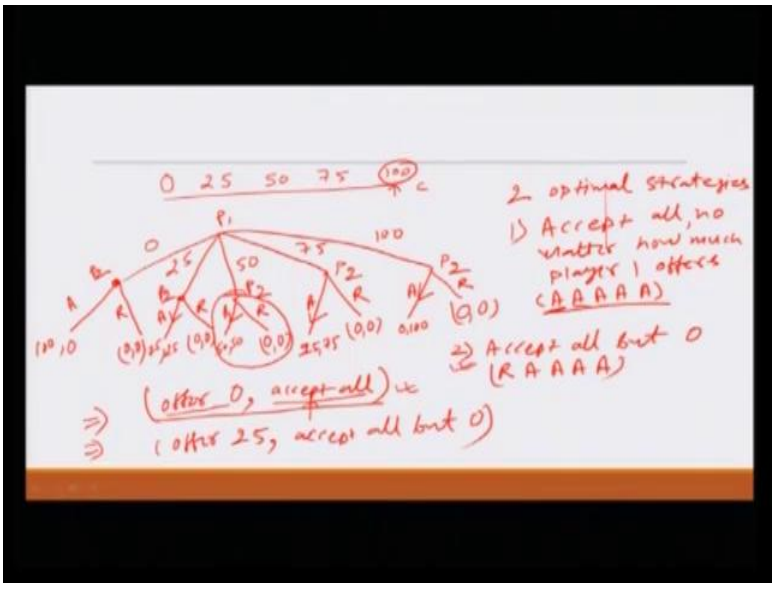

So, player 1 can offer either 0 or let us say 25 or 50 or 75 or100, let us say the earlier I was talking about c, this c is equal to 100 and this is, what player 1 and player 2 intent to divide. So, player 1 can offer one of these five offers to player 2 and then, player 2 can either accept or reject, the theta word is that player 2 gets to observe the offer of player 1 before he makes a decision to accept the offer or reject the offer. So, clearly the case is of extensive form game, it is the sequential game which should be modeled using extensive form, so we will be able to draw the game tree.

So, here is player 1 offering 0, offering 25, offering 50, offering 75 and offering 100,

here is player 1. And, after player 1 moves player 2 gets to move, in each of these cases and always player 2 have one of these two actions, accept reject, accept reject, accept reject and so on. Whenever player 2 rejects then no matter, how much player 1 is offering both of them get 0. And if player 1 player 2, player 2 accepts, then player 1 offers gets accepted and player 1 gets here, in this case player 1 offers 0 to player 2 accepts, player 2 gets 0 and all remaining that is 100 that goes to player 1, similarly here player 1 is offering 25.

So, player 2 gets if player 2 accepts, if player 2 gets 25 and remaining is 75 and so on, we can write everywhere the payoffs. Some of you may be thinking, why would player 1 offer 100, he would not that we will see an equilibrium he would not, but game tree has description of the all different possibilities. So, we have to draw all the possibilities, what happens we can of course, use backward induction technique to solve this game. What can happen here, if player 2 accepts and offer often rate player 2 gets 100 and if player 2 rejects then player 2 gets 0, 100 is more than 0, so player 2 will accept.

Player 2 will accept, similarly the offer of 75, offer of 50, offer of 25. How about the offer of 0? If player 1 offers player 2, 0 player 2 can either accept or reject like an any other scenario, but here accepting or rejecting both would get, player 2 0. So, player 2 is basically in different between accepting or rejecting. So, we can say that there are two optimal strategies for player 2, I just want to remain you, that whenever we are talking about strategy of player 2, the strategy is to describe an action for player 2 at all the information set, where he has a play in the game.

So, player 2 has a play in this, at this node, at this node, at this node, this node and this node 1 2 3 4 5 at five places. So, player 2 has strategies to describe five different actions for different information node. So, at all these four information nodes it is clear, that player 2 should accept and here player 2 is in different between accepting or rejecting.

So, two optimal strategies player 2 has, why two, one we can say accept all, no matter how much, no matter how much player 1 offers. Or in other word we can say accept, accept, accept, accept, accept the first accept corresponds to this information node, second accept to this information node and so on. So, these are the two different ways, we can write the same strategy.

Second optimal is strategy for player 2 is and, we will check again that, whether it is optimally strategy or not. If player 2 has adopted this is strategy no matter, what player 1

does, player 2 do as well as in comparison to any other strategy or in some time better, so this that is the reason it is the optimal strategy of player 2. The second is accept all, but 0 in other word, what we are saying reject accept, accept, accept, accept this is the strategy for player 2.

