Strategy: An Introduction to Game Theory
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Lecture – 29

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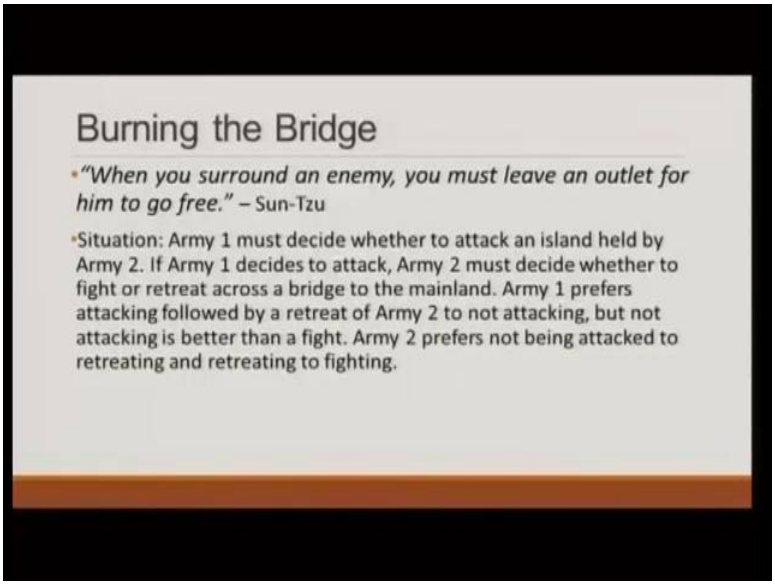

Welcome to mooc lectures on "Strategy an Introduction to Game Theory". In this module, I am going to talk about an application of extensive form game. This comes from "Art of War"; this book is by Sun-Tzu. What he had written in his book? Of course, he had written many things but one of the things that he had written that, "When you surround an enemy, you must leave an outlet for him to go free". So, in other words do not push your enemy in the corner. And, that we have similar saying in our country also.

So, I am checking a situation. That is again coming from Martin Oswald's book, "An introduction to game theory". The situation is that there is army one. And, army one must decide whether to attack an island held by army two. So, now it is clear to us that we have players such as army one and army two. And, army one can decide attack or not to attack army two. So if army one decides to attack, army two must decide whether to fight or retreat across a bridge to the mainland. So, since army two has two options; fight or retreat.

Now, let us look at their payoffs. Army one prefers attacking followed by a retreat of army two to not attacking. But, not attacking is better than fight. So, it gives the ranking.

There are different possibilities. If army one decides not to attack, the game ends. The strategic interaction ends there. Player two does not engage. In one way of modeling this; later on, we will try to model it slightly differently.

If army one; the best possible outcome for army one is to that army one attacks and army two retreats. The second best outcome is not attacking. And, the worst outcome for army one is to that army one attacks and army two starts fighting. For army two; army two prefers not being attacked to retreating and retreating to fighting. So, since fighting is the worst option for both the players.

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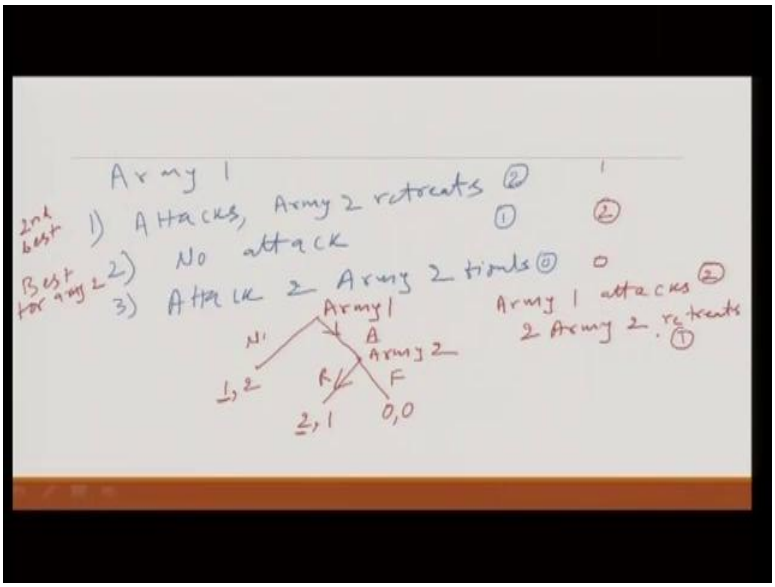

So, we can say; we can again write for army one. You can write for army one. Best is that army one attacks and army two retreats. And, second is no attack. And, the worst outcome is that attack and army two fights. We can give here. Again, these are the ranks. So, we can give here two for army one. If this happens, army one gets one; and if this happens, army gets zero. What is the best outcome for army two? Let me put it in the other color. Army two; the best outcome is not attack. This is best for army two.

So, we can say here we can give army two, two; if this happens. Second best is that; second best for army two; so, here army two gets one. And, in this case army two also gets zero. Of course, we can model this game as an extensive form game. So, how can we describe? First, army one is deciding not to attack or attack. If army one deciding not to attack, game ends. Army one gets one and army two gets two. First payoff is for

player one and second payoff is for player two. And as army one moves first, so by convention we can call army one as player one. And if army one attacks, then army two gets to move. And, army two can do one of these two things; retreat or fight. If army two retreats, this is the best outcome for player one. So, that is army one. Army one gets two and army two gets one and here both of them get zero, zero. This is a very simple game. We can use backward induction to solve. There is no imperfect information, so we do not have any problem in using backward induction. So army two here; at this point army two can see retreating gives one, while fighting gives zero.