Both, the strategy would give player 2, the same amount depending on, how much player 1 has offer and this is the best possible response from player 2. What should player 1 do? Let us look at, what should player 1 do. Given that player 2 he is playing accept all, what should he offer, if he offers 0, he gets 1, if he offers to 25 he get 75. So, since player 2 is going to accept all the offers, he is better off by offering only 0, so player 1 will offer 0 and player 2 will accept all. This is sub game perfect Nash equilibrium, why because, let us see can player 2 in the whole game, can player 2 do better by defeating.

Let us say, player 1 is offering 0 by defeating, can he do better, let us say here he accepts instead of accepting he rejects, then also his payoff is 0, so player 2 has no incentive to defeat, so this is clearly a Nash equilibrium. But, this is strategy profile is not only Nash equilibrium in the whole game, but this is strategy profile also induces the optimal behavior for both the players in all the sub games. Like this sub game will never reached, but what this is strategy for player 2 is saying accepts, so player 2 accepts, so player accepting is better than rejecting.

So, this is strategy profile gives Nash equilibrium not only in the whole game, but also in all sub games. Similarly, we have one another sub game perfect equilibrium and that is, let us say player 2, player 1 believes that player 2 is using accept all, but 0. In that case if player 1 offers 0, player 2 is going to reject and player 1 would get 0. And, if player 1 offers 25 player 2 is going to accept and then, player 1 would get 75.

Similarly, if player 1 offers 50, player 2 is going to accept and player 1 will get 50, so given that his better off by offering 25. So, this is another optimal strategy accept all, but 0, these are the two optimally strategy, what happens either player 1 offers nothing or player 1 offers their minimum, just to make player 2 accept. But, this was the special case in which we limited the option of player 1, player 1 could offer only 0, 25, 50, 75 or 100.

What if we allow player 1 to offer any amount, then what happens? So, we will do the same game, but here player 1 will be allowed to offer any amount, first of all now player 1 has infinite number of action.

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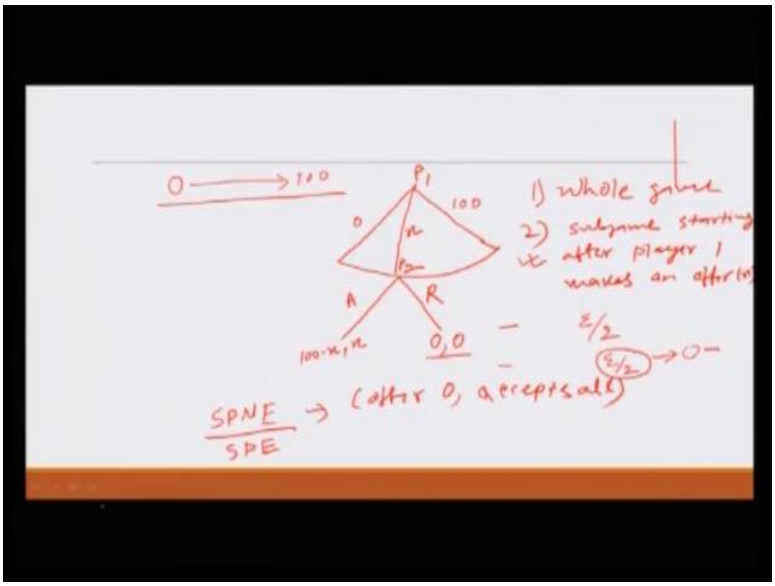

So, even between 0 to 1 we have infinite number of real numbers. How should we represent it in a game tree? So, here is a way we will represent, so player 1 moves and we can say we will use this to indicate, that there is a continuum of action available to player 1 starting from 0 to 100. And, after each of course, what do we have, we have infinite number of branches, we cannot draw all those branches, because we have really large number of branches.

And, every time player 2, once player 1 moves player 2 has one of these two action, accept or reject. And, if player 1 and player 2 accepts, then let us say this corresponds you offer x, player 2 gets x and player 1 gets 100 minus x and if player 2 rejects, both of them get 0, 0. Now, how can we obtain the sub game perfect Nash equilibrium of this game or in other word, what would be the outcome of this game notice, we have just learn, what happens, when player 1 has finite number of strategies and we obtain two cases one, in which player 1 offers nothing and second, in which player 1 offers pear minimum.

So, similarly here of course, we have infinite number of sub games, but we cannot in numerate all the sub games. But, we can say that there are two types of sub game, one is the whole game and second sub game starting, after player 1 makes and offer, let say x, so these are the two types of sub game. So, in the second type of sub game, because through backward induction we are starting, so in we have to start from the end.

So, in this type of sub game player 2 again have two optimal strategy, we look at the all

the sub games, one is in this type of sub game, now earlier, we saw that player 2 has two types of optimal strategy one, accept all second accept all but 0. But, we will see here that player 2 has only one optimal strategy that is accept all, you may be thinking, why not accept all, but something.

So, what we have here if we look at the second type of sub games player 2, would have only one optimal strategy. Earlier, we had seen that player 2 has two different optimal strategy and those two, where that either accept all are accept all, but 0, here what happens. If, we say accept all, but something less than if epsilon then, what happens, even, if player 1 has is offering epsilon by 2 player 2 is batter or by accepting this offer y.

Because, it player 2 accepts player 2 gets epsilon by 2 and if player 2 rejects player 2 gets 0, so epsilon 2 is clearly greater than 0. So, the optimal strategy only one optimal strategy just for player 2 and that is accept all the offer, given that player 2 accepts all the offer, player 1 has very nice choice, and nice choice for player 1 that he offers 0 as player 2 is going to accept all the offers.

So, we have only one S P any sub game perfect Nash equilibrium and some books it is write also written a S P E sub game perfect equilibrium and this optimal strategies that offer 0 and player 2 offer 0 and player 2 accepts all.. I, want to talk about, what happens in the real life, when two players face a situation similar to ultimatum game, but before we do that, let us play a simple game, in which player 2 does not have any strategic twice.

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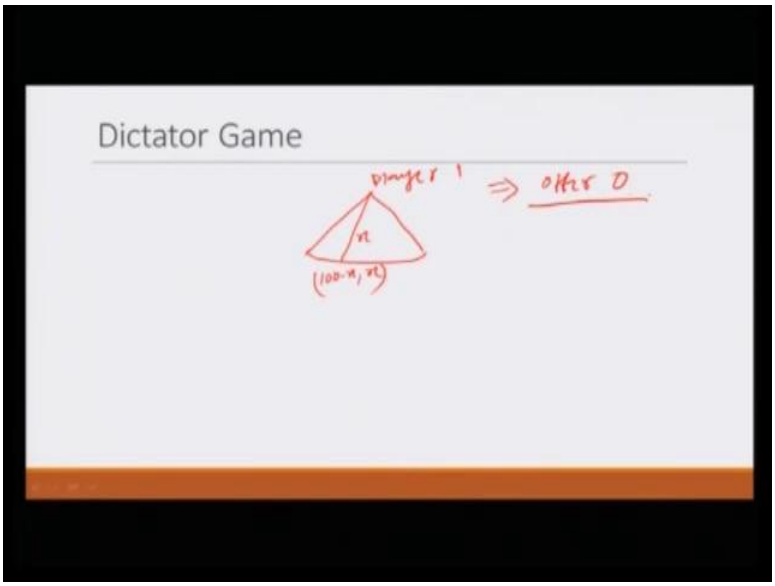

In dictator game is a variant of ultimatum game, what happens in this dictator game, is that player 1 offers and that offers get implemented player 2 does not have any choice. So, in true sense it is not a strategic interaction, but it has some significant, we will see, so player 1 and player 1 offers and it gets implemented.

So, it player 1 offers, player 1 gets 100 x and player 2 gets x the reason that it is called dictatorial game is that player 2 does not matter in this game player 1 decides the outcome. So, of course, here player 1 what is the optimal for player 1 to offer 0, because player 1 is interested in maximizing only his pay off.