So, of course army two would prefer to retreat rather than fight. So, army one sees this. If army one decides not to attack, then army one gets one and if army two decides to attack, then army one gets two. Of course, two is more than one. So, army one will decide to attack. So, what would be the outcome? Army one attacks and army two retreats. And, payoff for army one would be two and payoff for army two would be one.

Let us consider a slightly different scenario in which what army two can do. That army two; let us say if there is some mechanism through which army two forgo the option of retreating. Why I am talking about it? We will see later. So, let us say that army two cannot retreat. So, if we look at this payoff what we see? So, army two cannot retreat. It means army two has only one option is left; that is to attack, that is to fight. So, army two fights back when army one attacks.

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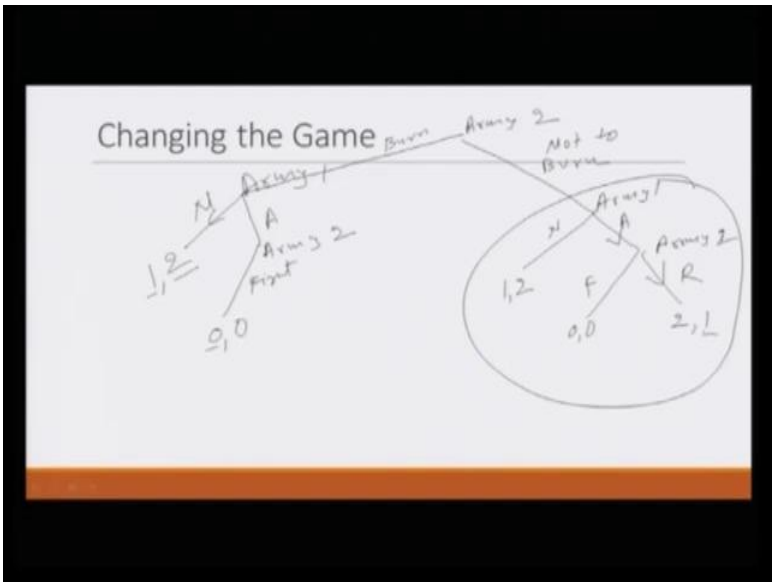

So, game tree looks like this. Army one; no attack. Payoff would be one for army one and two for army two. And if army two attacks, then army two can fight because retreating is not possible. And, here payoff would be; that is zero, zero. In this case, what happens? Army two has no strategic option left. As soon as army two gets attacked, army two fights back. So, army one knows this. So, let us compare the only decision that has to be made by army one.

So, army one not attacking would get army one as one and attacking would get zero. So, which one is the better option? Not attacking. So, game will move in this direction. So, what is happening here? That, we have built a super structure to the original game. So, what we had here? Let us say even before army one moves, we can say army two can decide whether to burn the bridge or not to burn the bridge and not to burn; here, not to burn. Again, then in this case army two would have both the option of retreating and fighting; fighting, retreating, not attacking, attacking. Here one, two; zero, zero; two, one.

So, of course original game was this sub game that we were discussing. And, what happens here? Army one attacks and army two retreats. And in the modified game, army one does not attack. So, now army two can see which option to pick; burn or not to burn. It is that burning will give army two a two payoff and not burning will give army two a payoff of one. So, two is greater than one. So, of course army two would burn. And, this is changing the game. When we model the scenario, burning the bridge was not the option described. But, that is in a way something beyond game theory. But, of course game theory helps in figuring out that what if a new action is introduced, how would it change the game and how would it change the outcome.

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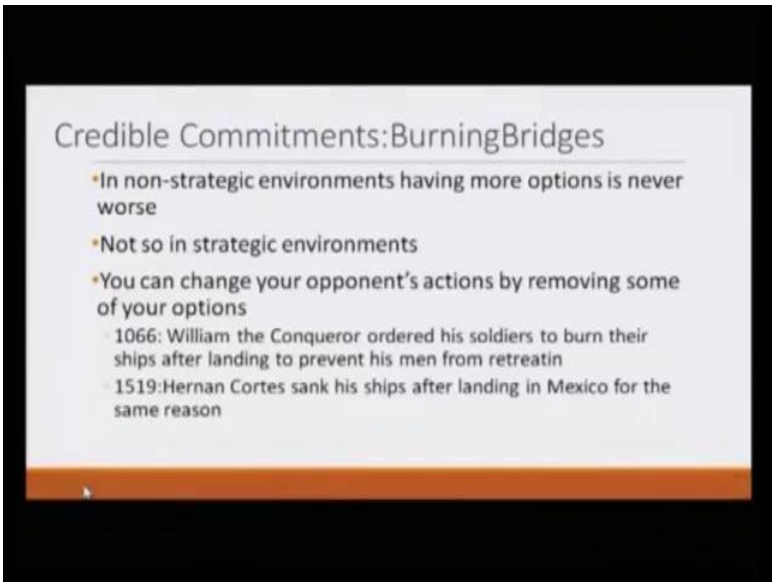

So, what we have here? By burning the bridge, army two makes a credible commitment that if I am attacked I am going to fight. And, fighting is the worst outcome for both army one and army two. Having known this, army one would not attack.

Typically, what happens in non-strategic environment? Having more options is never worse but, not so in strategic environment. We already have seen two examples. One right just now; another when we were talking about Braess paradox. Having one more option need everyone worse off. And, of course you can change opponent's optimal action by removing some of your choices.

And, history has some of the examples like, in 1066: William, the conqueror ordered his soldiers to burn their ships after landing to prevent his men from retreating. And, similarly Herman Cortes sank his ships after landing in Mexico for the same reason. So, of course game theory will not teach you how to change the game. But, of course game theory will give you the idea that if you are changing the game, how the outcome will change. Ok.

Thank you.