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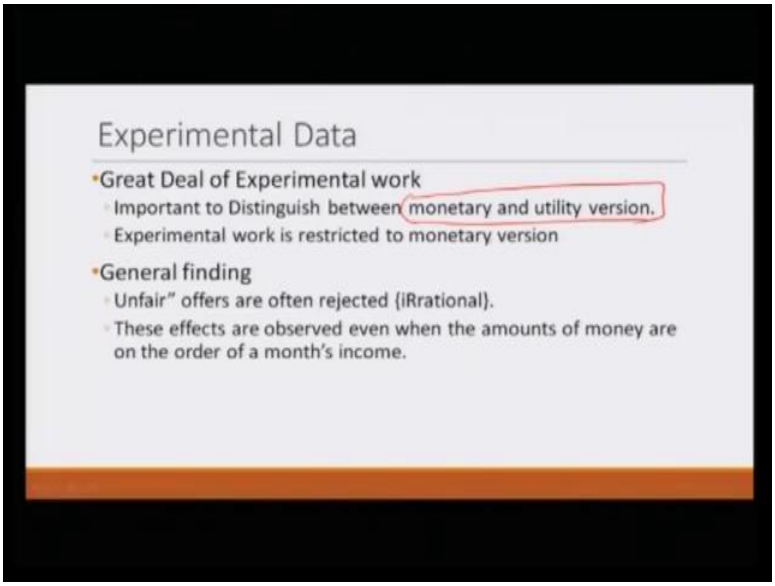

Now, let us look at the experimental data, because there is a great you know demand for verifying or the checking the result that is suggested by game theory. So, many economist ans psychologist have conducted experiments to see whether the prediction from theory comes out to be true in real life or not. But, we have to be causes that it is important in experimental work to distinguish between monetary and you to monetary and utility pay off.

For example, you know when we are talking about gain of 1 rupee, let us say a person who has 10 corers of rupees, if we gets one more his level increase in his level of happiness would not be same as the person, who has absolutely 0 and he gets 1 rupee. So, we have to understand see the people the players are interested in maximizing their utility not the monetary game. But, the problem is that happiness is difficult to major satisfaction is difficult to major, so experimental work is typically restricted to monetary

version.

So, when ultimatum game all direct dictator game is conducted in the lab setting, what do we see, that unfair offers and what do we by, what do we mean by unfair offer, let say in the dictator game player 1 offers a really low amount to player 2 or low amount such as 0, what do we observe in real life that unfair offers, such offers are often rejected. And, these effects are observed even the amount of money are on the order of months income. So, even when high amount of money is involve, because experimental wants to play with monetary and utility version. So, even if when the offer really high amount, then also the realize that unfair offers are often rejected.

So, in that action the production of game theory particularly in the case of ultimatum game and dictator game are not observed in the lab setting. Perhaps players are playing some did they have different notion in their mind, they also clear about with, whom they are playing this game, but experimental sub also taken care of this, they have pitted people anonymously against each other to control for this.

But, it sense our notion of fairness is very strong in all the players mind and they in the playing slightly different game, then the ultimatum game, because if they are playing the ultimatum game, then they should not be offering any amount to the other player.

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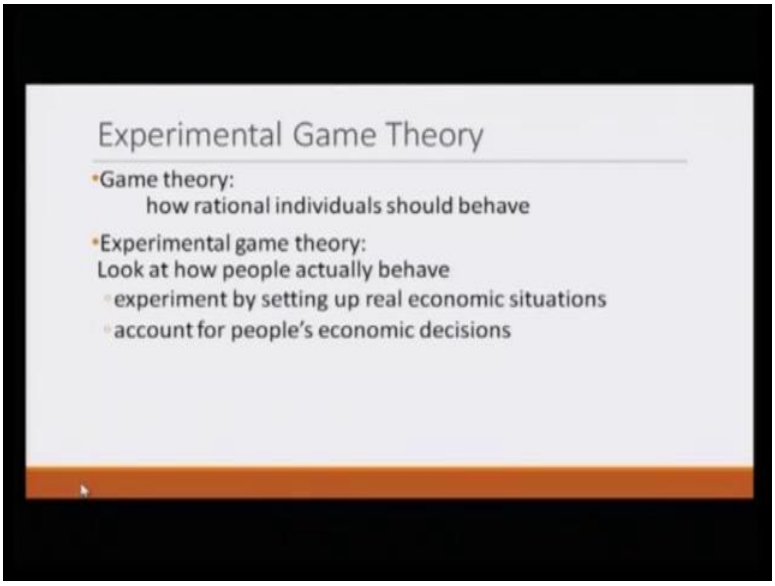

So, we will we should think about something call experimental game theory allow this course is not about experimental game theory, but I just want to introduce this notion game theory is about how rational individual should behave in strategic interaction. And,

experimental game theory looks at how people behave actually in real life, so this is the way game theory is progressing, we have there are this can be consider as one of the short comings of game theory, we will discuss such things in later modules also.

Thank you.